

Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2008 Paper
2 hours

The total number of marks available is 100.

You must write down all the stages in your working.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given correct to three significant figures where appropriate.

Section A

1. A driver of a car, initially moving at 30 ms^{-1} , applies the brakes so that the car comes to rest with constant deceleration in 10 seconds.
 - (a) Find the value of the deceleration. (2)
 - (b) Find the distance travelled in this time. (2)
2. The points A and B have coordinates $(0, 8)$ and $(6, 0)$ respectively.
 - (a) Find the equation of the line AB . (3)
 - (b) Find the equation of the line perpendicular to AB through its midpoint. (4)
3. Find the points of intersection of the line (5)

$$y = 5x + 13$$

with the circle

$$x^2 + y^2 = 13.$$

4. Glass marbles are produced in two colours, red and green, in the proportion $7 : 3$ respectively. From a large stock of the marbles, 5 are taken at random.

Find the probability that

- (a) all 5 are red, (2)
 - (b) exactly 3 are red. (3)
5. (a) Use calculus to find the stationary points on the curve (6)

$$y = x^3 - 3x + 1,$$

identifying which is a maximum and which is a minimum.

(b) Sketch the curve.

(1)

6. A speedboat accelerates from rest so that t seconds after starting its velocity, in ms^{-1} , is given by the formula

$$v = 0.36t^2 - 0.024t^3.$$

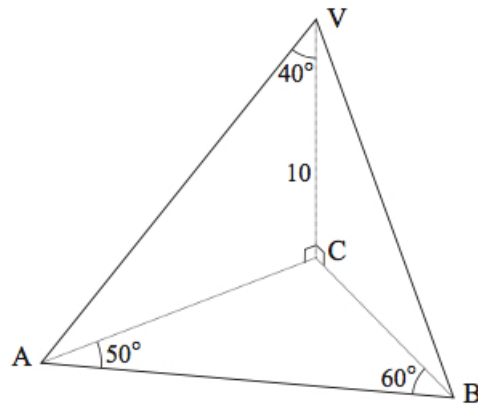
(a) Find the acceleration at time t .

(3)

(b) Find the distance travelled in the first 10 seconds.

(4)

7. A pyramid stands on a horizontal triangular base, ABC , as shown in the figure.



The angles CAB and ABC are 50° and 60° respectively.

The vertex, V , is directly above C with $VC = 10$ m.

The angle which the edge VA makes with the vertical is 40° .

(a) Calculate AC .

(2)

(b) Hence calculate AB .

(4)

8. It is required to solve the equation

$$2 \cos^2 x = 5 \sin x - 1.$$

(a) Show that this equation may be written as

(2)

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

(b) Hence solve the equation

(4)

$$2 \cos^2 x = 5 \sin x - 1$$

for values of x in the range $0^\circ \leq x \leq 360^\circ$.

9. The cubic equation

(5)

$$x^3 + ax^2 + bx - 26 = 0$$

has 3 positive, distinct, integer roots.

Find the values of a and b .

Section B

10. Simon and Gavin each drive a distance of 140 km along a motorway, both at constant speed. Simon drives at 5 km per hour faster than Gavin.

Let Gavin's speed be v km per hour.

- (a) Write down expressions in terms of v for the times, in hours, taken by Gavin and Simon. (2)

Simon completes the journey in 15 minutes less than Gavin.

- (b) Explain why (5)

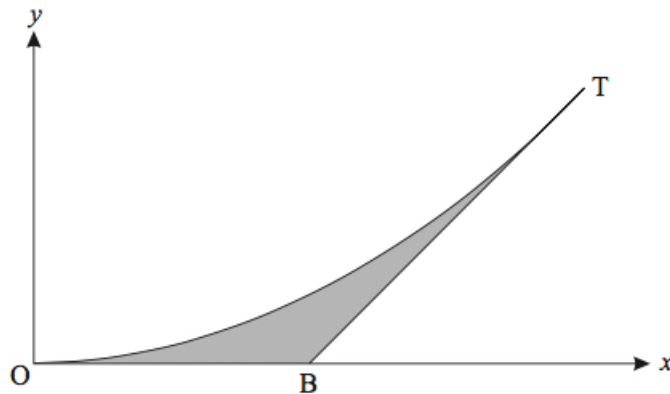
$$\frac{140}{v} - \frac{140}{v+5} = \frac{1}{4}$$

and show that this equation reduces to the equation

$$v^2 + 5v - 2800 = 0.$$

- (c) Solve this equation to find v and hence find the times taken by Simon and Gavin. Give your answers correct to the nearest minute. (5)

11. The side of a fairground slide is in the shaded shape as shown in the figure below. Units are metres.



The curve has equation $y = \lambda x^2$.

T has coordinates $(4, 2)$.

The line BT is a tangent to the curve at T .

It meets the x -axis at the point B .

- (a) Find the value of λ . (1)

- (b) Find the equation of the tangent BT and hence find the coordinates of the point B . (6)

(c) Find the area of the shaded portion of the graph. (5)

12. A furniture manufacturer produces tables and chairs.

In each week the following constraints apply.

- There are 24 workers, each working for 40 hours (i.e., there are 960 worker-hours available).
- There is a maximum of £1 800 available for the purchase of materials.
- Each table requires £30 worth of materials and 12 worker-hours.
- Each chair requires £10 worth of materials and 6 worker-hours.
- It is necessary to make at least 3 times as many chairs as tables.

Let x be the number of tables produced each week and y be the number of chairs produced each week.

(a) Show that the worker-hour constraint reduces to the inequality $2x + y \leq 160$. (2)

(b) Find the inequality relating to the cost of materials constraint and the inequality relating to the numbers of tables and chairs. (3)

(c) Plot these three inequalities on a graph, using 1 cm to represent 10 tables on the x -axis and 1 cm to represent 20 chairs on the y -axis. Indicate the region for which these inequalities hold. You should shade the region which is **not** required. (4)

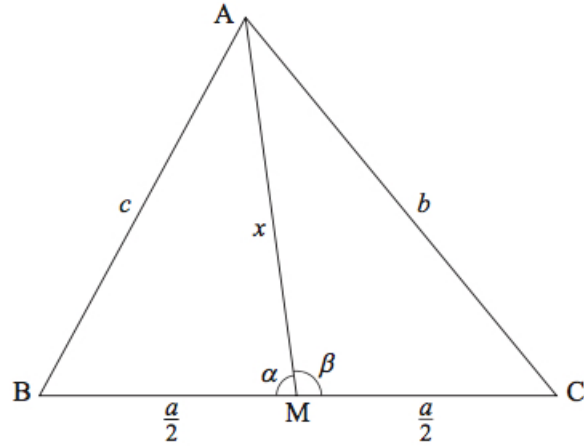
When finished, each table is sold for a profit of £20 and each chair is sold for a profit of £5.

(d) The manufacturer wishes to maximise the profit. Explain why the objective function is given by (1)

$$P = 20x + 5y.$$

(e) Find the number of tables and chairs that should be made in order to maximise the profit. (2)

13. In the triangle shown below, M is the midpoint of BC .



- (a) Explain why (2)

$$\cos \alpha = -\cos \beta.$$

- (b) Using the cosine rule in the triangle BMA , show that (2)

$$\cos \alpha = \frac{4x^2 + a^2 - 4c^2}{4ax}.$$

- (c) Find a similar expression for $\cos \beta$. (1)

- (d) Using the results in parts (a), (b), and c), show that (5)

$$4x^2 + a^2 = 2(c^2 + b^2).$$

A triangular lawn has sides 46 m, 29 m and 27 m.

- (e) Find the distance from the midpoint of the longest side to the opposite corner. (2)