Dr Oliver Mathematics
Applied Mathematics: Mechanics or Statistics
Section B
2014 Paper
1 hour

The total number of marks available is 32.
You must write down all the stages in your working.

1. Find the gradient of the tangent to the curve (4)

$$y = 2x\sqrt{x-1}$$

at the point where $x = 10$.

2. Matrices are given as

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 4 \\ k & 0 & -1 \\ 5 & 3 & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 & -10 & 2 \\ -3 & 9 & 0 \\ 0 & -2 & 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 & 2 & -6 \\ 1 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix}.$$

(a) Calculate $\mathbf{A} + \mathbf{B}$. (1)

(b) Find the determinant of $\mathbf{A}$. (2)

(c) Calculate $\mathbf{BC}$. (1)

(d) Describe the relationship between $\mathbf{B}$ and $\mathbf{C}$. (2)

3. Find the exact value of (5)

$$\int_0^{2\pi} x \sin 3x \, \mathrm{d}x.$$

4. Evaluate (2)

$$\sum_{r=1}^{80} 3r^2.$$

5. (a) Write down and simplify the binomial expansion of (3)

$$(\mathrm{e}^x + 2)^4.$$

(b) Hence obtain (2)

$$\int (\mathrm{e}^x + 2)^4 \, \mathrm{d}x.$$

6. A flu-like virus starts to spread through the 20 000 inhabitants of Dumbarton. The situation can be modelled by the differential equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(20\,000 - N)}{10\,000},$$

where N is the number of people infected after t days and $0 < N < 20\,000$.

(a) How many people are infected when the infection is spreading most rapidly? (1)

(b) Express (5)

$$\frac{10\,000}{N(20\,000 - N)}$$

in partial fractions and show that

$$\ln\left(\frac{N}{20\,000 - N}\right) = 2t + c,$$

for some constant c.

Initially there were 100 people infected.

(c) Show that (4)

$$N = \frac{20\,000\,\mathrm{e}^{2t}}{199 + \mathrm{e}^{2t}}.$$