Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2011 November Paper 2 Variant 1: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Solve the equation (3)

$$|4x - 5| = 21.$$

Solution

$\underline{4x - 5 = 21}$:

$$\begin{aligned} 4x - 5 = 21 &\Rightarrow 4x = 26 \\ &\Rightarrow x = 6\tfrac{1}{2}. \end{aligned}$$

$\underline{-(4x - 5) = 21}$:

$$\begin{aligned} -(4x - 5) = 21 &\Rightarrow -4x + 5 = 21 \\ &\Rightarrow -4x = 16 \\ &\Rightarrow x = -4. \end{aligned}$$

Hence, the solutions are

$$\underline{\underline{x = -4 \text{ or } x = 6\tfrac{1}{2}}}.$$

2. Given that the straight line (4)

$$y = 3x + c$$

is a tangent to the curve

$$y = x^2 + 9x + k,$$

express k in terms of c.

Solution

Well,

$$x^2 + 9x + k = 3x + c \Rightarrow x^2 + 6x + (k - c) = 0$$

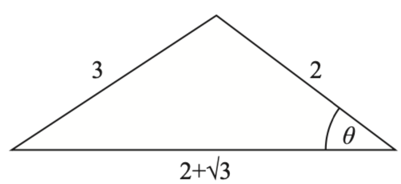
and

$$\begin{aligned} b^2 - 4ac = 0 &\Rightarrow 6^2 - 4(1)(k - c) = 0 \\ &\Rightarrow 4(k - c) = 36 \\ &\Rightarrow k - c = 9 \\ &\Rightarrow \underline{\underline{k = c + 9}}. \end{aligned}$$

3. Without using a calculator, find the value of $\cos\theta$, giving your answer in the form (5)

$$\frac{a + b\sqrt{3}}{c},$$

where a, b, and c are integers.



Solution

Cosine rule:

$$(2 + \sqrt{3})^2 + 2^2 - 2 \times (2 + \sqrt{3}) \times 2 \times \cos\theta = 3^2$$

$\times$	2	$+\sqrt{3}$
2	4	$+2\sqrt{3}$
$+\sqrt{3}$	$+2\sqrt{3}$	$+3$

$$\begin{aligned}
&\Rightarrow \quad (7+4\sqrt{3}) + 4 - 4(2+\sqrt{3})\cos\theta = 9 \\
&\Rightarrow \quad (7+4\sqrt{3}) - 4(2+\sqrt{3})\cos\theta = 5 \\
&\Rightarrow \quad -4(2+\sqrt{3})\cos\theta = -2-4\sqrt{3} \\
&\Rightarrow \quad \cos\theta = \frac{2+4\sqrt{3}}{4(2+\sqrt{3})} \\
&\Rightarrow \quad \cos\theta = \frac{2+4\sqrt{3}}{4(2+\sqrt{3})} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}
\end{aligned}$$

$\times$	2	$+4\sqrt{3}$
2	4	$+8\sqrt{3}$
$-\sqrt{3}$	$-2\sqrt{3}$	-12

$\times$	2	$+\sqrt{3}$
2	4	$+2\sqrt{3}$
$-\sqrt{3}$	$-2\sqrt{3}$	-3

$$\begin{aligned}
&\Rightarrow \quad \cos\theta = \frac{-8+6\sqrt{3}}{4(4-3)} \\
&\Rightarrow \quad \cos\theta = \frac{-8+6\sqrt{3}}{4} \\
&\Rightarrow \quad \cos\theta = \frac{-4+3\sqrt{3}}{2}
\end{aligned}$$

hence, $\underline{\underline{a=-4}}$, $\underline{\underline{b=3}}$, and $\underline{\underline{c=2}}$.

4. (a) Given that (2)

$$y = \frac{1}{x^2+3},$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{kx}{(x^2+3)^2},$$

where k is a constant to be found.

Solution

Well,

$$\begin{aligned} y = \frac{1}{x^2+3} &\Rightarrow y = (x^2+3)^{-1} \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -(x^2+3)^{-2} \times 2x \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x}{(x^2+3)^2}}}; \end{aligned}$$

hence, $\underline{\underline{k = -2}}$.

(b) Hence find (3)

$$\int \frac{6x}{(x^2+3)^2}\,\mathrm{d}x$$

and evaluate

$$\int_1^3 \frac{6x}{(x^2+3)^2}\,\mathrm{d}x.$$

Solution

Now,

$$\begin{aligned} \int \frac{6x}{(x^2+3)^2}\,\mathrm{d}x &= -3\int \left(-\frac{2x}{(x^2+3)^2}\right)\,\mathrm{d}x \\ &= \underline{\underline{-\frac{3}{x^2+3} + c}} \end{aligned}$$

and

$$\begin{aligned} \int_1^3 \frac{6x}{(x^2+3)^2}\,\mathrm{d}x &= \left[-\frac{3}{x^2+3} + c\right]_{x=1}^{3} \\ &= \left(-\tfrac{1}{4} + c\right) - \left(-\tfrac{3}{4} + c\right) \\ &= \underline{\underline{\tfrac{1}{2}}}. \end{aligned}$$

5. (a) The functions f and g are defined, for $x \in \mathbb{R}$, by (2)

$$\mathrm{f}: x \mapsto 2x + 3,$$
$$\mathrm{g}: x \mapsto x^2 - 1.$$

Find $\mathrm{f\,g}(4)$.

Solution
Well,

$$\begin{aligned}\mathrm{f\,g}(4) &= \mathrm{f}(\mathrm{g}(4)) \\ &= \mathrm{f}(15) \\ &= \underline{\underline{33}}.\end{aligned}$$

(b) The functions h and k are defined, for $x > 0$, by

$$\mathrm{h}: x \mapsto x + 4,$$
$$\mathrm{k}: x \mapsto \sqrt{x}.$$

Express each of the following in terms of h and k.

(i) $x \mapsto \sqrt{x+4}$, (1)

Solution
$\underline{\underline{\mathrm{k\,h}(x)}}$.

(ii) $x \mapsto x + 8$, (1)

Solution
$\underline{\underline{\mathrm{h}^2(x)}}$.

(iii) $x \mapsto x^2 - 4$. (2)

Solution
Well,

$$y = x + 4 \Rightarrow y - 4 = x$$

and

$$\mathrm{h}^{-1}(x) = x - 4.$$

Now,

$$y = \sqrt{x} \Rightarrow y^2 = x$$

and

$$\mathrm{k}^{-1}(x) = x^2.$$

Hence, $\underline{\underline{\mathrm{h}^{-1}\,\mathrm{k}^{-1}(x)}}$.

6. **Solutions to this question by accurate drawing will not be accepted.** (6)

The points $A(1,4)$, $B(3,8)$, $C(13,13)$, and D are the vertices of a trapezium in which AB is parallel to DC and angle BAD is 90°.

Find the coordinates of D.

Solution

Well,

$$\begin{aligned} m_{AB} &= \frac{8-4}{3-1} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

and so $m_{AD} = -\frac{1}{2}$. Now, the equation of AD is

$$y - 4 = -\tfrac{1}{2}(x-1) \quad (1).$$

Next, as AB is parallel to DC, $m_{CD} = 2$ and the equation of CD is

$$y - 13 = 2(x-13) \quad (2).$$

Do (1) − (2):

$$\begin{aligned} 9 = -\tfrac{1}{2}(x-1) - 2(x-13) &\Rightarrow 9 = -\tfrac{1}{2}x + \tfrac{1}{2} - 2x + 26 \\ &\Rightarrow -\tfrac{35}{2} = -\tfrac{5}{2}x \\ &\Rightarrow x = 7 \end{aligned}$$

insert into (1):

$$\begin{aligned} &\Rightarrow y - 4 = -\tfrac{1}{2}(7-1) \\ &\Rightarrow y - 4 = -3 \\ &\Rightarrow y = 1. \end{aligned}$$

[Check in (2): $2(7-13) = -12 = 1 - 13$ ✓]
Hence, the coordinates of $\underline{\underline{D(7,1)}}$.

7. (a) Given that (3)

$$\tan x = p,$$

find an expression, in terms of p, for $\operatorname{cosec}^2 x$.

Solution

Now,

$$\begin{aligned}\operatorname{cosec}^2 x &= 1 + \cot^2 x \\ &= 1 + \frac{1}{\tan^2 x} \\ &= \underline{\underline{1 + \frac{1}{p^2}}}.\end{aligned}$$

(b) Prove that (4)

$$(1 + \sec\theta)(1 - \cos\theta) \equiv \sin\theta \tan\theta.$$

Solution

Well,

$\times$	1	$+\sec\theta$
1	1	$+\sec\theta$
$-\cos\theta$	$-\cos\theta$	-1

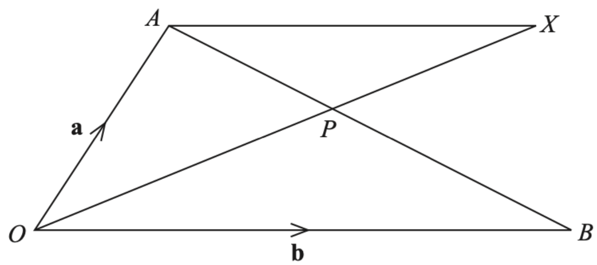
and so

$$\begin{aligned}(1 + \sec\theta)(1 - \cos\theta) &\equiv \sec\theta - \cos\theta \\ &\equiv \frac{1}{\cos\theta} - \cos\theta \\ &\equiv \frac{1 - \cos^2\theta}{\cos\theta} \\ &\equiv \frac{\sin^2\theta}{\cos\theta} \\ &\equiv \sin\theta \cdot \frac{\sin\theta}{\cos\theta} \\ &\equiv \underline{\underline{\sin\theta \tan\theta}},\end{aligned}$$

as required.

8. In the diagram,

- $\overrightarrow{OA} = \mathbf{a}$,
- $\overrightarrow{OB} = \mathbf{b}$, and
- $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$.



(a) Given that $\overrightarrow{OX} = \mu\overrightarrow{OP}$, where μ is a constant, express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$, and $\mathbf{b}$. (3)

Solution

Well,

$$\begin{aligned}\overrightarrow{AP} &= \tfrac{2}{5}\overrightarrow{AB} \\ &= \tfrac{2}{5}(\overrightarrow{AO} + \overrightarrow{OB}) \\ &= \tfrac{2}{5}(-\overrightarrow{OA} + \overrightarrow{OB}) \\ &= \tfrac{2}{5}(-\mathbf{a} + \mathbf{b})\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{OX} &= \mu\overrightarrow{OP} \\ &= \mu(\overrightarrow{OA} + \overrightarrow{AP}) \\ &= \mu[\mathbf{a} + \tfrac{2}{5}(-\mathbf{a} + \mathbf{b})] \\ &= \mu(\mathbf{a} - \tfrac{2}{5}\mathbf{a} + \tfrac{2}{5}\mathbf{b}) \\ &= \underline{\underline{\tfrac{3}{5}\mu\mathbf{a} + \tfrac{2}{5}\mu\mathbf{b}}}.\end{aligned}$$

(b) Given also that $\overrightarrow{AX} = \lambda\overrightarrow{OB}$, where λ is a constant, use a vector method to find the value of μ and of λ. (5)

Solution

Well,

$$\begin{aligned}\overrightarrow{OX} &= \overrightarrow{OA} + \overrightarrow{AX} \\ &= \overrightarrow{OA} + \lambda\overrightarrow{OB} \\ &= \mathbf{a} + \lambda\mathbf{b}.\end{aligned}$$

As the result in part (a) equals the the result in part (b):

$$\begin{aligned}\mathbf{a} : \text{so } \tfrac{3}{5}\mu &= 1 \\ \Rightarrow \underline{\underline{\mu = \tfrac{5}{3}}}\end{aligned}$$

$$\begin{aligned}\mathbf{b} : \text{so } \tfrac{2}{5}\mu &= \lambda \\ \Rightarrow \underline{\underline{\lambda = \tfrac{2}{3}}}.\end{aligned}$$

9. The table shows experimental values of two variables x and y.

x	1	2	3	4	5
y	3.40	2.92	2.93	3.10	3.34

It is known that x and y are related by the equation

$$y = \frac{a}{\sqrt{x}} + bx,$$

where a and b are constants.

(a) Complete the following table. (1)

$x\sqrt{x}$	
$y\sqrt{x}$	

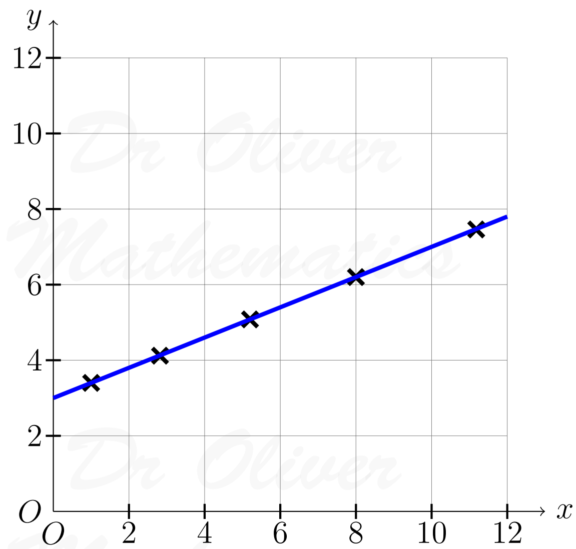
Solution

All answers are to 2 decimal places:

$x\sqrt{x}$	1	2.83	5.20	8	11.18
$y\sqrt{x}$	3.40	4.13	5.07	6.20	7.47

(b) Plot $y\sqrt{x}$ against $x\sqrt{x}$ and draw a straight line graph. (2)

Solution



(c) Use your graph to estimate the value of a and of b. (3)

Solution

The line of best fit goes through $(0, 3)$ and $(12, 7.8)$:

$$\begin{aligned} m &= \frac{7.8 - 3}{12 - 0} \\ &= 0.4 \end{aligned}$$

and the equation is

$$\begin{aligned} y\sqrt{x} - 3 = 0.4(x\sqrt{x} - 0) &\Rightarrow y\sqrt{x} - 3 = 0.4x\sqrt{x} + 3 \\ &\Rightarrow \underline{\underline{y = \frac{3}{\sqrt{x}} + 0.4x}}; \end{aligned}$$

hence, $\underline{\underline{a = 3}}$ and $\underline{\underline{b = 0.4}}$.

(d) Estimate the value of y when x is 1.5. (1)

Solution

Well,

$$x = 1.5 \Rightarrow y = 3.049\,489\,743 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{y = 3.05 \text{ (2 dp)}}}.$$

10. It is given that

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}.$$

(a) Find $2\mathbf{A} - \mathbf{B}$. (2)

Solution

Well,

$$\begin{aligned} 2\mathbf{A} - \mathbf{B} &= 2\begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix} - \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 6 & 4 \\ 2 & -10 \end{pmatrix} - \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} 5 & 0 \\ 4 & -13 \end{pmatrix}}}. \end{aligned}$$

(b) Find $\mathbf{BA}$. (2)

Solution

$$\begin{aligned} \mathbf{BA} &= \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} 7 & -18 \\ -3 & -19 \end{pmatrix}}}. \end{aligned}$$

(c) Find the inverse matrix, $\mathbf{A}^{-1}$. (2)

Solution

Now,

$$\det \mathbf{A} = -15 - 2 = -17$$

and so

$$\mathbf{A}^{-1} = -\tfrac{1}{17}\begin{pmatrix} -5 & -2 \\ -1 & 3 \end{pmatrix}.$$

(d) Use your answer to part (c) to solve the simultaneous equations (2)

$$3x + 2y = 23$$
$$x - 5y = 19.$$

Solution

Convert it into matrices:

$$\begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 23 \\ 19 \end{pmatrix}$$
$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = -\tfrac{1}{17}\begin{pmatrix} -5 & -2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 23 \\ 19 \end{pmatrix}$$
$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = -\tfrac{1}{17}\begin{pmatrix} -153 \\ 34 \end{pmatrix}$$
$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \end{pmatrix};$$

hence, the solutions are

$$x = 9 \text{ and } y = -2.$$

11. (a) (i) Solve (3)

$$\frac{5^{2x+3}}{25^{2x}} = \frac{25^{2-x}}{125^{x}}.$$

Solution

We make everything a power of 5:

$$\begin{aligned}
\frac{5^{2x+3}}{25^{2x}} = \frac{25^{2-x}}{125^x} &\Rightarrow \frac{5^{2x+3}}{(5^2)^{(2x)}} = \frac{(5^2)^{(2-x)}}{(5^3)^x} \\
&\Rightarrow \frac{5^{2x+3}}{5^{4x}} = \frac{5^{2(2-x)}}{5^{3x}} \\
&\Rightarrow 5^{(2x+3)-4x} = 5^{2(2-x)-3x} \\
&\Rightarrow 5^{2x+3-4x} = 5^{4-2x-3x} \\
&\Rightarrow 5^{-2x+3} = 5^{4-5x} \\
&\Rightarrow -2x+3 = 4-5x \\
&\Rightarrow 3x = 1 \\
&\Rightarrow \underline{\underline{x = \tfrac{1}{3}}}.
\end{aligned}$$

(ii) Solve (4)

$$\log_{10} y + \log_{10}(y-15) = 2.$$

Solution

$$\begin{aligned}
\log_{10} y + \log_{10}(y-15) = 2 &\Rightarrow \log_{10} y(y-15) = 2 \\
&\Rightarrow y(y-15) = 10^2 \\
&\Rightarrow y^2 - 15y = 100 \\
&\Rightarrow y^2 - 15y - 100 = 0
\end{aligned}$$

$$\left.\begin{matrix} \text{add to:} & -15 \\ \text{multiply to:} & -100 \end{matrix}\right\} -20, +5$$

$$\begin{aligned}
&\Rightarrow (y-20)(y+5) = 0 \\
&\Rightarrow y = 20 \text{ or } y = -5.
\end{aligned}$$

But $y \neq -5$ (why?) so $\underline{\underline{y = 20}}$.

(b) Without using a calculator, and showing each stage of your working, find the value of (3)

$$2\log_{12} 4 - \tfrac{1}{2}\log_{12} 81 + 4\log_{12} 3.$$

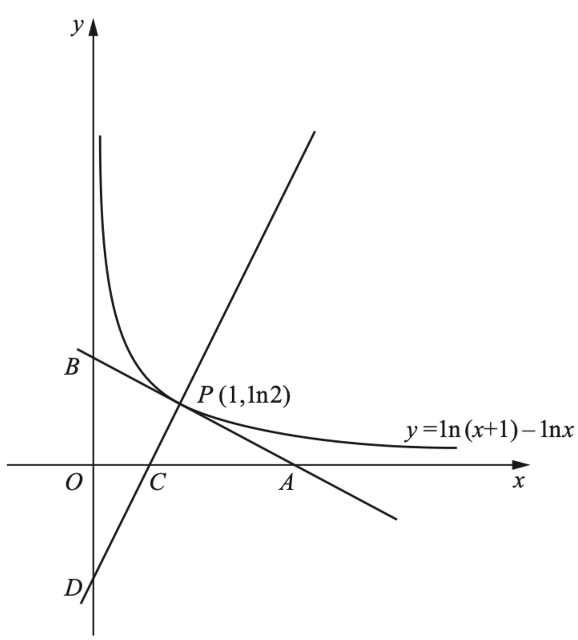
Solution

$$\begin{aligned}
& 2\log_{12}4 - \tfrac{1}{2}\log_{12}81 + 4\log_{12}3 \\
= \; & \log_{12}4^2 - \log_{12}81^{\frac{1}{2}} + \log_{12}3^4 \\
= \; & \log_{12}16 - \log_{12}9 + \log_{12}81 \\
= \; & \log_{12}\left(\frac{16\times 81}{9}\right) \\
= \; & \log_{12}144 \\
= \; & \log_{12}12^2 \\
= \; & 2\log_{12}12 \\
= \; & \underline{\underline{2}}.
\end{aligned}$$

EITHER

12. The diagram shows part of the curve

$$y = \ln(x+1) - \ln x.$$



- The tangent to the curve at the point $P(1, \ln 2)$ meets the x-axis at A and the y-axis at B.

- The normal to the curve at P meets the x-axis at C and the y-axis at D.

(a) Find, in terms of $\ln 2$, the coordinates of A, B, C, and D. (8)

Solution

$\underline{APC}$:

$$y = \ln(x+1) - \ln x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x+1} - \frac{1}{x}.$$

Now,

$$x = 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\tfrac{1}{2}$$

and the equation of AB is

$$y - \ln 2 = -\tfrac{1}{2}(x-1).$$

Next,

$$\begin{aligned} x = 0 &\Rightarrow y - \ln 2 = -\tfrac{1}{2}(0-1) \\ &\Rightarrow y = \ln 2 + \tfrac{1}{2}; \\ y = 0 &\Rightarrow 0 - \ln 2 = -\tfrac{1}{2}(x-1) \\ &\Rightarrow 2\ln 2 = x - 1 \\ &\Rightarrow x = 2\ln 2 + 1; \end{aligned}$$

so, $\underline{\underline{A(2\ln 2 + 1, 0)}}$ and $\underline{\underline{B(0, \ln 2 + \tfrac{1}{2})}}$.

$\underline{PCD}$:

$$m_{\text{normal}} = 2$$

and the equation of CD is

$$y - \ln 2 = 2(x-1).$$

Now,

$$\begin{aligned} x = 0 &\Rightarrow y - \ln 2 = 2(0-1) \\ &\Rightarrow y = \ln 2 - 2; \\ y = 0 &\Rightarrow 0 - \ln 2 = 2(x-1) \\ &\Rightarrow -\tfrac{1}{2}\ln 2 = x - 1 \\ &\Rightarrow x = -\tfrac{1}{2}\ln 2 + 1; \end{aligned}$$

so, $\underline{\underline{C(-\tfrac{1}{2}\ln 2 + 1, 0)}}$ and $\underline{\underline{D(0, \ln 2 - 2)}}$.

(b) Given that

$$\frac{\text{area of triangle } BPD}{\text{area of triangle } APC} = \frac{1}{k},$$

express k in terms of $\ln 2$. (3)

Solution

Well,

$$\begin{aligned}\text{area of triangle } BPD &= \tfrac{1}{2} \times \tfrac{5}{2} \times 1 \\ &= \tfrac{5}{4}\end{aligned}$$

and

$$\begin{aligned}\text{area of triangle } APC &= \tfrac{1}{2} \times \tfrac{5}{2} \ln 2 \times \ln 2 \\ &= \tfrac{5}{4}(\ln 2)^2.\end{aligned}$$

Finally,

$$\begin{aligned}\frac{\text{area of triangle } BPD}{\text{area of triangle } APC} &= \frac{\frac{5}{4}}{\frac{5}{4}(\ln 2)^2} \\ &= \underline{\underline{\frac{1}{(\ln 2)^2}}};\end{aligned}$$

hence, $\underline{\underline{k = (\ln 2)^2}}$.

OR

13. A curve has equation

$$y = x\mathrm{e}^x.$$

The curve has a stationary point at P.

(a) Find, in terms of e, the coordinates of P and determine the nature of this stationary point. (5)

Solution

Product rule:

$$\begin{aligned}u = x &\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \\ v = \mathrm{e}^x &\Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x\end{aligned}$$

and

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x\mathrm{e}^x + \mathrm{e}^x.$$

Now,

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = 0 &\Rightarrow x\mathrm{e}^x + \mathrm{e}^x = 0\\ &\Rightarrow \mathrm{e}^x(x+1) = 0\\ &\Rightarrow x + 1 = 0\\ &\Rightarrow x = -1\\ &\Rightarrow y = -\mathrm{e}^{-1};\end{aligned}$$

so, $\underline{\underline{P(-1, -\mathrm{e}^{-1})}}$.

Product rule again:

$$\begin{aligned}u = x + 1 &\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1\\ v = \mathrm{e}^x &\Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x\end{aligned}$$

and

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = (x+1)\mathrm{e}^x + \mathrm{e}^x.$$

Next,

$$x = -1 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{-1} > 0$$

and the stationary point is a $\underline{\underline{\text{minimum}}}$.

The normal to the curve at the point $Q(1, \mathrm{e})$ meets the x-axis at R and the y-axis at S.

(b) Find, in terms of e, the area of triangle ORS, where O is the origin. (6)

Solution

Well,

$$x = 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}$$

and

$$m_{\text{normal}} = -\tfrac{1}{2\mathrm{e}}.$$

Now, the normal is

$$y - \mathrm{e} = -\tfrac{1}{2\mathrm{e}}(x - 1)$$

and

$$
\begin{aligned}
x = 0 &\Rightarrow y - \mathrm{e} = -\tfrac{1}{2\mathrm{e}}(0-1) \\
&\Rightarrow y - \mathrm{e} = \tfrac{1}{2\mathrm{e}} \\
&\Rightarrow y = \mathrm{e} + \tfrac{1}{2\mathrm{e}}; \\
y = 0 &\Rightarrow 0 - \mathrm{e} = -\tfrac{1}{2\mathrm{e}}(x-1) \\
&\Rightarrow 2\mathrm{e}^2 = x - 1 \\
&\Rightarrow x = 2\mathrm{e}^2 + 1;
\end{aligned}
$$

so, $R(2\mathrm{e}^2+1)$ and $S(0, \mathrm{e} + \frac{1}{2\mathrm{e}})$. Finally,

$$
\begin{aligned}
\text{area of triangle } ORS &= \tfrac{1}{2} \times (2\mathrm{e}^2+1) \times (\mathrm{e} + \tfrac{1}{2\mathrm{e}}) \\
&= \tfrac{1}{2} \times (2\mathrm{e}^2+1) \times (\tfrac{2\mathrm{e}^2+1}{2\mathrm{e}}) \\
&= \underline{\underline{\frac{(2\mathrm{e}^2+1)^2}{4\mathrm{e}}}}.
\end{aligned}
$$