Dr Oliver Mathematics
Tangents: Part 1

1. What is the equation for the tangent to the graph of

$$y = \arcsin\left(\frac{x}{2}\right)$$

at the origin?

Solution

$$y = \arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \cdot \tfrac{1}{2}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1 - \frac{x^2}{4}}} \cdot \tfrac{1}{2}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{\frac{1}{4}(4 - x^2)}} \cdot \tfrac{1}{2}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{1}{2}\sqrt{4 - x^2}} \cdot \tfrac{1}{2}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{4 - x^2}}.$$

Now,

$$x = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2}$$

and the equation of the tangent is

$$y - 0 = \tfrac{1}{2}(x - 0) \Rightarrow \underline{\underline{y = \tfrac{1}{2}x}}.$$