Dr Oliver Mathematics
Mathematics: National Qualifications N5
2015 Paper 2: Calculator
1 hour 20 minutes

The total number of marks available is 50.
You must write down all the stages in your working.

1. A house is valued at £240 000.   (3)
   Its value is predicted to rise by 2.8% per annum.
   Calculate its predicted value after 2 years.

   Solution

   $$240\,000 \times (1.028)^2 = \underline{£253\,628.16}.$$

2. A function is defined as   (2)
   $$\mathrm{f}(x) = 3x + 2.$$

   Given that
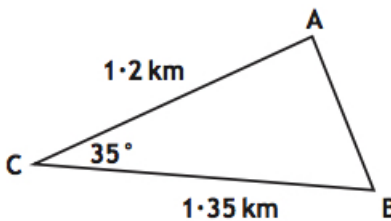 $$\mathrm{f}(a) = 23,$$

   calculate a.

   Solution

   $$3a + 2 = 23 \Rightarrow 3a = 21$$
   $$\Rightarrow \underline{\underline{a = 7}}.$$

3. Triangle ABC is shown below.   (3)

Calculate the length of AB.

Solution

$$AB = \sqrt{1.2^2 + 1.35^2 - 2 \cdot 1.2 \cdot 1.35 \cdot \cos 35°}$$
$$= 0.780\,030\,369 \text{ (FCD)}$$
$$= \underline{\underline{0.780 \text{ km}^2}} \text{ (3 sf)}.$$

4. Find $|\mathbf{u}|$, the magnitude of vector (2)

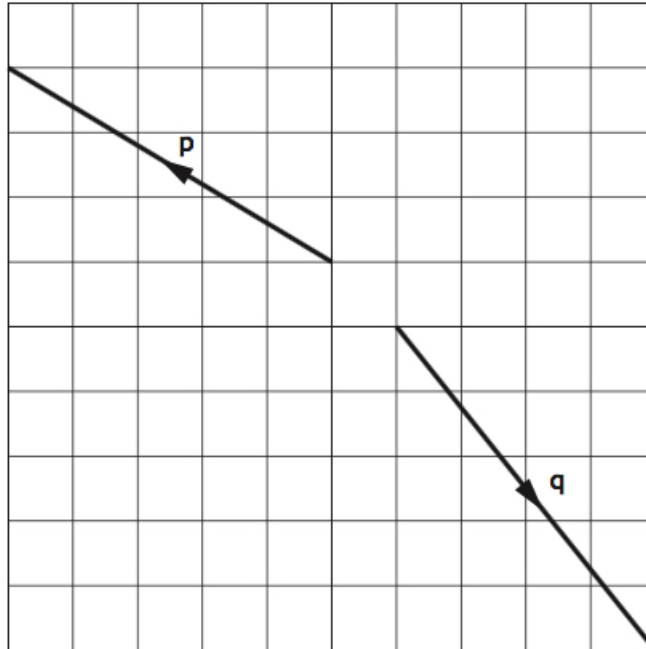$$\mathbf{u} = \begin{pmatrix} 6 \\ -13 \\ 18 \end{pmatrix}.$$

Solution

$$|\mathbf{u}| = \sqrt{6^2 + (-13)^2 + 18^2}$$
$$= \sqrt{529}$$
$$= \underline{\underline{23}}.$$

5. The vectors $\mathbf{p}$ and $\mathbf{q}$ are shown in the diagram below. (2)

Find the resultant vector $\mathbf{p} + \mathbf{q}$.
Express your answer in component form.

Solution

$$\mathbf{p} + \mathbf{q} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$
$$= \underline{\underline{\begin{pmatrix} -1 \\ -2 \end{pmatrix}}}.$$

6. The Earth is approximately spherical with a radius of $6\,400$ kilometres.

    (a) Calculate the volume of the Earth giving your answer in scientific notation, correct to 2 significant figures. (3)

    Solution

    $$\text{Volume} = \tfrac{4}{3} \times \pi \times (6\,400)^3$$
    $$= 1.098\,066\,219 \times 10^{12} \ \text{(FCD)}$$
    $$= \underline{\underline{1.1 \times 10^{12} \ \text{km}^3 \ \text{(2 sf)}}}.$$

The approximate volume of the Moon is 2.2×10^{10} cubic kilometres.

(b) Calculate how many times the Earth's volume is greater than the Moon's. (2)

> **Solution**
>
> $$\frac{1.098\ldots \times 10^{12}}{2.2 \times 10^{10}} = 49.912\,100\,88 \text{ (FCD)}$$
> $$= \underline{\underline{50}} \text{ (2 sf)}.$$

7. Express (3)

$$\frac{5t}{s} \div \frac{t}{2s^2}$$

in its simplest form.

> **Solution**
>
> $$\frac{5t}{s} \div \frac{t}{2s^2} = \frac{5t}{s} \times \frac{2s^2}{t}$$
> $$= \frac{5}{1} \times \frac{2s}{1}$$
> $$= \underline{\underline{10s}}.$$

8. James paid £297.50 for a laptop in a sale. (3)
The discount in the sale was 15%.
Calculate the original price of the laptop.

> **Solution**
>
> $$100 - 15 = 0.85$$
>
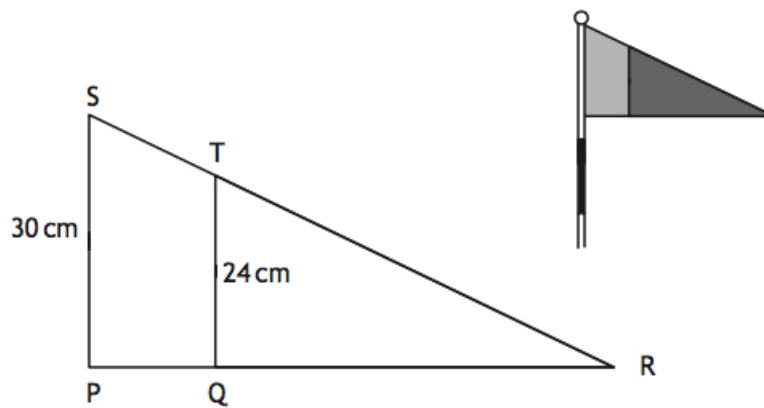> and, hence, the original price of the laptop is
>
> $$\frac{297.50}{0.85} = \underline{\underline{£350}}.$$

9. The flag at each hole on a golf course is coloured red and blue. (4)
The diagram below represents a flag.

Triangle QRT represents the red section.
$PQTS$ represents the blue section.

Triangles PRS and QRT are mathematically similar.
The area of triangle QRT is 400 square centimetres.
Calculate the area of $PQTS$, the blue section of the flag.

Solution

The length scale factor (LSF) is
$$\frac{30}{24} = \tfrac{5}{4}$$
and the area scale factor (ASF) is
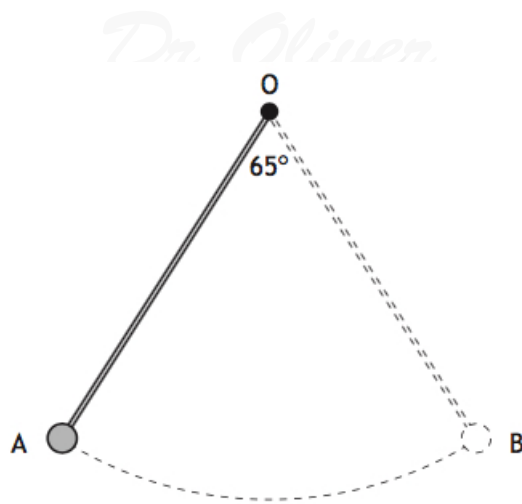$$(\tfrac{5}{4})^2 = \tfrac{25}{16}.$$

Now,
$$\text{area of } PSR = \tfrac{25}{16} \times 400$$
$$= 625$$

which means

$$\text{Area of } PQTS = \text{area of } PSR - \text{area of } QRT$$
$$= 625 - 400$$
$$= \underline{\underline{225}} \text{ square centimetres.}$$

10. The pendulum of a clock swings along an arc of a circle, centre O. (4)

The pendulum swings through an angle of 65°, travelling from A to B.
The length of the arc AB is 28.4 centimetres.
Calculate the length of the pendulum.

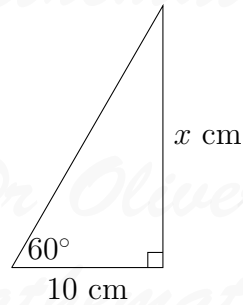**Solution**

Let the length of the pendulum be x cm. Then

$$\frac{65}{360} \times 2 \times \pi \times x = 28.4 \Rightarrow x = \frac{360 \times 28.4}{130\pi}$$
$$\Rightarrow x = 25.033\,848\,28 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 25.0 \text{ (3 sf)}}}.$$

11. The top of a table is in the shape of a regular hexagon. (4)
    The three diagonals of the hexagon which are shown as dotted lines in the diagram below
    each have length 40 centimetres.
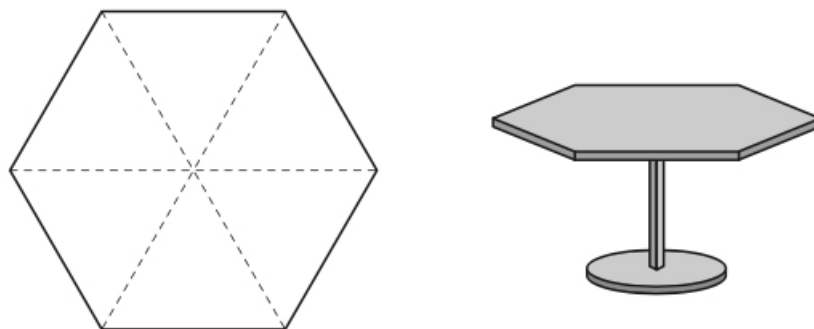
Calculate the area of the top of the table.

Solution

$$\frac{x}{10} = \tan 60° \Rightarrow x = 10\sqrt{3}$$
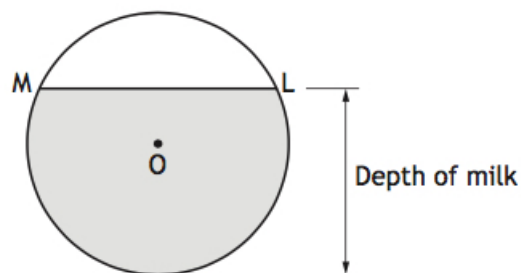
and the area of one-sixth of the table top is

$$\tfrac{1}{2} \times 20 \times 10\sqrt{3} = 100\sqrt{3}.$$

Hence,

$$\text{whole table top} = 6 \times 100\sqrt{3}$$
$$= \underline{\underline{600\sqrt{3} \text{ cm}^2}}.$$

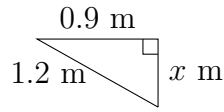12. The diagram below shows the circular cross-section of a milk tank. (4)

The radius of the circle, centre O, is 1.2 metres.
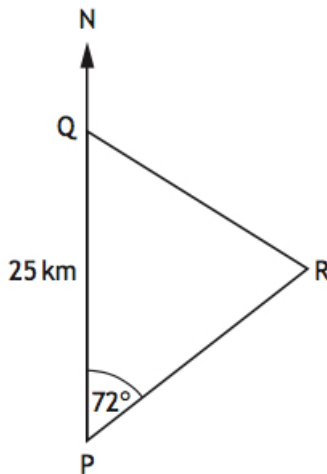The width of the surface of the milk in the tank, represented by ML in the diagram, is 1.8 metres.

Calculate the depth of the milk in the tank.

Solution

0.9 m

1.2 m

x m

So, the depth of the milk in the tank is

$$1.2 + \sqrt{1.2^2 - 0.9^2} = 1.2 + \tfrac{3}{7}\sqrt{10}$$
$$= 1.993\,725\,393 \text{ (FCD)}$$
$$= \underline{\underline{1.99 \text{ m } (3 \text{ sf})}}.$$

13. In the diagram below P, Q, and R represent the positions of Portlee, Queenstown, and Rushton respectively. (4)

N

Q

25 km

R

72°

P

Portlee is 25 kilometres due South of Queenstown.
From Portlee, the bearing of Rushton is 072°.
From Queenstown, the bearing of Rushton is 128°.

Calculate the distance between Portlee and Rushton.
Do not use a scale drawing.

Solution

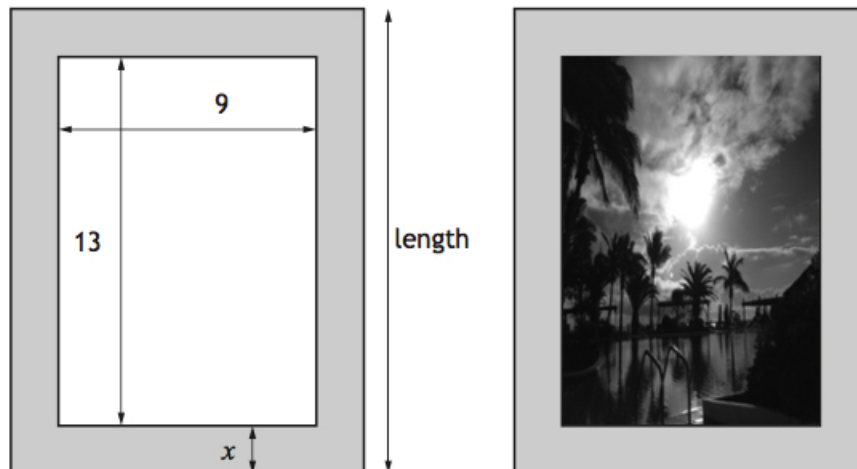$$\angle PQR = 180 - 128$$
$$= 52°$$

and

$$\angle PRQ = 180 - (72 + 52)$$
$$= 56°.$$

Now, we apply the sine rule:

$$\frac{PR}{\sin PQR} = \frac{PQ}{\sin PRQ} \Rightarrow \frac{PR}{\sin 52°} = \frac{25}{\sin 56°}$$
$$\Rightarrow PR = \frac{25 \sin 52°}{\sin 56°}$$
$$\Rightarrow PR = 23.762\,817\,87 \text{ (FCD)}$$
$$\Rightarrow PR = 23.8 \text{ km (3 sf)}.$$

14. A rectangular picture measuring 9 centimetres by 13 centimetres is placed on a rectangular piece of card.
    The area of the card is 270 square centimetres.
    There is a border x centimetres wide on all sides of the picture.

(a) (i) Write down an expression for the length of the card in terms of x. (1)

> **Solution**
> $(2x + 13)$ cm.

(ii) Hence show that (2)
$$4x^2 + 44x - 153 = 0.$$

> **Solution**
>
> | $\times$ | $2x$ | $+9$ |
> |---|---|---|
> | $2x$ | $4x^2$ | $+18x$ |
> | $+13$ | $+26x$ | $+117$ |
>
> $$(2x + 13)(2x + 9) = 270 \Rightarrow 4x^2 + 44x + 177 = 270$$
> $$\Rightarrow 4x^2 + 44x - 153 = 0,$$
>
> as required.

(b) Calculate x, the width of the border. (4)
Give your answer correct to one decimal place.

> **Solution**
> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
> $$= \frac{-44 \pm \sqrt{(-44)^2 - 4 \times 4 \times (-153)}}{2 \times 4}$$
> $$= \frac{-44 \pm \sqrt{4\,384}}{8}$$
> $$= -13.776\,472\,68,\ 2.776\,472\,679\ \text{(FCD)};$$
>
> as $x > 0$,
> $$x = 2.78\ \text{cm (3 sf)}.$$