Dr Oliver Mathematics
GCSE Mathematics
2005 November Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. A school snack bar offers a choice of four snacks. (2)
   The four snacks are burgers, pizza, pasta, and salad.
   Students can choose one of these four snacks.
   The table shows the probability that a student will choose burger or pizza or salad.

Snack	Burger	Pizza	Pasta	Salad
Probability	0.35	0.15		0.2

   300 students used the snack bar on Tuesday.
   Work out an estimate for the number of students who chose pizza.

Solution

An estimate for the number of students who chose pizza is

$$300 \times 0.15 = \underline{\underline{45 \text{ people}}}.$$

2. Emma repairs bicycles. (2)
   She keeps records of the cost of the repairs.
   The table gives information about the costs of all repairs which she carried out in one week.

Cost (£C)	Frequency
$0 < C \leqslant 10$	3
$10 < C \leqslant 20$	7
$20 < C \leqslant 30$	6
$30 < C \leqslant 40$	8
$40 < C \leqslant 50$	9

   Find the class interval in which the median lies.

Solution

Complete the cumulative frequency table:

Cost (£C)	Cumulative Frequency
$0 < C \leqslant 10$	3
$0 < C \leqslant 20$	$7 + 3 = 10$
$0 < C \leqslant 30$	$10 + 6 = 16$
$0 < C \leqslant 40$	$16 + 8 = 24$
$0 < C \leqslant 50$	$24 + 9 = 33$

Now, the median is in

$$\frac{33+1}{2} = 17\text{th piece of information}$$

and so the class interval in which the median lies is $\underline{\underline{30 < C \leqslant 40}}$.

3. Jenny and Kath hire the canal boat for 14 days. (2)
They share the hire cost of £1 785.00 in the ratio 2 : 3.
Work out the smaller share.

Solution

$$2 + 3 = 5$$

and the smaller share is

$$\tfrac{2}{5} \times 1\,785 = 2 \times 357 = \underline{\underline{£714}}.$$

4. (a) Expand and simplify (2)

$$(x - y)^2.$$

Solution

$\times$	x	$-y$
x	x^2	$-xy$
$-y$	$-xy$	$+y^2$

Hence,

$$(x-y)^2 = \underline{\underline{x^2 - 2xy + y^2}}.$$

(b) Rearrange (3)

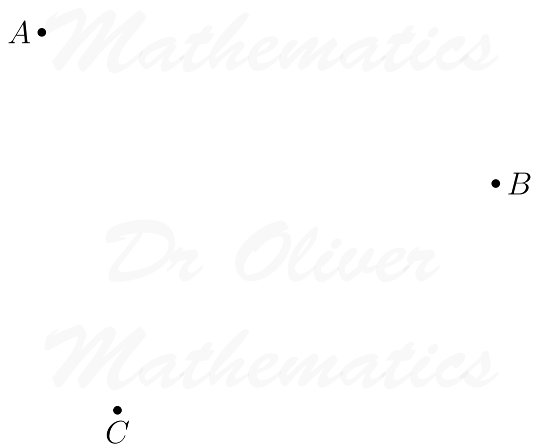
$$a(q-c) = d$$

to make q the subject.

Solution

$$\begin{aligned} a(q-c) = d &\Rightarrow aq - ac = d \\ &\Rightarrow aq = ac + d \\ &\Rightarrow \underline{\underline{q = \frac{ac+d}{a}}}. \end{aligned}$$

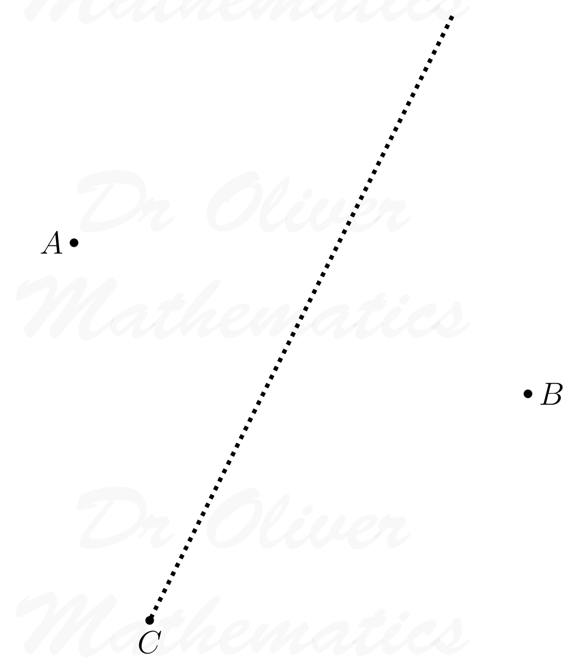
5. Jill rolls a ball from point C.



At any point on its path, the ball is the same distance from point A and point B.

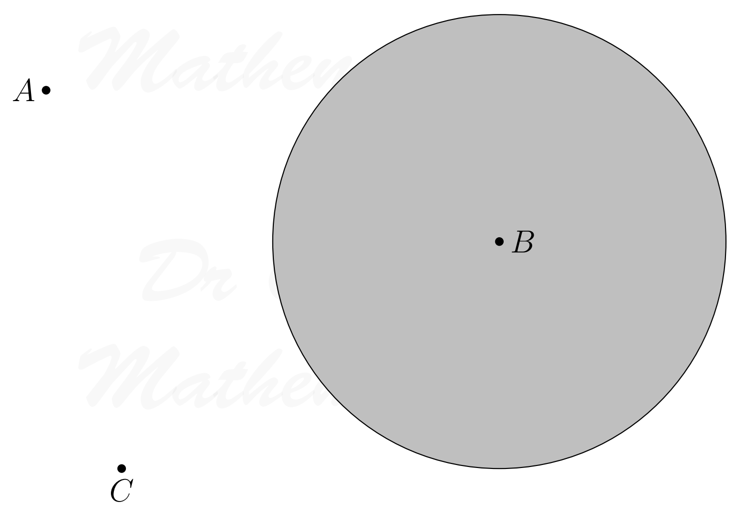
(a) Draw accurately the path that the ball will take. (2)

Solution



(b) Shade the region that contains all the points that are no more than 3 cm from point B. (2)

Solution



6. Work out an estimate for the value of (3)

$$\frac{5.79 \times 312}{0.523}.$$

Solution

Round to 1 significant figure:

$$\frac{5.79 \times 312}{0.523} \approx \frac{6 \times 300}{0.5} = \frac{1\,800}{0.5} = \underline{\underline{3\,600}}.$$

7. The sizes of the angles, in degrees, of the quadrilateral are
$x + 10$,
$2x$,
$x + 90$, and
$x + 20$.

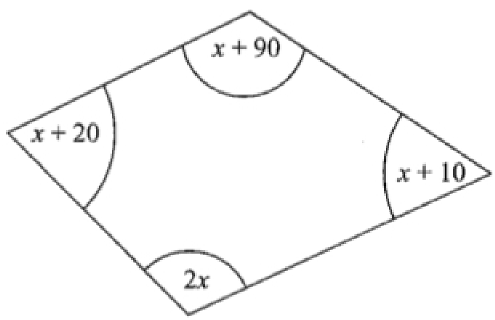


Diagram **NOT** accurately drawn

(a) Use this information to write down an equation in terms of x. (2)

Solution

$$\underline{\underline{(x + 20) + 2x + (x + 90) + (x + 10) = 360}}$$

or

$$\underline{\underline{5x + 120 = 360}}.$$

(b) Use your answer to part (a) to work out the size of the smallest angle of the quadrilateral. (3)

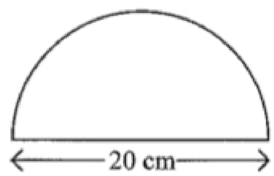
Solution

$$5x + 120 = 360 \Rightarrow 5x = 240$$
$$\Rightarrow x = 48$$

and the size of the smallest angle of the quadrilateral is

$$48 + 10 = \underline{\underline{58^\circ}}.$$

8. A semicircle has a diameter of 20 cm. (3)



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Work out the perimeter of the semicircle.
Take the value of π to be 3.14.

Solution

The radius of the circle is 10 cm and so

$$\begin{aligned} \text{perimeter} &= 20 + \pi \times 10 \\ &\approx 20 + 3.14 \times 10 \\ &= 20 + 31.4 \\ &= \underline{\underline{51.4 \text{ cm}}}. \end{aligned}$$

9. (a) Write the number 40 000 000 in standard form. (1)

Solution

$$40\,000\,000 = \underline{\underline{4 \times 10^7}}.$$

(b) Write 1.4×10^{-5} as an ordinary number. (1)

Solution

$$1.4 \times 10^{-5} = \underline{\underline{0.000\,014}}.$$

(c) Work out (2)

$$(5 \times 10^4) \times (6 \times 10^9).$$

Give your answer in standard form.

Solution

$$\begin{aligned}(5 \times 10^4) \times (6 \times 10^9) &= 30 \times 10^{13} \\ &= \underline{\underline{3 \times 10^{14}}}.\end{aligned}$$

10. QRS is a straight line. (3)

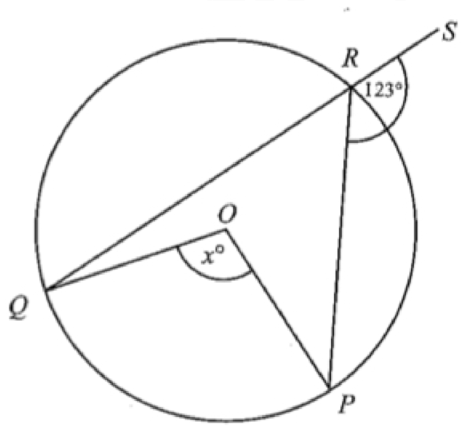


Diagram **NOT** accurately drawn

QR and PR are chords of a circle, centre O.
Angle $PRS = 123°$.
Angle $QOP = x°$.
Calculate the size of the angle marked $x°$.
Give reasons for your answer.

Solution

Angle $QRP = 180 - 123 = 57°$ (supplementary angles).
Angle $QOP = 2 \times 57° = \underline{\underline{114°}}$ (the angle at the circumference is twice the angle at the centre).

11. Here are some expressions. (2)

$\dfrac{\pi r^3}{x}$	$\dfrac{r^3}{\pi}$	$\pi x + r$	$\pi r^2 + rx$	$\pi(x + r)$	$\dfrac{\pi^3}{x^2}$

The letters r and x represent lengths.
π is a number that has no dimensions.
Tick (✓) the boxes underneath the two expressions that can represent areas.

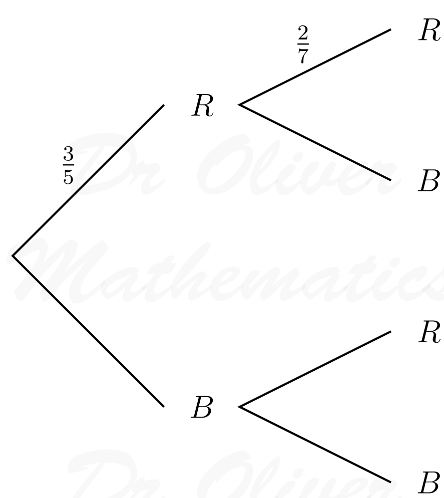
Solution

$\dfrac{\pi r^3}{x}$	$\dfrac{r^3}{\pi}$	$\pi x + r$	$\pi r^2 + rx$	$\pi(x + r)$	$\dfrac{\pi^3}{x^2}$
✓			✓		

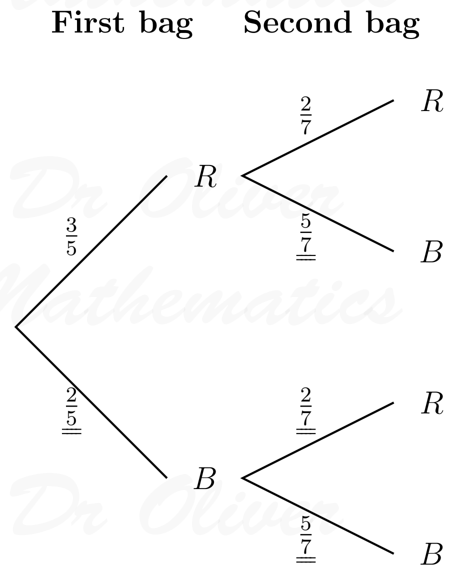
12. Loren has two bags.
The first bag contains 3 red counters and 2 blue counters.
The second bag contains 2 red counters and 5 blue counters.
Loren takes one counter at random from each bag.

(a) Complete the probability tree diagram. (2)

First bag **Second bag**



Solution



(b) Work out the probability that Loren takes one counter of each colour. (3)

Solution

$$\begin{aligned}\text{P(different colours)} &= \text{P}(RB) + \text{P}(BR) \\ &= \left(\tfrac{3}{5} \times \tfrac{5}{7}\right) + \left(\tfrac{2}{5} \times \tfrac{2}{7}\right) \\ &= \tfrac{15}{35} + \tfrac{4}{35} \\ &= \underline{\underline{\tfrac{19}{35}}}.\end{aligned}$$

13. Bill buys a new machine.
The value of the machine depreciates by 20% each year.

(a) Bill says, "After 5 years, the machine will have no value." (1)
Bill is **wrong**.
Explain why.

Solution
Bill should multiply it by $\underline{\underline{0.8^5}}$ and not 100%.

Bill wants to work out the value of the machine after 2 years.

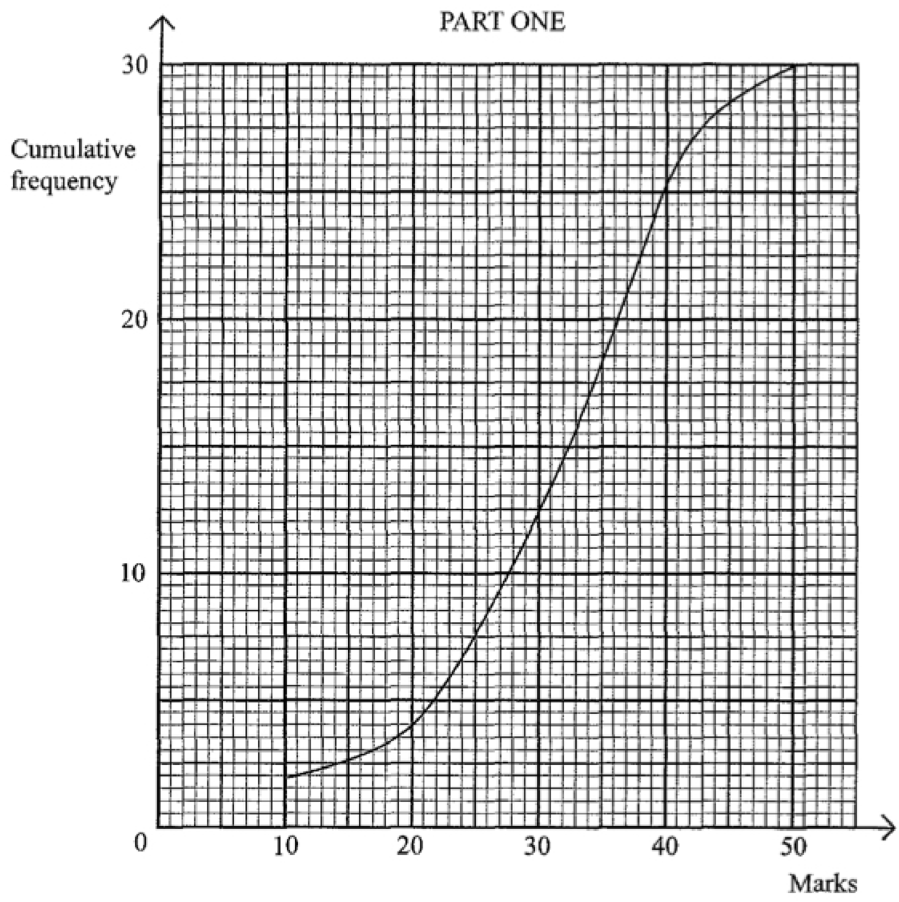
(b) By what single decimal number should Bill multiply the value of the machine when new? (2)

Solution

Bill should multiply it by

$$0.8^2 = \underline{\underline{0.64}}.$$

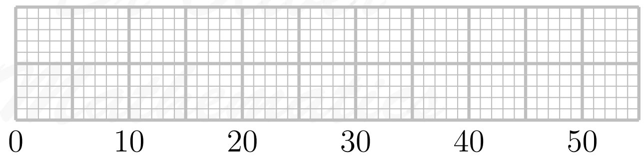
14. 30 students took part in a National Science quiz.
The quiz was in two parts.
The cumulative frequency graph on the grid opposite gives information about the marks scored in Part One.



The lowest mark was 5 and the highest mark was 47.

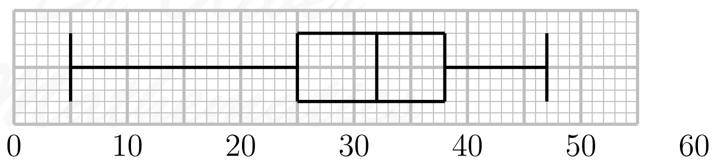
(a) In the space provided on the grid, draw a box plot using the cumulative frequency graph for the results of Part One. (3)

Part One



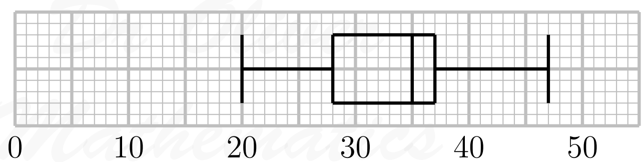
Solution

Part One



The diagram also shows a box plot for the results of Part Two.

Part Two



Use the box plots to compare the two distributions.

(b) Give **two** differences between them. (2)

Solution

Median: They scored 32 in Part One and they scored 35 in Part Two which means that the median was higher in Part Two.

Range: In Part One, they scored $47 - 5 = 42$ and, in Part Two, they scored $47 - 20 = 27$ which means the range was less in Part Two.

or

IQR: In Part One, they scored $38 - 25 = 13$ and, in Part Two, they scored $37 - 28 = 9$ which means the IQR was less in Part Two.

15. A straight line has equation $y = 2x - 3$.
The point P lies on the straight line.
They coordinate of P is -4.

(a) Find the x-coordinate of P. (2)

Solution

$$-4 = 2x - 3 \Rightarrow 2x = -1$$
$$\Rightarrow \underline{\underline{x = -\tfrac{1}{2}}}.$$

A straight line **L** is parallel to $y = 2x - 3$ and passes through the point $(3, 4)$.

(b) Find the equation of line **L**. (3)

Solution

$$y - 4 = 2(x - 3) \Rightarrow y - 4 = 2x - 6$$
$$\Rightarrow \underline{\underline{y = 2x - 2}}.$$

$y = 2x - 3$	$y = 3 - 2x$	$y = \frac{1}{2}x - 3$	$y = 3 - \frac{1}{2}x$	$y = 2x + 3$

(c) Put a tick (✓) underneath the equation which is the equation of a straight line that is perpendicular to the line with equation $y = 2x - 3$. (1)

Solution

$y = 2x - 3$	$y = 3 - 2x$	$y = \frac{1}{2}x - 3$	$y = 3 - \frac{1}{2}x$	$y = 2x + 3$
			✓	

16. The table shows the number of students in each year group at a school. (3)

Year Group	7	8	9	10	11
Number of students	190	145	145	140	130

Jenny is carrying out a survey for her GCSE Mathematics project.
She uses a stratified sample of 60 students according to year group.
Calculate the number of Year 11 students that should be in her sample.

Solution

The total number of students is

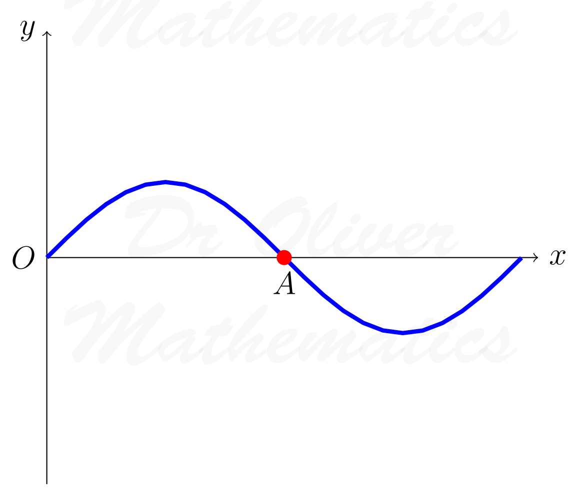
$$190 + 145 + 145 + 140 + 130 = 750$$

and the number of Year 11 students that should be in her sample is

$$\begin{aligned}\frac{60}{750} \times 130 &= \frac{2}{25} \times 130 \\ &= \frac{260}{25} \\ &= 10\tfrac{10}{25};\end{aligned}$$

hence, 10 students.

17. The diagram shows a sketch of part of the curve $y = \sin x^\circ$.



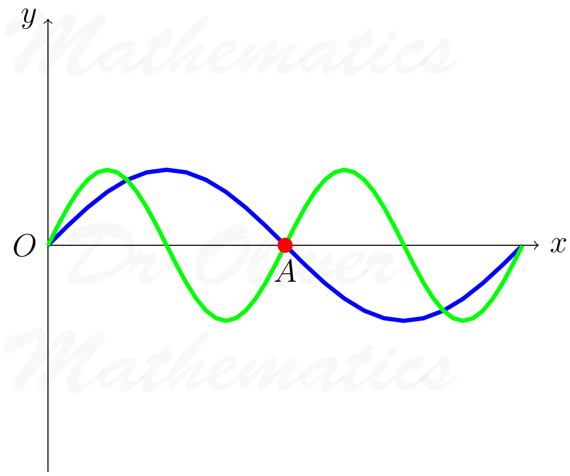
(a) Write down the coordinates of the point A. (1)

Solution

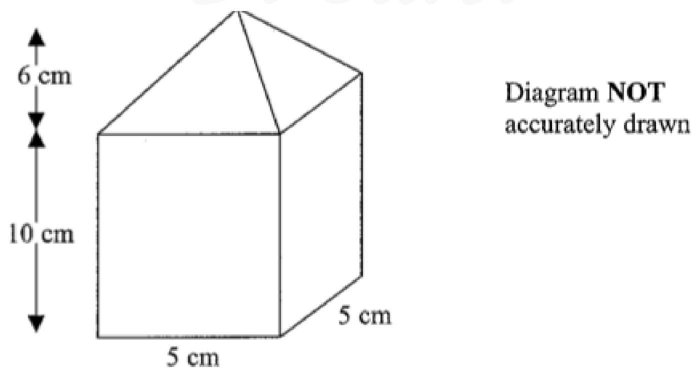
$(180, 0)$.

(b) Sketch the graph of $y = \sin 2x^\circ$. (2)

Solution



18. The diagram shows a model.



The model is a cuboid with a pyramid on top.
The base of the model is a square with sides of length 5 cm.
The height of the cuboid in the model is 10 cm.
The height of the pyramid in the model is 6 cm.

(a) Calculate the volume of the model. (3)

Solution

$$\begin{aligned}\text{Volume} &= \text{cuboid} + \text{pyramid}\\ &= (10 \times 5 \times 5) + (\tfrac{1}{3} \times 6 \times 5 \times 5)\\ &= 250 + 50\\ &= \underline{\underline{300 \text{ cm}^3}}.\end{aligned}$$

The model represents a concrete post.
The model is built to a scale of 1 : 30.
The surface area of the model is 290 cm^2.

(b) Calculate the surface area of the post. (3)
Give your answer in square metres.

Solution

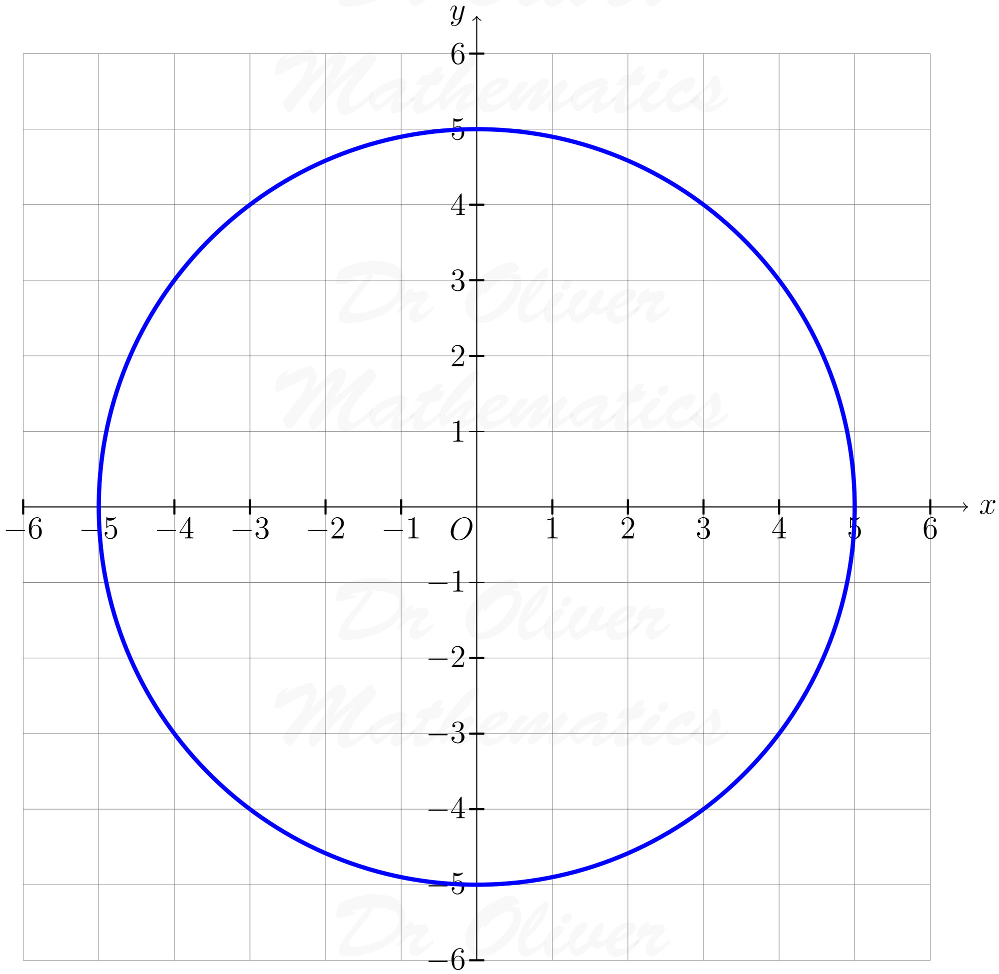
The length scale ratio (LSR) is 1 : 30 and so the area scale ratio (ASR) is

$$1^2 : 30^2 = 1 : 900.$$

Finally, the surface area of the post is

$$\begin{aligned} 290 \times 900 &= 261\,000 \text{ cm}^2 \\ &= 26.1 \times 100^2 \text{ cm}^2 \\ &= \underline{\underline{26.1 \text{ m}^2}}. \end{aligned}$$

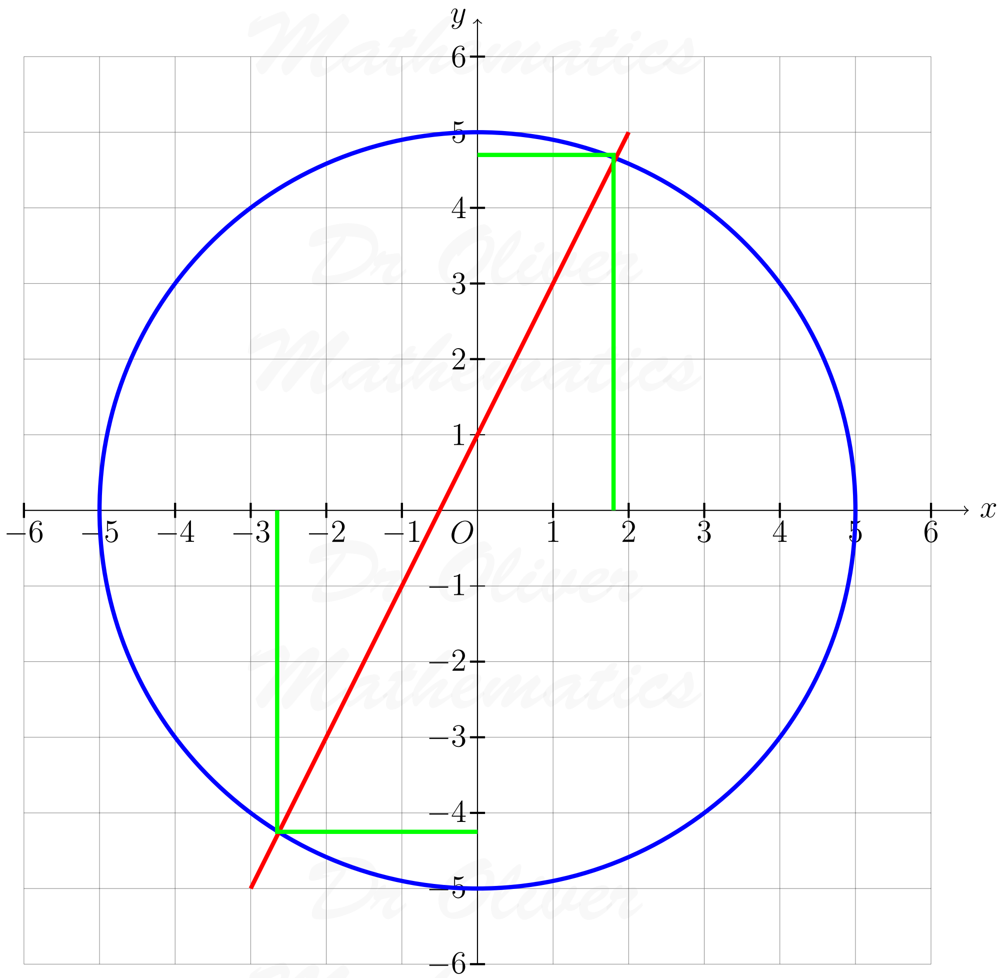
19. The diagram shows a circle of radius 5 cm, centre the origin. (3)



Draw a suitable straight line on the diagram to find estimates of the solutions to the pair of equations of

$$x^2 + y^2 = 25$$
$$y = 2x + 1.$$

Solution



Correct read-off: approximately

$$\underline{\underline{x = -2.65, y = -4.25 \text{ or } x = 1.8, y = 4.7.}}$$

20. (a) Rationalise $\dfrac{1}{\sqrt{7}}$. (2)

Solution

$$\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$
$$= \frac{\sqrt{7}}{7}.$$

(b) (i) Expand and simplify (5)

$$(\sqrt{3} + \sqrt{15})^2.$$

Give your answer in the form $n + m\sqrt{5}$, where n and m are integers.

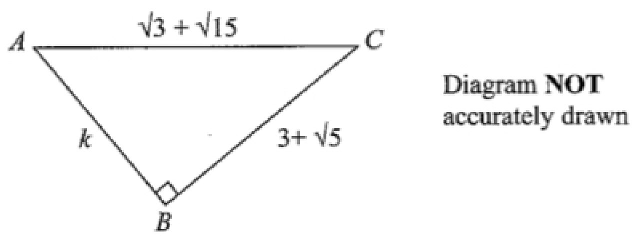
Solution

$\times$	$\sqrt{3}$	$+\sqrt{15}$
$\sqrt{3}$	3	$+3\sqrt{5}$
$+\sqrt{15}$	$+3\sqrt{5}$	$+15$

Hence,

$$(\sqrt{3} + \sqrt{15})^2 = \underline{\underline{18 + 6\sqrt{5}}}.$$

(ii) All measurements on the triangle are in centimetres.



ABC is a right-angled triangle.
k is a positive integer.
Find the value of k.

Solution

$\times$	3	$+\sqrt{5}$
3	9	$+3\sqrt{5}$
$+\sqrt{5}$	$+3\sqrt{5}$	$+5$

Hence

$$\begin{aligned} k &= \sqrt{(\sqrt{3}+\sqrt{15})^2 - (3+\sqrt{5})^2} \\ &= \sqrt{(18+6\sqrt{5}) - (14+6\sqrt{5})} \\ &= \sqrt{4} \\ &= \underline{\underline{2}}. \end{aligned}$$

21. (a) Simplify (4)

(i) $(3x^2y)^3$,

Solution

$$(3x^2y)^3 = \underline{\underline{27x^6y^3}}.$$

(ii) $(2t^{-3})^{-2}$.

Solution

$$\begin{aligned} (2t^{-3})^{-2} &= \frac{1}{(2t^{-3})^2} \\ &= \underline{\underline{\frac{1}{4t^{-6}} \text{ or } \tfrac{1}{4}t^6}}. \end{aligned}$$

(b) Show that (3)

$$x^2 - 4x + 15$$

can be written as

$$(x+p)^2 + q$$

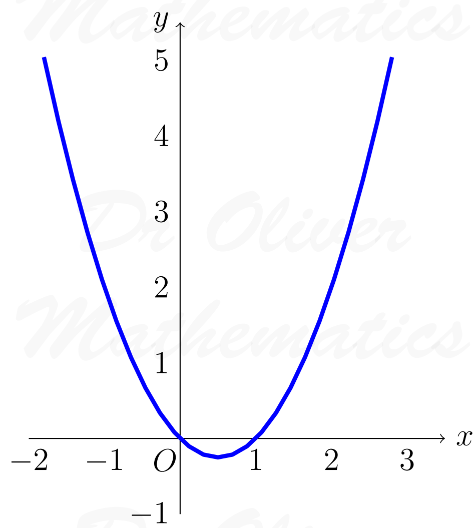
for all values of x.
State the values of p and q.

Solution

$$\begin{aligned} x^2 - 4x + 15 &= (x^2 - 4x + 4) + 11 \\ &= (x-2)^2 + 11; \end{aligned}$$

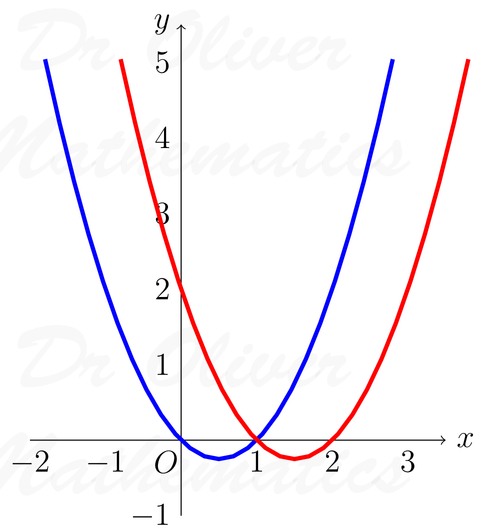
hence, $\underline{\underline{p = -2}}$ and $\underline{\underline{q = 11}}$.

22. The diagram shows a sketch of the graph of $y = x^2 - x$.



(a) Sketch and label the graph of $y = (x-1)^2 - (x-1)$. (3)
Show clearly where this graph crosses the x-axis and where it crosses the y-axis

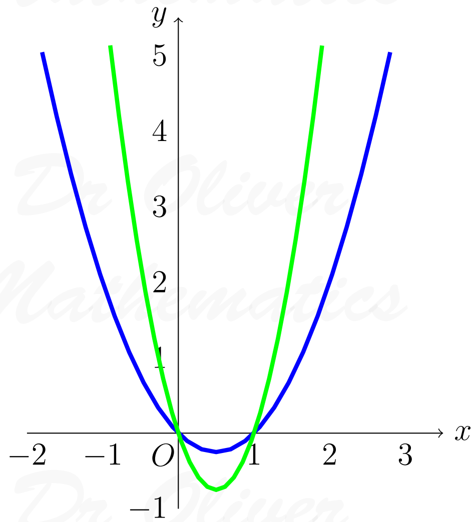
Solution
The graph of $y = (x-1)^2 - (x-1)$ is a translation, one unit to the right.



$\underline{\underline{(1, 0)}}$, $\underline{\underline{(2, 0)}}$, and $\underline{\underline{(0, 2)}}$

(b) Sketch and label the graph of $y = 3(x^2 - x)$. (1)

Solution



The line $y = 4 - 4x$ intersects the curve $y = 3(x^2 - x)$ at the points A and B.

(c) Use an algebraic method to find the coordinates of A and B. (5)

Solution

$$\begin{aligned}3(x^2 - x) = 4 - 4x &\Rightarrow 3x^2 - 3x = 4 - 4x \\ &\Rightarrow 3x^2 + x - 4 = 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & +1 \\ \text{multiply to:} & (+3) \times (-4) = -12\end{array}\right\} -3,\ +4$$

$$\begin{aligned}&\Rightarrow 3x^2 + 4x - 3x - 4 = 0 \\ &\Rightarrow x(3x + 4) - (3x + 4) = 0 \\ &\Rightarrow (x - 1)(3x + 4) = 0 \\ &\Rightarrow 3x + 4 = 0 \text{ or } x - 1 = 0 \\ &\Rightarrow x = -\tfrac{4}{3} \text{ or } x = 1 \\ &\Rightarrow y = \tfrac{28}{3} \text{ or } y = 0;\end{aligned}$$

hence,

$$\underline{\underline{(-\tfrac{4}{3}, \tfrac{28}{3}) \text{ or } (1, 0)}}.$$

23. Prove algebraically that the sum of the squares of any two odd numbers leaves a remainder of 2 when divided by 4. (3)

Solution

Let $(2m+1)$ and $(2n+1)$ be any odd squares where m and $n \in \mathbb{Z}$. Then

$$\begin{aligned}(2m+1)^2 + (2n+1)^2 &= (4m^2+4m+1) + (4n^2+4n+1) \\ &= 4(m^2+n^2+m+n)+2;\end{aligned}$$

hence, the sum of the squares of any two odd numbers leaves a remainder of 2 when divided by 4.

24. ABC is an equilateral triangle.

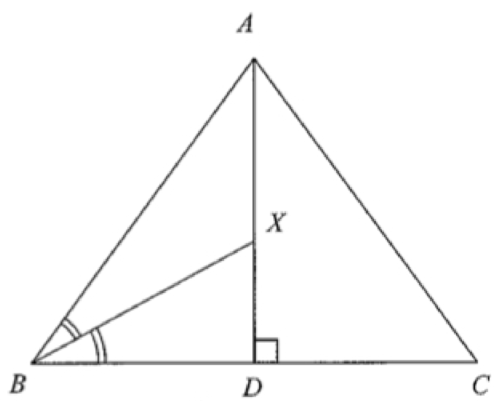


Diagram **NOT** accurately drawn

AD is the perpendicular bisector of BC.
BX is the angle bisector of angle ABC.

(a) Show that triangle BXD is similar to triangle ACD. (2)

Solution

$\angle DBX = \angle DAC = 30°$.
$\angle BDX = \angle ADC = 90°$.
$\angle DXB = \angle DCA = 60°$.
So, the triangle BXD is similar to triangle ACD.

In triangle ACD, $AC = 2$ cm and $AD = \sqrt{3}$ cm.

(b) Show that $XD = \dfrac{1}{\sqrt{3}}$ cm. (3)

Solution

$$\frac{XD}{BD} = \frac{CD}{AD} \Rightarrow \frac{XD}{1} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \underline{\underline{XD = \frac{1}{\sqrt{3}}}},$$

as required.