Dr Oliver Mathematics
AQA GCSE Mathematics
2018 November Paper 3: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. A shape is translated by the vector (1)

$$\begin{pmatrix} 0 \\ 4 \end{pmatrix}.$$

In which direction does the shape move?

up down left right

Circle your answer.

Solution

$\underline{\underline{\text{up}}}$ down left right

2. What is 1.75 kilometres as a fraction of 700 metres? (1)
Circle your answer.

$\frac{5}{2}$ $\frac{1}{4}$ $\frac{4}{1}$ $\frac{2}{5}$

Solution

Well,

$$\frac{1.75 \text{ kilometres}}{700 \text{ metres}} = \frac{1\,750 \text{ metres}}{700 \text{ metres}} = \frac{5}{2}$$

so

$\underline{\underline{\frac{5}{2}}}$ $\frac{1}{4}$ $\frac{4}{1}$ $\frac{2}{5}$

3. The first 4 terms of a linear sequence are (1)

$$3 \quad 11 \quad 19 \quad 27.$$

Circle the expression for the nth term.

$$8-5n \quad n+8 \quad 8n+3 \quad 8n-5$$

Solution

Let the

$$n\text{th term} = an + b.$$

Now,

$$\begin{array}{cccccccc} 3 & & 11 & & 19 & & 27 \\ & 8 & & 8 & & 8 & \\ a+b & & 2a+b & & 3a+b & & 4a+b \\ & a & & a & & a & \end{array}$$

We compare terms:

$$a = 8$$

and

$$\begin{aligned} a + b = 3 &\Rightarrow 8 + b = 3 \\ &\Rightarrow b = -5 \end{aligned}$$

so

$$8-5n \quad n+8 \quad 8n+3 \quad \underline{\underline{8n-5}}$$

4. Work out the lowest common multiple (LCM) of 20, 30, and 40. (1)
Circle your answer.

$$10 \quad 120 \quad 240 \quad 24\,000$$

Solution

Well,

$$20 = 2^2 \times 5$$
$$30 = 2 \times 3 \times 5$$
$$40 = 2^3 \times 5$$

and we are left with the LCM being equal to

$$2^3 \times 3 \times 5 = 120$$

so

$$10 \quad \underline{\underline{120}} \quad 240 \quad 24\,000$$

5. The length of a table is 110 cm to the nearest cm. (2)
Complete the error interval.

$$\ldots\ldots\ldots \text{ cm} \leqslant \text{length} < \ldots\ldots\ldots$$

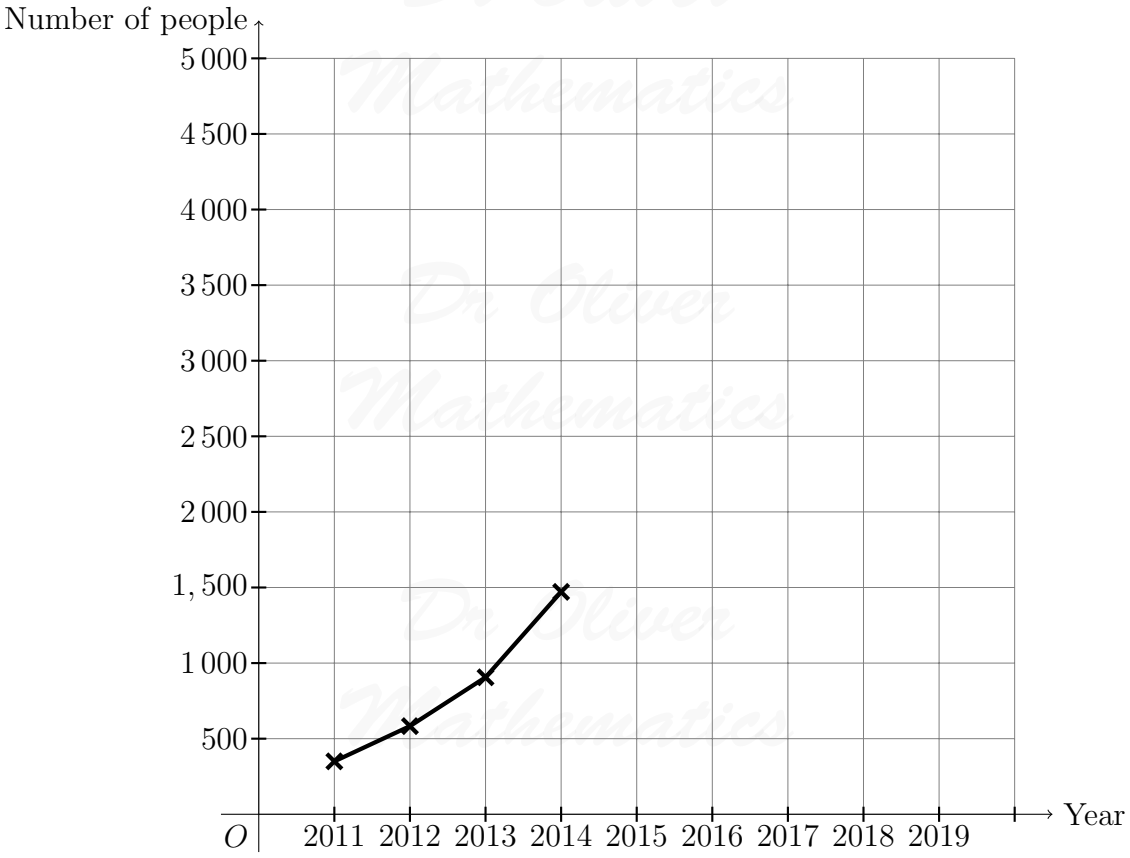
Solution

$$\underline{\underline{109.5}} \text{ cm} \leqslant \text{length} < \underline{\underline{110.5}}$$

6. A music festival has taken place each year from 2011.
The table shows the number of people who attended each year.

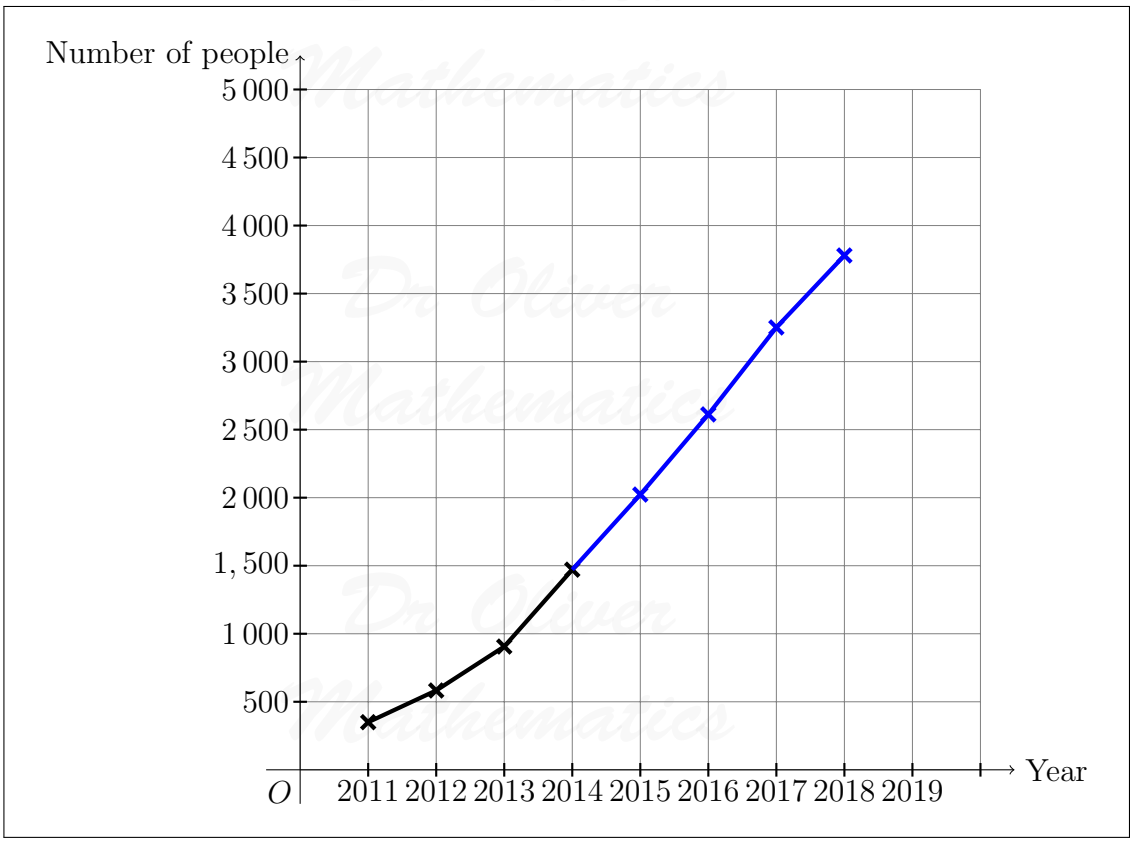
Year	2011	2012	2013	2014	2015	2016	2017	2018
Number of people	350	583	906	1 471	2 023	2 612	3 251	3 780

The festival organisers draw a time series graph to represent the data.
The first four years have been plotted.



(a) Complete the graph. (2)

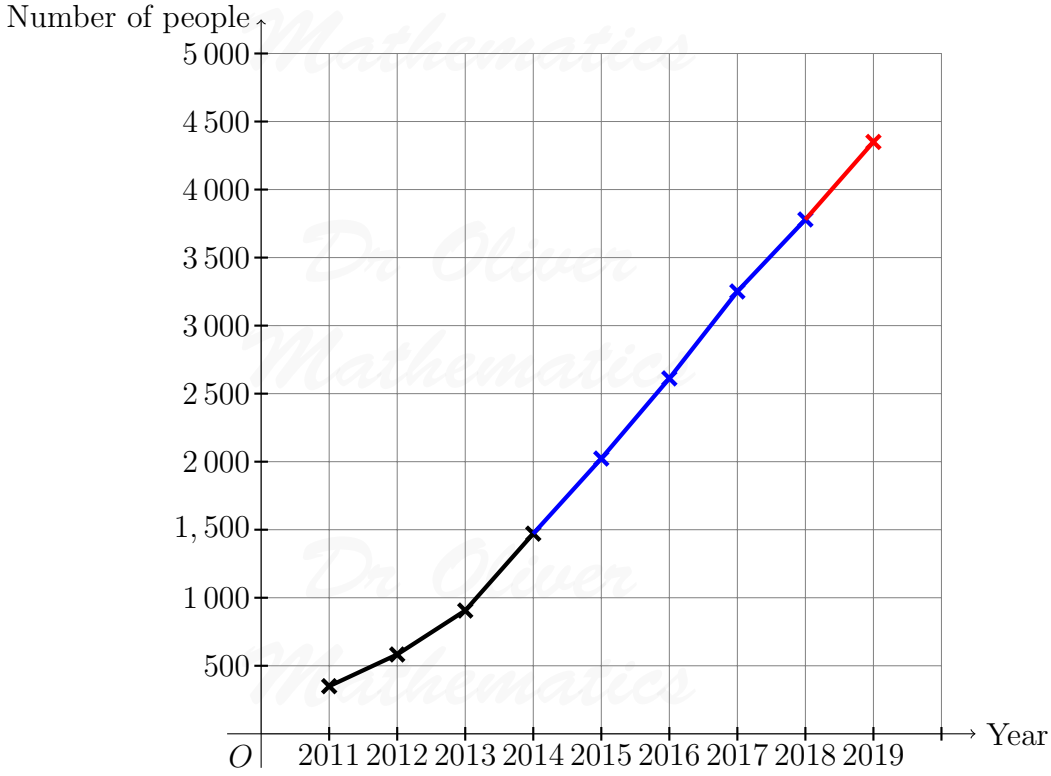
Solution



(b) Use the graph to estimate the number of people who will attend the festival in 2019. (2)

Solution

Well, past 2013, we seem to be going up linearly:



So, about 4 350.

7. (3)

$$k = n^2 + 9n + 1.$$

Mo says, "k will be a prime number for all integer values of n from 1 to 9."

Show that Mo is wrong.
You **must** show that your value of k is **not** prime.

Solution

Well,

$$n = 1 \Rightarrow k = 1^2 + (9 \times 1) + 1 = 11 \text{ (prime)}$$
$$n = 2 \Rightarrow k = 2^2 + (9 \times 2) + 1 = 23 \text{ (prime)}$$
$$n = 3 \Rightarrow k = 3^2 + (9 \times 3) + 1 = 37 \text{ (prime)}$$
$$n = 4 \Rightarrow k = 4^2 + (9 \times 4) + 1 = 53 \text{ (prime)}$$
$$n = 5 \Rightarrow k = 5^2 + (9 \times 5) + 1 = 71 \text{ (prime)}$$
$$n = 6 \Rightarrow k = 6^2 + (9 \times 6) + 1 = 91 = 7 \times 13 \text{ (not prime);}$$

hence, taking $n = 6$, k is not prime.

8. Doug owes an amount of £600. (3)

He wants to pay off this amount in five months.

He says, "Each month, I will pay back 20% of the amount I still owe."

Show working to check if his method is correct.

Solution

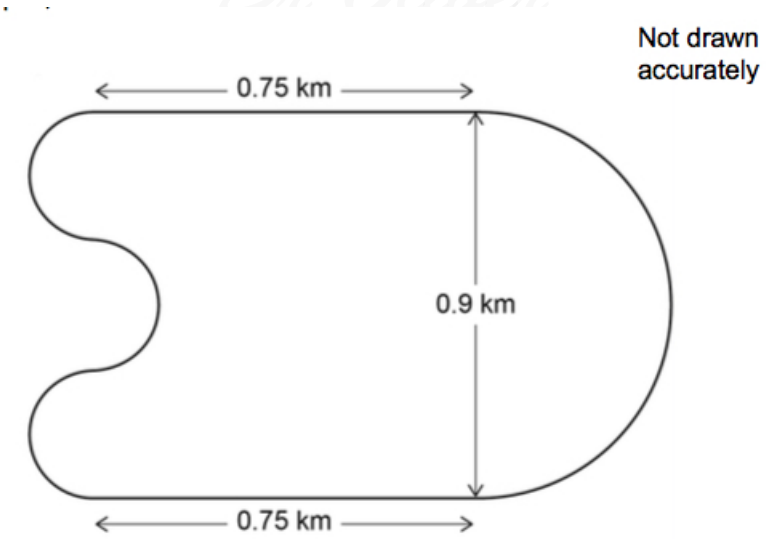
Well,

Month	Still to pay back
1	$\frac{4}{5} \times 600 = 480$
2	$\frac{4}{5} \times 480 = 384$
3	$\frac{4}{5} \times 384 = 307.2$
4	$\frac{4}{5} \times 307.2 = 245.76$
5	$\frac{4}{5} \times 245.76 = 196.608$

So, his method is not correct

9. A motor racing circuit consists of (5)

- two parallel straight sections, each of length 0.75 km,
- a semicircle of diameter 0.9 km, and
- three equal, smaller semicircles.



The length of a motor race must be greater than 305 km.

What is the lowest number of **full** laps needed at this circuit?
You **must** show your working.

Solution

Well,

$$\text{radius of the semicircle} = \frac{0.9}{2} = 0.45$$

and

$$\text{radius of the smaller semicircle} = \frac{0.9}{2 \times 3} = 0.15.$$

$$\begin{aligned}
\text{each lap} &= (2 \times 0.75) + (3 \times \tfrac{1}{2} \times 2 \times \pi \times 0.15) + (\tfrac{1}{2} \times 2 \times \pi \times 0.45) \\
&= 1.5 + \tfrac{9}{20}\pi + \tfrac{9}{20}\pi \\
&= 1.5 + \tfrac{9}{10}\pi \\
&= 4.327\,433\,388 \text{ km (FCD)}.
\end{aligned}$$

Now,

$$\begin{aligned}
\text{full laps} &= \frac{305}{4.327\ldots} \\
&= 70.480\ldots;
\end{aligned}$$

hence, he needs to do 71 laps.

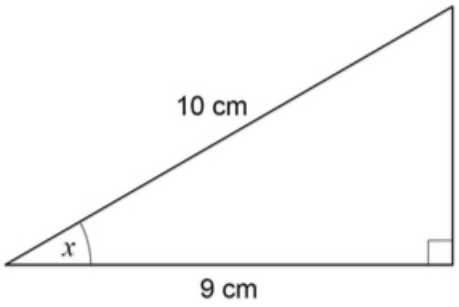
10. Solve (2)

$$8 > 3 - \tfrac{1}{2}x.$$

Solution

$$8 > 3 - \tfrac{1}{2}x \Rightarrow \tfrac{1}{2}x > -5$$
$$\Rightarrow \underline{\underline{x > -10}}.$$

11. Use trigonometry to work out the size of angle x. (2)

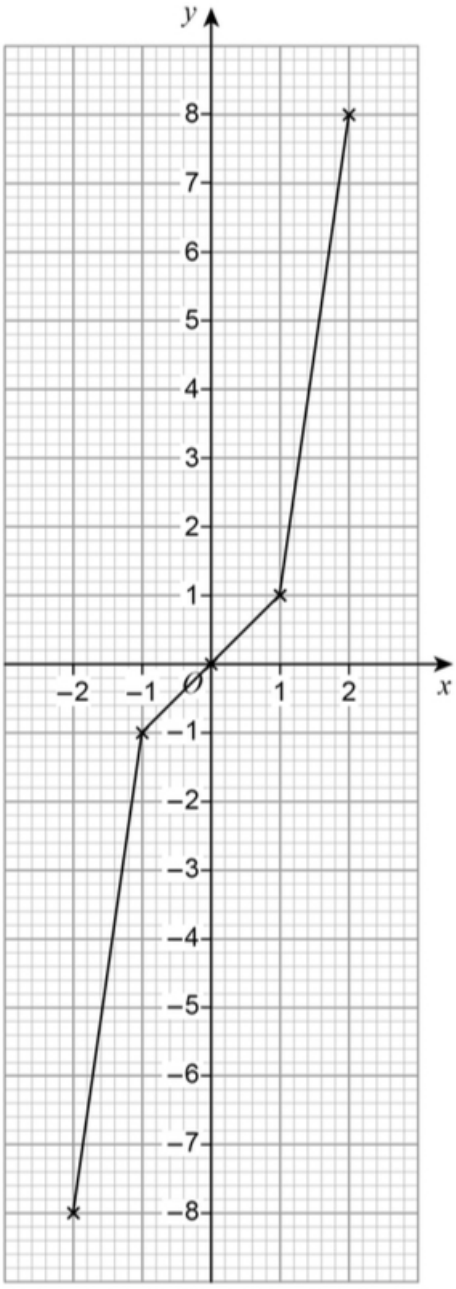


Not drawn accurately

Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos x = \tfrac{9}{10}$$
$$\Rightarrow x = 25.841\,932\,76 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 25.8^\circ \text{ (3 sf)}}}.$$

12. Lewis wants to draw the graph $y = x^3$ for values of x from -2 to 2. (1)
Here is his graph.



Make **one** criticism of his graph.

Solution

E.g., he has constructed the graph using straight lines, it should be curved, etc.

13. The probability of Heads when a biased coin is thrown is 0.6. (1)

The coin is thrown 500 times.

Circle the expected number of Tails.

20 200 250 300

Solution

$$\begin{aligned}\text{Expected number} &= (1 - 0.6) \times 500 \\ &= 0.4 \times 500 \\ &= 200\end{aligned}$$

so

20 $\underline{\underline{200}}$ 250 300

14. The mean mass of a squad of 19 hockey players is 82 kg. (3)

A player of mass 93 kg joins the squad.

Work out the mean mass of the squad now.

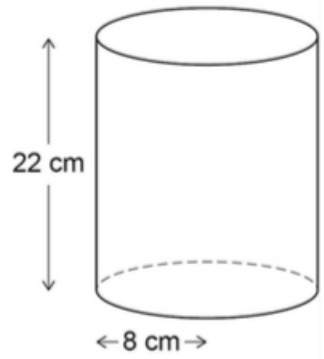
Solution

$$\begin{aligned}\text{Mean mass} &= \frac{(19 \times 82) + 93}{19 + 1} \\ &= \frac{1\,558 + 93}{20} \\ &= \frac{1\,651}{20} \\ &= \underline{\underline{82.55 \text{ kg}}}.\end{aligned}$$

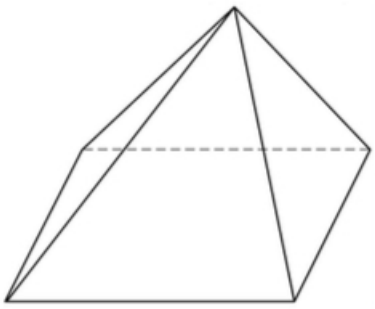
15. A company makes two types of lampshade using fabric on wire frames. (5)

Lampshade A

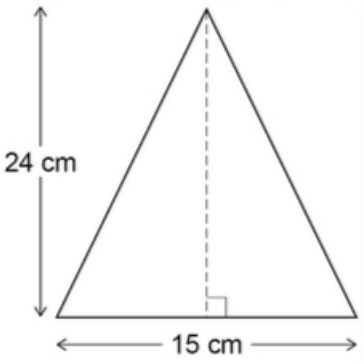
Fabric is used to make the curved surface of a cylinder.
The cylinder has radius 8 cm and height 22 cm.



Lampshade B
Fabric is used to make the four triangular faces of a pyramid.



Each triangular face has base 15 cm and perpendicular height 24 cm.



Cost of fabric	£400 per square metre
Other costs for A	£3.50 per lampshade
Other costs for B	£7.50 per lampshade

Work out the ratio

$$\text{cost of one lampshade } A : \text{cost of one lampshade } B.$$

Give your answer in the form $n : 1$.

Solution

Lampshade A:

$$\begin{aligned}\text{Fabric} &= 2 \times \pi \times 0.08 \times 0.22 \\ &= 0.035\,2\pi \text{ m}^2\end{aligned}$$

and

$$\begin{aligned}\text{cost of fabric} &= 400 \times 0.035\,2\pi \\ &= 14.08\pi \text{ m}^2\end{aligned}$$

Lampshade B:

$$\begin{aligned}\text{Fabric} &= 4 \times \tfrac{1}{2} \times 0.15 \times 0.24 \\ &= 0.072 \text{ m}^2\end{aligned}$$

and

$$\begin{aligned}\text{cost of fabric} &= 400 \times 0.072 \\ &= 28.8 \text{ m}^2.\end{aligned}$$

Hence,

$$\begin{aligned}&\text{cost of one lampshade } A : \text{cost of one lampshade } B \\ = \;& (14.08\pi + 3.5) : (28.8 + 7.5) \\ = \;& 47.733\ldots : 36.3 \\ = \;& 1.314\,975\,883 \text{ (FCD)} : 1 \\ = \;& \underline{\underline{1.31 \text{ (3 sf)} : 1}}.\end{aligned}$$

16. In a running club there are 50 females and 80 males. (4)

- If a female is chosen at random, the probability she has blue eyes is 0.38.
- If a male is chosen at random, the probability he has blue eyes is 0.6.

One person is chosen at random.

Show that the probability the person has blue eyes is **more than** 0.5.

Solution

$$\begin{aligned}\mathrm{P}(\text{blue eyes}) &= \mathrm{P}(\text{man, blue eyes}) + \mathrm{P}(\text{woman, blue eyes}) \\ &= (\tfrac{80}{130} \times 0.6) + (\tfrac{50}{130} \times 0.38) \\ &= \tfrac{24}{65} + \tfrac{19}{130} \\ &= \tfrac{67}{130} \\ &> 0.515\ldots;\end{aligned}$$

hence, the probability the person has blue eyes is more than 0.5.

17. (1)

$$w = \frac{3}{5\sqrt{x}}.$$

Circle the expression for w^2.

$$\frac{6}{10x^2} \quad \frac{9}{25x^2} \quad \frac{6}{10x} \quad \frac{9}{25x}$$

Solution

Well,

$$\left(\frac{3}{5\sqrt{x}}\right)^2 = \frac{9}{25x}$$

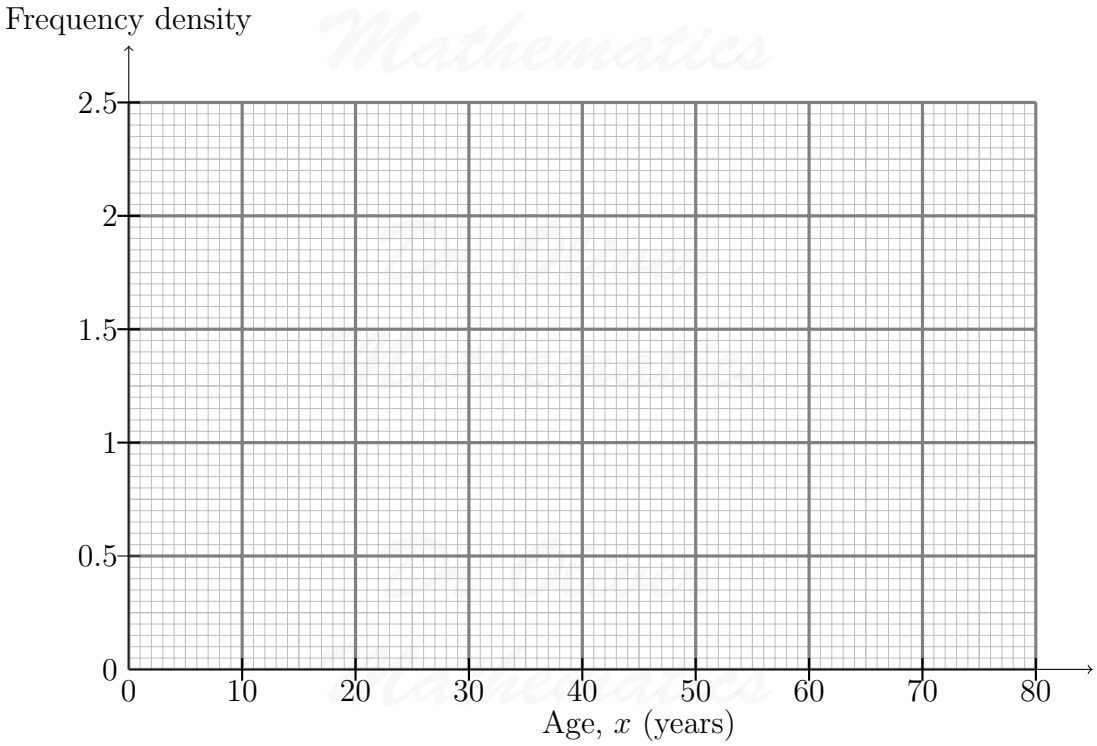
so

$$\frac{6}{10x^2} \quad \frac{9}{25x^2} \quad \frac{6}{10x} \quad \underline{\underline{\frac{9}{25x}}}$$

18. Here is some information about the ages of people at a concert. (3)

Age, x (years)	Frequency
$10 \leqslant x < 15$	8
$15 \leqslant x < 25$	24
$25 \leqslant x < 40$	30
$40 \leqslant x < 75$	39

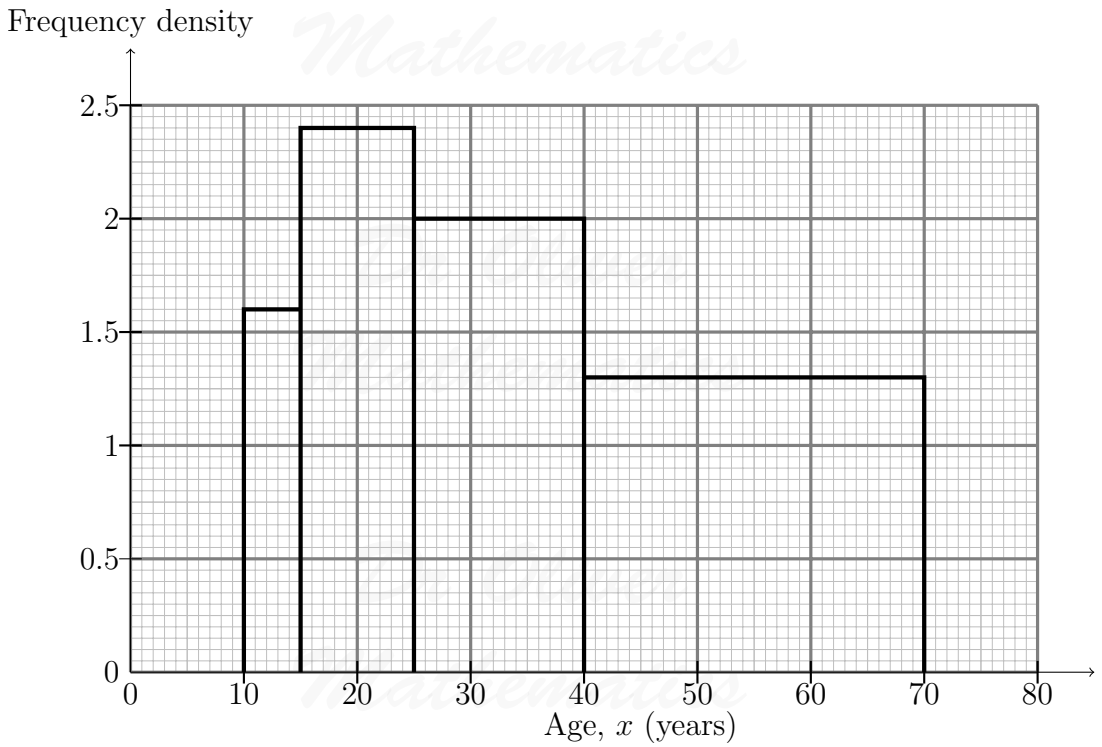
Draw a histogram to represent the information.



Solution

Age, x (years)	Frequency	Width	Frequency density
$10 \leqslant x < 15$	8	5	$\frac{8}{5} = 1.6$
$15 \leqslant x < 25$	24	10	$\frac{24}{10} = 2.4$
$25 \leqslant x < 40$	30	15	$\frac{30}{15} = 2$
$40 \leqslant x < 70$	39	30	$\frac{39}{35} = 1.3$

so



19. The length of a roll of ribbon is 30 metres, correct to the nearest half-metre. (3)

A piece of length 5.8 metres, correct to the nearest 10 centimetres, is cut from the roll.

Work out the maximum possible length of ribbon left on the roll.

Solution

Well,

$$29.75 \leqslant \text{length of a roll} < 30.25$$

and

$$5.75 \leqslant \text{piece} < 5.85.$$

Finally,

$$\begin{aligned} \text{length of a roll} - \text{piece} &= 30.25 - 5.75 \\ &= \underline{\underline{24.5 \text{ metres}}}. \end{aligned}$$

20. Curve P has equation (3)

$$y = 2(x-1)^2 - 5.$$

Curve Q is a reflection in the y-axis of curve P.

Work out the equation of curve Q.

Give your answer in the form

$$y = ax^2 + bx + c,$$

where a, b, and c are integers.

Solution

Well, we go

$$y = \mathrm{f}(x) \to y = \mathrm{f}(-x)$$

and the equation of curve Q is

$$y = 2(x+1)^2 - 5$$

$\times$	x	$+1$
x	x^2	$+x$
$+1$	$+x$	$+1$

$$\begin{aligned} &= 2(x^2 + 2x + 1) - 5 \\ &= 2x^2 + 4x + 2 - 5 \\ &= \underline{\underline{2x^2 + 4x - 3}}; \end{aligned}$$

hence, $\underline{\underline{a = 2}}$, $\underline{\underline{b = 4}}$, and $\underline{\underline{c = -3}}$.

21. Priya and Joe travel the same 16.8 km route. (5)

- Priya starts at 9.00 am and walks at a constant speed of 6 km/h.
- Joe starts at 9.30 am and runs at a constant speed.

Joe overtakes Priya at 10.20 am.

At what time does Joe finish the route?

Solution

Let the speed of Joe be v km/h.
50 minutes is the time for Joe to pass Priya:

$$6 \times \frac{4}{3} = v \times \frac{5}{6} \Rightarrow v = \frac{6 \times \frac{4}{3}}{\frac{5}{6}}$$
$$\Rightarrow v = 9.6 \text{ km/h}.$$

He takes

$$\frac{16.8}{9.6} = 1.75 \text{ hours},$$

and, hence, Joe finishes the route is

$$9.30 + 1 \text{ h } 45 \text{ mins} = \underline{\underline{11.15 \text{ am}}}.$$

22. An approximate solution to an equation is found using the iterative formula

$$x_{n+1} = \frac{(x_n)^3 - 2}{10},$$

with $x_1 = -1$.

(a) Work out the values of x_2 and x_3. (2)

Solution

$$x_2 = \frac{(-1)^3 - 2}{10}$$
$$= \frac{-1 - 2}{10}$$
$$= \underline{\underline{-0.3}}$$
$$x_3 = \frac{(-0.3)^3 - 2}{10}$$
$$= \frac{-0.027 - 2}{10}$$
$$= \underline{\underline{-0.202\,7}}.$$

(b) Work out the solution to 5 decimal places. (1)

Solution

Well,

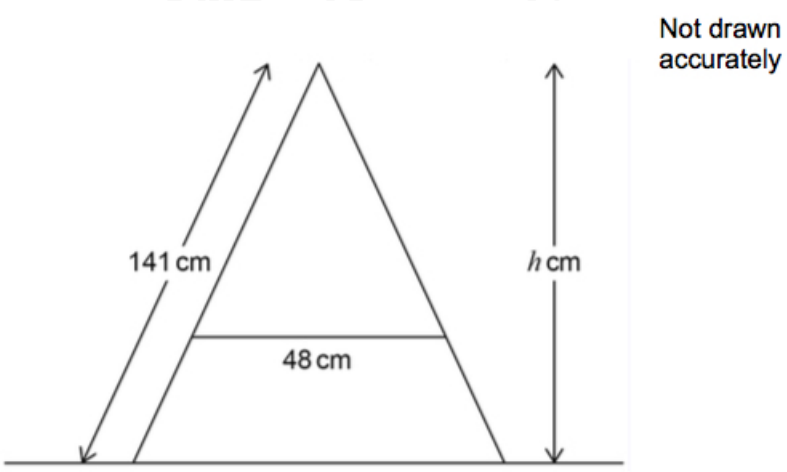
$$\begin{aligned} x_4 &= -0.200\,000\,887 \text{ (FCD)} \\ x_5 &= -0.200\,800\,010\,6 \text{ (FCD)} \\ x_6 &= -0.200\,809\,638\,6 \text{ (FCD)} \\ x_6 &= -0.200\,809\,755 \text{ (FCD)}, \end{aligned}$$

and so the solution is $\underline{\underline{-0.200\,81 \text{ (5 dp)}}}$.

23. The diagram shows the side view of a step ladder with a horizontal strut of length 48 cm. (5)

The strut is one third of the way up the ladder.

The symmetrical cross section of the ladder shows two similar triangles.



Work out the vertical height, h cm, of the ladder.

Solution

Let x cm be the separation of the ends. Now, similar triangles:

$$\begin{aligned} 48 : x := \tfrac{2}{3}h : h &\Rightarrow \frac{48}{x} = \frac{\frac{2}{3}}{1} \\ &\Rightarrow \frac{x}{48} = \frac{1}{\frac{2}{3}} \\ &\Rightarrow x = 48 \times \tfrac{3}{2} \\ &\Rightarrow x = 72. \end{aligned}$$

Divide the triangle into two pieces:

$$\begin{aligned} \text{opp}^2 + \text{adj}^2 = \text{hyp}^2 &\Rightarrow 36^2 + h^2 = 141^2 \\ &\Rightarrow h^2 = 141^2 - 36^2 \\ &\Rightarrow h^2 = 18\,585 \\ &\Rightarrow h = 136.326813\,2 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{h = 136 \text{ cm (3 sf)}}}. \end{aligned}$$

24. A sphere has radius $2x$ cm. (4)

A cone has

- radius $3x$ cm and
- perpendicular height h cm.

The sphere and the cone have the same volume.

Volume of a sphere = $\frac{4}{3}\pi r^3$		where r is the radius
Volume of a cone	= $\frac{1}{3}\pi r^2 h$	where r is the radius and h is the perpendicular height

Work out

radius of cone : perpendicular height of cone.

Give your answer in the form $a : b$, where a and b are integers.

Solution

Well,

$$\begin{aligned}\text{volume of the sphere} &= \tfrac{4}{3} \times \pi \times (2x)^3 \\ &= \tfrac{32}{3}x^3\pi\end{aligned}$$

and

$$\begin{aligned}\text{volume of the cone} &= \tfrac{1}{3} \times \pi \times (3x)^2 \times h \\ &= 3hx^2\pi.\end{aligned}$$

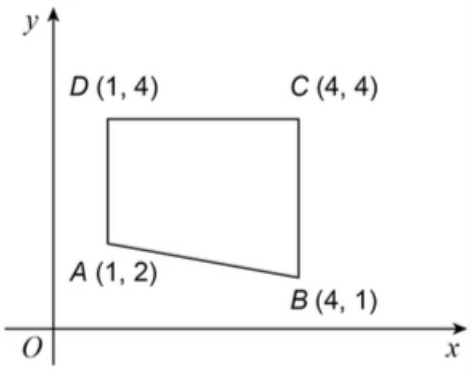
Now, the sphere and the cone have the same volume so

$$\begin{aligned}\tfrac{32}{3}x^3\pi = 3hx^2\pi &\Rightarrow \tfrac{32}{3}x = 3h \\ &\Rightarrow \tfrac{32}{9}x = h.\end{aligned}$$

Next,

$$\begin{aligned}\text{radius of cone : perpendicular height of cone} &\Rightarrow 3x = \tfrac{32}{9}x \\ &\Rightarrow \underline{\underline{27 = 32}}.\end{aligned}$$

25. $ABCD$ is a quadrilateral. (1)



Not drawn accurately

The quadrilateral is reflected in the line $x = 4$.

Which vertices are invariant?
Circle your answer.

A and D C and D B and C B and D

Solution

A and D $\quad$ C and D $\quad$ $\underline{\underline{B \text{ and } C}}$ $\quad$ B and D

26. (4)

$$\mathrm{f}(x) = \frac{2x+3}{x-4}.$$

Work out $\mathrm{f}^{-1}(x)$.

Solution

$$\begin{aligned} y = \frac{2x+3}{x-4} &\Rightarrow y(x-4) = 2x+3 \\ &\Rightarrow xy - 4y = 2x+3 \\ &\Rightarrow xy - 2x = 3+4y \\ &\Rightarrow x(y-2) = 3+4y \\ &\Rightarrow x = \frac{3+4y}{y-2} \end{aligned}$$

and

$$\underline{\underline{\mathrm{f}^{-1}(x) = \frac{3+4x}{x-2}.}}$$

27. The line

$$y = 3x + p$$

and the circle

$$x^2 + y^2 = 53$$

intersect at points A and B.

p is a positive integer.

(a) Show that the x-coordinates of points A and B satisfy the equation (3)

$$10x^2 + 6px + p^2 - 53 = 0.$$

Solution
Well,

$\times$	$3x$	$+p$
$3x$	$9x^2$	$+3px$
$+p$	$+3px$	$+p^2$

and, inserting the linear equation into the non-linear equation one:

$$\begin{aligned} x^2 + y^2 = 53 &\Rightarrow x^2 + (3x + p)^2 = 53 \\ &\Rightarrow x^2 + (9x^2 + 6px + p^2) = 53 \\ &\Rightarrow \underline{\underline{10x^2 + 6px + p^2 - 53 = 0}}, \end{aligned}$$

as required.

The coordinates of A are $(2, 7)$.

(b) Work out the coordinates of B. (5)
You **must** show your working.

Solution
Well,

$$\begin{aligned} x = 2, y = 7 &\Rightarrow 7 = 3(2) + p \\ &\Rightarrow 7 = 6 + p \\ &\Rightarrow p = 1 \end{aligned}$$

and

$$\begin{aligned} 10x^2 + 6px + p^2 - 53 = 0 &\Rightarrow 10x^2 + 6(1)x + (1^2) - 53 = 0 \\ &\Rightarrow 10x^2 + 6x - 52 = 0 \\ &\Rightarrow 2(5x^2 + 3x - 26) = 0 \end{aligned}$$

$$\left.\begin{array}{rc} \text{add to:} & +3 \\ \text{multiply to:} & (+5) \times (-26) = -130 \end{array}\right\} +13, -10$$

e.g.,

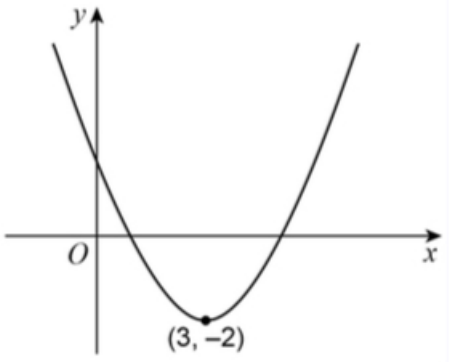
$$\Rightarrow 2[5x^2 + 13x - 10x - 26] = 0$$
$$\Rightarrow 2[x(5x + 13) - 2(5x + 13)] = 0$$
$$\Rightarrow 2(x - 2)(5x + 13) = 0$$
$$\Rightarrow x - 2 = 0 \text{ or } 5x + 13 = 0$$
$$\Rightarrow x = 2 \text{ or } x = -\tfrac{13}{5}.$$

Now,

$$x = -\tfrac{13}{5} \Rightarrow y = -\tfrac{34}{5}$$

so $\underline{\underline{B(-2\tfrac{3}{5}, -6\tfrac{4}{5})}}$.

28. Here is a sketch of a quadratic curve. (1)
The turning point is $(3, -2)$.



Not drawn accurately

Circle the correct statement about the gradient of the curve for $x < 3$.

gradient is positive | gradient is negative

gradient is zero | gradient could be any value

Solution

gradient is positive | $\underline{\underline{\text{gradient is negative}}}$

gradient is zero | gradient could be any value