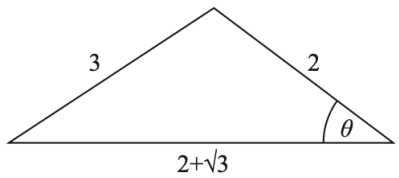
Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2011 November Paper 2 Variant 1: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Solve the equation (3)
$$|4x - 5| = 21.$$

2. Given that the straight line (4)
$$y = 3x + c$$
is a tangent to the curve
$$y = x^2 + 9x + k,$$
express k in terms of c.

3. Without using a calculator, find the value of $\cos\theta$, giving your answer in the form (5)
$$\frac{a + b\sqrt{3}}{c},$$
where a, b, and c are integers.



4. (a) Given that (2)
$$y = \frac{1}{x^2 + 3},$$
show that
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{kx}{(x^2 + 3)^2},$$
where k is a constant to be found.

(b) Hence find (3)

$$\int \frac{6x}{(x^2+3)^2}\,\mathrm{d}x$$

and evaluate

$$\int_1^3 \frac{6x}{(x^2+3)^2}\,\mathrm{d}x.$$

5. (a) The functions f and g are defined, for $x \in \mathbb{R}$, by (2)

$$\begin{aligned} \mathrm{f} &: x \mapsto 2x+3, \\ \mathrm{g} &: x \mapsto x^2-1. \end{aligned}$$

Find $\mathrm{f\,g}(4)$.

(b) The functions h and k are defined, for $x > 0$, by

$$\begin{aligned} \mathrm{h} &: x \mapsto x+4, \\ \mathrm{k} &: x \mapsto \sqrt{x}. \end{aligned}$$

Express each of the following in terms of h and k.

(i) $x \mapsto \sqrt{x+4}$, (1)

(ii) $x \mapsto x+8$, (1)

(iii) $x \mapsto x^2-4$. (2)

6. **Solutions to this question by accurate drawing will not be accepted.** (6)

The points $A(1,4)$, $B(3,8)$, $C(13,13)$, and D are the vertices of a trapezium in which AB is parallel to DC and angle BAD is 90°.

Find the coordinates of D.

7. (a) Given that (3)

$$\tan x = p,$$

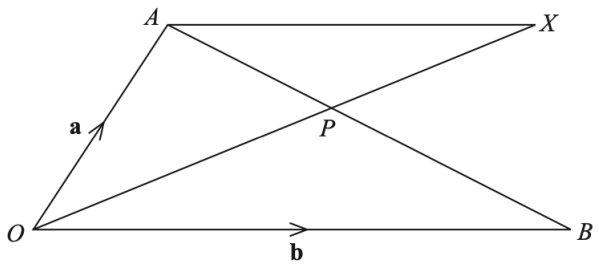
find an expression, in terms of p, for $\mathrm{cosec}^2 x$.

(b) Prove that (4)

$$(1+\sec\theta)(1-\cos\theta) \equiv \sin\theta\tan\theta.$$

8. In the diagram,

- $\overrightarrow{OA} = \mathbf{a}$,
- $\overrightarrow{OB} = \mathbf{b}$, and
- $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$.



(a) Given that $\overrightarrow{OX} = \mu\overrightarrow{OP}$, where μ is a constant, express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$, and $\mathbf{b}$. (3)

(b) Given also that $\overrightarrow{AX} = \lambda\overrightarrow{OB}$, where λ is a constant, use a vector method to find the value of μ and of λ. (5)

9. The table shows experimental values of two variables x and y.

x	1	2	3	4	5
y	3.40	2.92	2.93	3.10	3.34

It is known that x and y are related by the equation

$$y = \frac{a}{\sqrt{x}} + bx,$$

where a and b are constants.

(a) Complete the following table. (1)

$x\sqrt{x}$	
$y\sqrt{x}$	

(b) Plot $y\sqrt{x}$ against $x\sqrt{x}$ and draw a straight line graph. (2)

(c) Use your graph to estimate the value of a and of b. (3)

(d) Estimate the value of y when x is 1.5. (1)

10. It is given that

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}.$$

(a) Find $2\mathbf{A} - \mathbf{B}$. (2)

(b) Find $\mathbf{BA}$. (2)

(c) Find the inverse matrix, $\mathbf{A}^{-1}$. (2)

(d) Use your answer to part (c) to solve the simultaneous equations (2)

$$3x + 2y = 23$$
$$x - 5y = 19.$$

11. (a) (i) Solve (3)

$$\frac{5^{2x+3}}{25^{2x}} = \frac{25^{2-x}}{125^{x}}.$$

(ii) Solve (4)

$$\log_{10} y + \log_{10}(y - 15) = 2.$$

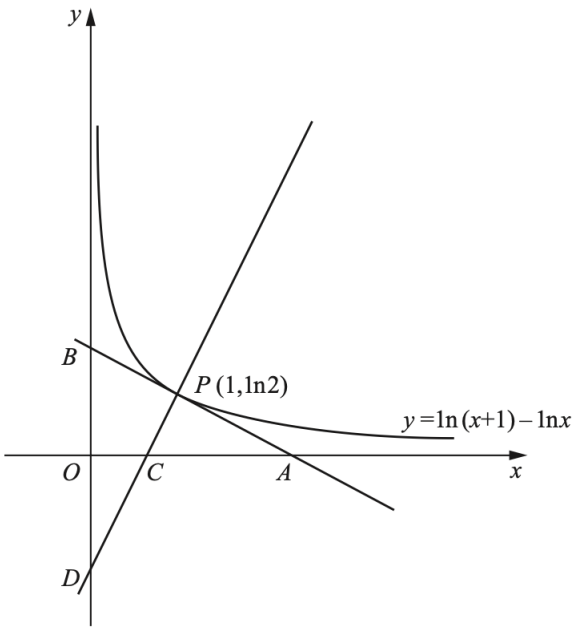
(b) Without using a calculator, and showing each stage of your working, find the value of (3)

$$2\log_{12} 4 - \tfrac{1}{2}\log_{12} 81 + 4\log_{12} 3.$$

EITHER

12. The diagram shows part of the curve

$$y = \ln(x + 1) - \ln x.$$



- The tangent to the curve at the point $P(1, \ln 2)$ meets the x-axis at A and the y-axis at B.
- The normal to the curve at P meets the x-axis at C and the y-axis at D.

(a) Find, in terms of $\ln 2$, the coordinates of A, B, C, and D. (8)

(b) Given that (3)

$$\frac{\text{area of triangle } BPD}{\text{area of triangle } APC} = \frac{1}{k},$$

express k in terms of $\ln 2$.

OR

13. A curve has equation

$$y = x\mathrm{e}^x.$$

The curve has a stationary point at P.

(a) Find, in terms of e, the coordinates of P and determine the nature of this stationary point. (5)

The normal to the curve at the point $Q(1, \mathrm{e})$ meets the x-axis at R and the y-axis at S.

(b) Find, in terms of e, the area of triangle ORS, where O is the origin. (6)