

**Dr Oliver Mathematics**  
**Cambridge O Level Additional Mathematics**  
**2012 November Paper 2 Variant 1: Calculator**  
**2 hours**

The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You must write down all the stages in your working.

1. Solve the equation

$$|7x + 5| = |3x - 13|.$$

(4)

**Solution**

Well,

$$7x + 5 = 0 \Rightarrow x = -\frac{5}{7}$$

and

$$3x - 13 = 0 \Rightarrow x = \frac{13}{3}.$$

$$\underline{-\frac{5}{7} < x < \frac{13}{3} :}$$

$$7x + 5 = 3x - 13 \Rightarrow 4x = -18$$

$$\Rightarrow \underline{\underline{x = -4\frac{1}{2}}}.$$

$$\underline{x < -\frac{5}{7} \text{ or } x > \frac{13}{3} :}$$

$$7x + 5 = -(3x - 13) \Rightarrow 7x + 5 = -3x + 13$$

$$\Rightarrow 10x = 8$$

$$\Rightarrow \underline{\underline{x = \frac{4}{5}}}.$$

2. The total surface area,  $A \text{ cm}^2$ , of a solid cylinder with radius  $r \text{ cm}$  and height  $5 \text{ cm}$  is given by

$$A = 2\pi r^2 + 10\pi r.$$

(4)

Given that  $r$  is increasing at a rate of  $0.2 \text{ cm s}^{-1}$ , find the rate of increase of  $A$  when  $r = 6$ .

**Solution**

Well,

$$A = 2\pi r^2 + 10\pi r \Rightarrow \frac{dA}{dr} = 4\pi r + 10\pi$$

and

$$\begin{aligned} \frac{dA}{dr} &= \frac{dA}{dt} \times \frac{dr}{dt} \Rightarrow 34\pi = \frac{dA}{dt} \times 0.2 \\ &\Rightarrow \underline{\underline{\frac{dA}{dt} = 6.8 \text{ cm}^2 \text{ s}^{-1}}}. \end{aligned}$$

3. Solve the inequality

$$4x(4 - x) > 7.$$

(4)

**Solution**

Now,

$$\begin{aligned} 4x(4 - x) > 7 &\Rightarrow 16x - 4x^2 > 7 \\ &\Rightarrow 4x^2 - 16x + 7 < 0 \end{aligned}$$

$$\left. \begin{array}{l} \text{add to:} \quad -16 \\ \text{multiply to: } (+4) \times (+7) = +28 \end{array} \right\} -14, -2$$

e.g.,

$$\begin{aligned} &\Rightarrow 4x^2 - 14x - 2x + 7 < 0 \\ &\Rightarrow 2x(2x - 7) - 1(2x - 7) < 0 \\ &\Rightarrow (2x - 1)(2x - 7) < 0. \end{aligned}$$

We need a ‘table of signs’:

|                    | $x < \frac{1}{2}$ | $x = \frac{1}{2}$ | $\frac{1}{2} < x < \frac{7}{2}$ | $x = \frac{7}{2}$ | $x > \frac{7}{2}$ |
|--------------------|-------------------|-------------------|---------------------------------|-------------------|-------------------|
| $2x - 1$           | −                 | 0                 | +                               | +                 | +                 |
| $2x - 7$           | −                 | −                 | −                               | 0                 | +                 |
| $(2x - 1)(2x - 7)$ | +                 | 0                 | −                               | 0                 | +                 |

Hence,

$$\underline{\underline{\frac{1}{2} < x < \frac{7}{2}}}.$$

4. (a) Find the coefficient of  $x^5$  in the expansion of  $(2 - x)^8$ . (2)

**Solution**

Reading part (b), we want the coefficient of  $x^4$  and the coefficient of  $x^5$ :

$$\begin{aligned}(2 - x)^8 &= \dots + \binom{8}{4}(2)^4(-x)^4 + \binom{8}{5}(2)^3(-x)^5 + \dots \\ &= \dots + 1\,120x^4 - 448x^5 + \dots;\end{aligned}$$

hence, the coefficient of  $x^5$  is -448.

- (b) Find the coefficient of  $x^5$  in the expansion of  $(1 + 2x)(2 - x)^8$ . (3)

**Solution**

Well,

|          |              |           |
|----------|--------------|-----------|
| $\times$ | $1\,120x^4$  | $-448x^5$ |
| $1$      | $\dots$      | $-448x^5$ |
| $2x$     | $+2\,240x^4$ | $\dots$   |

and the coefficient of  $x^5$  is

$$-448 + 2\,240 = \underline{\underline{1\,792}}.$$

5. A 4-digit number is formed by using four of the six digits 2, 3, 4, 5, 6, and 8; no digit may be used more than once in any number.

How many different 4-digit numbers can be formed if

- (a) there are no restrictions, (2)

**Solution**

$$6 \times 5 \times 4 \times 3 = \underline{\underline{360}}.$$

- (b) the number is even and more than 6 000? (3)

**Solution**

There are two choice for the first number (6 or 8) and three choices (2, 4, 6, or 8 - whichever is unchosen). So

$$2 \times 3 \times 4 \times 3 = \underline{\underline{72}}.$$

6. (a) Given that (2)

$$\frac{2^{x-3}}{8^{2y-3}} = 16^{x-y},$$

show that

$$3x + 2y = 6.$$

**Solution**

Well,

$$\begin{aligned} \frac{2^{x-3}}{8^{2y-3}} = 16^{x-y} &\Rightarrow \frac{2^{x-3}}{(2^3)^{2y-3}} = (2^4)^{x-y} \\ &\Rightarrow \frac{2^{x-3}}{2^{3(2y-3)}} = 2^{4(x-y)} \\ &\Rightarrow \frac{2^{x-3}}{2^{6y-9}} = 2^{4x-4y} \\ &\Rightarrow 2^{(x-3)-(6y-9)} = 2^{4x-4y} \\ &\Rightarrow 2^{x-6y+6} = 2^{4x-4y} \end{aligned}$$

take  $\log_2$ :

$$\begin{aligned} &\Rightarrow x - 6y + 6 = 4x - 4y \\ &\Rightarrow \underline{\underline{3x + 2y = 6}} \quad (1), \end{aligned}$$

as required.

(b) Given also that

(4)

$$\frac{5^y}{125^{x-2}} = 25,$$

find the value of  $x$  and of  $y$ .

**Solution**

Now,

$$\begin{aligned}\frac{5^y}{125^{x-2}} = 25 &\Rightarrow \frac{5^y}{(5^3)^{x-2}} = 5^2 \\ &\Rightarrow \frac{5^y}{5^{3(x-2)}} = 5^2 \\ &\Rightarrow \frac{5^y}{5^{3x-6}} = 5^2 \\ &\Rightarrow 5^{y-(3x-6)} = 5^2 \\ &\Rightarrow 5^{-3x+y+6} = 5^2\end{aligned}$$

take  $\log_5$ :

$$\begin{aligned}&\Rightarrow -3x + y + 6 = 2 \\ &\Rightarrow -3x + y = -4 \quad (2).\end{aligned}$$

Do (1) + (2):

$$3y = 2 \Rightarrow \underline{\underline{y = \frac{2}{3}}}$$

and insert into (1):

$$\begin{aligned}3x + 2\left(\frac{2}{3}\right) &= 6 \Rightarrow 3x + \frac{4}{3} = 6 \\ &\Rightarrow 3x = \frac{14}{3} \\ &\Rightarrow \underline{\underline{x = \frac{14}{9}}}.\end{aligned}$$

[Check:  $-3\left(\frac{14}{9}\right) + \frac{2}{3} = -4 \checkmark$ ]

7. (a) Find

(2)

$$\frac{d}{dx}(\tan 4x).$$

**Solution**

$$\frac{d}{dx}(\tan 4x) = \underline{\underline{4 \sec^2 4x}}.$$

(b) Hence find

(3)

$$\int (1 + \sec^2 4x) \, dx.$$

**Solution**

$$\int (1 + \sec^2 4x) \, dx = \underline{\underline{x + \frac{1}{4} \tan 4x + c.}}$$

(c) Hence show that

(2)

$$\int_{-\frac{1}{16}\pi}^{\frac{1}{16}\pi} (1 + \sec^2 4x) \, dx = k(\pi + 4),$$

where  $k$  is a constant to be found.

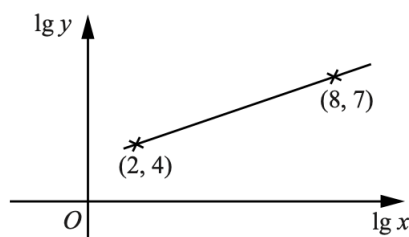
**Solution**

Well,

$$\begin{aligned} \int_{-\frac{1}{16}\pi}^{\frac{1}{16}\pi} (1 + \sec^2 4x) \, dx &= \left[ x + \frac{1}{4} \tan 4x \right]_{x=-\frac{1}{16}\pi}^{\frac{1}{16}\pi} \\ &= \left( \frac{1}{16}\pi + \frac{1}{4} \right) - \left( -\frac{1}{16}\pi - \frac{1}{4} \right) \\ &= \frac{1}{8}\pi + \frac{1}{2} \\ &= \underline{\underline{\frac{1}{8}(\pi + 4)}}; \end{aligned}$$

hence,  $\underline{\underline{k = \frac{1}{8}}}$ .

8. The variables  $x$  and  $y$  are related in such a way that when  $\log_{10} y$  is plotted against  $\log_{10} x$  a straight line graph is obtained as shown in the diagram.



The line passes through the points (2, 4) and (8, 7).

- (a) Express  $y$  in terms of  $x$ , giving your answer in the form

(5)

$$y = ax^b,$$

where  $a$  and  $b$  are constants.

**Solution**

Well,

$$\begin{aligned} m &= \frac{7-4}{8-2} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

and the equation is

$$\begin{aligned} \log_{10} y - 4 &= \frac{1}{2}(\log_{10} x - 2) \Rightarrow \log_{10} y - 4 = \frac{1}{2} \log_{10} x - 1 \\ &\Rightarrow \log_{10} y - \frac{1}{2} \log_{10} x = 3 \\ &\Rightarrow \log_{10} y - \log_{10} x^{\frac{1}{2}} = 3 \\ &\Rightarrow \log_{10} \left( \frac{y}{x^{\frac{1}{2}}} \right) = 3 \\ &\Rightarrow \frac{y}{x^{\frac{1}{2}}} = 10^3 \\ &\Rightarrow \underline{\underline{y = 1\,000\,x^{\frac{1}{2}}}}. \end{aligned}$$

Another method of drawing a straight line graph for the relationship

$$y = ax^b,$$

found in part (a), involves plotting  $\log_{10} x$  on the horizontal axis and  $\log_{10}(y^2)$  on the vertical axis.

For this straight line graph what is

- (b) the gradient,

(1)

**Solution**

Well,

$$\begin{aligned}y &= 1\,000 x^{\frac{1}{2}} \Rightarrow \log_{10} y = \log_{10}(1\,000 x^{\frac{1}{2}}) \\&\Rightarrow \log_{10} y = \log_{10} 1\,000 + \log_{10} x^{\frac{1}{2}} \\&\Rightarrow \log_{10} y = \log_{10} 1\,000 + \frac{1}{2} \log_{10} x \\&\Rightarrow 2 \log_{10} y = 2 \log_{10} 1\,000 + \log_{10} x \\&\Rightarrow \log_{10} y^2 = 2 \log_{10} 1\,000 + \log_{10} x.\end{aligned}$$

The gradient is 1.

(c) the intercept on the vertical axis?

(1)

**Solution**

The intercept is

$$\begin{aligned}2 \log_{10} 1\,000 &= 2 \log_{10} 10^3 \\&= 6 \log_{10} 10 \\&= \underline{6}.\end{aligned}$$

9. A plane, whose speed in still air is  $420 \text{ kmh}^{-1}$ , travels directly from  $A$  to  $B$ , a distance of  $1\,000 \text{ km}$ .

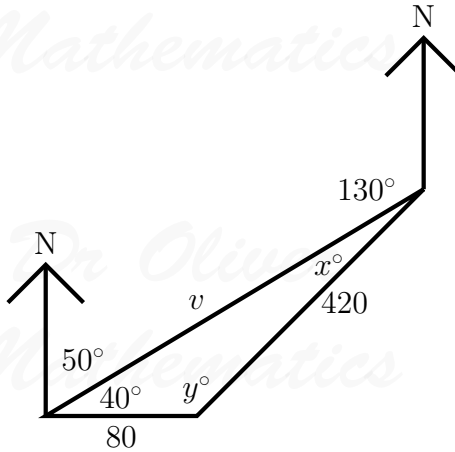
The bearing of  $B$  from  $A$  is  $230^\circ$  and there is a wind of  $80 \text{ kmh}^{-1}$  from the east.

(a) Find the bearing on which the plane was steered.

(4)

**Solution**

We need a picture:



Sine rule:

$$\frac{\sin x^\circ}{80} = \frac{\sin 40^\circ}{420} \Rightarrow \sin x^\circ = \frac{80 \sin 40^\circ}{420}$$

$$\Rightarrow x = 7.032\,696\,713 \text{ (FCD).}$$

Hence, the bearing on which the plane was steered is

$$360 - (130 + x) = 222.967\,303\,7 \text{ (FCD)}$$

$$= \underline{\underline{223^\circ \text{ (3 sf)}}}.$$

(b) Find the time taken for the journey.

(4)

### Solution

Sine rule:

$$\frac{v}{\sin y^\circ} = \frac{420}{\sin 40^\circ} \Rightarrow \frac{v}{\sin[180 - (40 + x)]^\circ} = \frac{420}{\sin 40^\circ}$$

$$\Rightarrow v = \frac{420 \sin 132.967 \dots^\circ}{\sin 40^\circ}$$

$$\Rightarrow v = 478.123\,661\,7 \text{ (FCD).}$$

Finally,

$$\text{time taken} = \frac{1\,000}{478.123 \dots}$$

$$= 2.091\,509\,122 \text{ (FCD)}$$

$$= \underline{\underline{2.09 \text{ hours (3 sf)}}}.$$

10. The acceleration,  $a \text{ ms}^{-2}$ , of a particle,  $t \text{ s}$  after passing through a fixed point  $O$ , is given by

$$a = 4 - 2t, \text{ for } t > 0.$$

The particle, which moves in a straight line, passes through  $O$  with a velocity of  $12 \text{ ms}^{-1}$ .

- (a) Find the value of  $t$  when the particle comes to instantaneous rest.

(5)

**Solution**

Well,

$$a = 4 - 2t \Rightarrow v = 4t - t^2 + c,$$

for some constant  $c$ . Now,

$$\begin{aligned} t = 0, v = 12 &\Rightarrow 12 = 0 - 0 + c \\ &\Rightarrow c = 12, \end{aligned}$$

and

$$v = 4t - t^2 + 12.$$

Next,

$$\begin{aligned} v = 0 &\Rightarrow 4t - t^2 + 12 = 0 \\ &\Rightarrow t^2 - 4t - 12 = 0 \end{aligned}$$

$$\begin{array}{l} \text{add to:} \quad -4 \\ \text{multiply to:} \quad -12 \end{array} \left. \vphantom{\begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array}} \right\} -6, +2$$

$$\begin{aligned} &\Rightarrow (t - 6)(t + 2) = 0 \\ &\Rightarrow t - 6 = 0 \text{ or } t + 2 = 0 \\ &\Rightarrow t = 6 \text{ or } t = -2. \end{aligned}$$

Hence, as  $t$  needs to be non-negative,  $t = 6$ .

- (b) Find the distance from  $O$  of the particle when it comes to instantaneous rest.

(3)

**Solution**

Well,

$$\begin{aligned}\text{distance} &= \int_0^6 (4t - t^2 + 12) \, dt \\ &= \left[ 2t^2 - \frac{1}{3}t^3 + 12t \right]_{t=0}^6 \\ &= (72 - 72 + 72) - (0 - 0 + 0) \\ &= \underline{\underline{72 \text{ m}}}.\end{aligned}$$

11. (a) Solve

$$4 \sin x + 9 \cos x = 0, \text{ for } 0^\circ < x < 360^\circ.$$

(3)

**Solution**

$$\begin{aligned}4 \sin x + 9 \cos x = 0 &\Rightarrow 4 \sin x = -9 \cos x \\ &\Rightarrow \tan x = -\frac{9}{4} \\ &\Rightarrow x = 113.962\,489, 293.962\,489 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 114, 294 \text{ (3 sf)}}}.\end{aligned}$$

(b) Solve

$$\operatorname{cosec} y - 1 = 12 \sin y, \text{ for } 0^\circ < y < 360^\circ.$$

(5)

**Solution**

$$\operatorname{cosec} y - 1 = 12 \sin y \Rightarrow \frac{1}{\sin y} - 1 = 12 \sin y$$

multiply by  $\sin y$ :

$$\begin{aligned}&\Rightarrow 1 - \sin y = 12 \sin^2 y \\ &\Rightarrow 12 \sin^2 y + \sin y - 1 = 0\end{aligned}$$

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array} \right\} \begin{array}{l} +1 \\ (+12) \times (-1) = -12 \end{array} + 4, -3$$

e.g.,

$$\begin{aligned} &\Rightarrow 12 \sin^2 y + 4 \sin y - 3 \sin -1 = 0 \\ &\Rightarrow 4 \sin y(3 \sin y + 1) - 1(3 \sin y + 1) = 0 \\ &\Rightarrow (4 \sin y - 1)(3 \sin y + 1) = 0 \\ &\Rightarrow 4 \sin y - 1 = 0 \text{ or } 3 \sin y + 1 = 0 \\ &\Rightarrow \sin y = \frac{1}{4} \text{ or } \sin y = -\frac{1}{3}. \end{aligned}$$

$\sin y = \frac{1}{4} :$

$$\begin{aligned} \sin y = \frac{1}{4} &\Rightarrow y = 14.477\,512\,19, 165.522\,487\,8 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{y = 14.5, 166 \text{ (3 sf)}}}. \end{aligned}$$

$\sin y = -\frac{1}{3} :$

$$\begin{aligned} \sin y = -\frac{1}{3} &\Rightarrow y = 199.471\,220\,6, 340.528\,779\,4 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{y = 199, 341 \text{ (3 sf)}}}. \end{aligned}$$

(c) Solve

$$3 \sec\left(\frac{1}{3}z\right) = 5, \text{ for } 0 < z < 6\pi \text{ radians.}$$

(4)

**Solution**

Well,

$$0 < z < 6\pi \Rightarrow 0 < \frac{1}{3}z < 2\pi$$

and

$$\begin{aligned} 3 \sec\left(\frac{1}{3}z\right) = 5 &\Rightarrow \sec\left(\frac{1}{3}z\right) = \frac{5}{3} \\ &\Rightarrow \cos\left(\frac{1}{3}z\right) = \frac{3}{5} \\ &\Rightarrow \frac{1}{3}z = 0.927\,295\,218, 5.355\,890\,089 \text{ (FCD)} \\ &\Rightarrow z = 2.781\,885\,654, 16.067\,670\,27 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{z = 2.78, 16.1 \text{ (3 sf)}}}. \end{aligned}$$

**EITHER**

12. The point  $A(0, 10)$  lies on the curve for which

$$\frac{dy}{dx} = e^{-\frac{1}{4}x}.$$

The point  $B$ , with  $x$ -coordinate  $-4$ , also lies on the curve.

- (a) Find, in terms of  $e$ , the  $y$ -coordinate of  $B$ .

(5)

**Solution**

Now,

$$\frac{dy}{dx} = e^{-\frac{1}{4}x} \Rightarrow y = -4e^{-\frac{1}{4}x} + c,$$

for some constant  $c$ . Next,

$$\begin{aligned} x = 0, y = 10 &\Rightarrow 10 = -4 + c \\ &\Rightarrow c = 14, \end{aligned}$$

and

$$y = -4e^{-\frac{1}{4}x} + 14.$$

Finally,

$$x = -4 \Rightarrow \underline{\underline{y = -4e + 14.}}$$

The tangents to the curve at the points  $A$  and  $B$  intersect at the point  $C$ .

- (b) Find, in terms of  $e$ , the  $x$ -coordinate of the point  $C$ .

(5)

**Solution**

Well,

$$x = 0 \Rightarrow \frac{dy}{dx} = 1$$

and the equation of the tangent at  $A$  is

$$y - 10 = (x - 0) \Rightarrow y = x + 10 \quad (1).$$

Next,

$$x = -4 \Rightarrow \frac{dy}{dx} = e$$

and the equation of the tangent at  $B$  is

$$\begin{aligned} y - (-4e + 14) &= e(x + 4) \Rightarrow y + 4e - 14 = ex + 4e \\ &\Rightarrow y = ex + 14 \quad (2). \end{aligned}$$

They intersect at

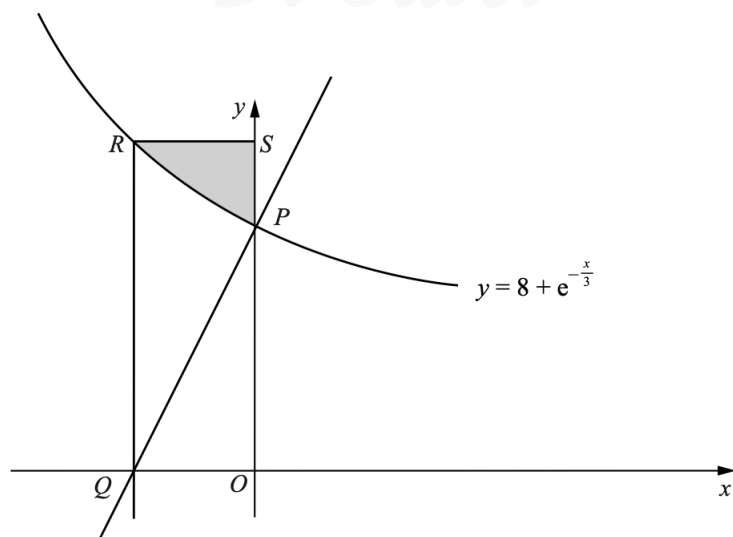
$$\begin{aligned}x + 10 &= ex + 14 \Rightarrow -4 = ex - x \\&\Rightarrow -4 = x(e - 1) \\&\Rightarrow x = -\frac{4}{e - 1}.\end{aligned}$$

**OR**

13. The diagram shows part of the curve

$$y = 8 + e^{-\frac{1}{3}x}$$

crossing the  $y$ -axis at  $P$ .



The normal to the curve at  $P$  meets the  $x$ -axis at  $Q$ .

(a) Find the coordinates of  $Q$ .

(4)

**Solution**

Well,

$$y = 8 + e^{-\frac{1}{3}x} \Rightarrow \frac{dy}{dx} = -\frac{1}{3}e^{-\frac{1}{3}x}$$

and

$$x = 0 \Rightarrow \frac{dy}{dx} = -\frac{1}{3}.$$

Now,

$$m_{\text{normal}} = 3$$

and

$$x = 0 \Rightarrow y = 9.$$

So the to the curve at  $P$  has the equation

$$y - 9 = 3(x - 0).$$

Finally,

$$\begin{aligned} y = 0 &\Rightarrow -9 = 3(x - 0) \\ &\Rightarrow x = -3; \end{aligned}$$

hence,  $Q(-3, 0)$ .

The line through  $Q$ , parallel to the  $y$ -axis, meets the curve at  $R$  and  $OQRS$  is a rectangle.

(b) Find

$$\int (8 + e^{-\frac{1}{3}x}) \, dx$$

(6)

and hence find the area of the shaded region.

**Solution**

Well,

$$\int (8 + e^{-\frac{1}{3}x}) \, dx = \underline{\underline{8x - 3e^{-\frac{1}{3}x} + c.}}$$

Now,

$$x = -3 \Rightarrow y = 8 + e;$$

so,  $R(-3, 8 + e)$  and  $S(0, 8 + e)$ .

Finally,

$$\begin{aligned}\text{area of the shaded region} &= \int_{-3}^0 [(8 + e) - (8 + e^{-\frac{1}{3}x})] \, dx \\ &= \int_{-3}^0 (e - e^{-\frac{1}{3}x}) \, dx \\ &= \left[ ex + 3e^{-\frac{1}{3}x} \right]_{x=-3}^0 \\ &= (0 + 3) - (-3e + 3e) \\ &= \underline{\underline{3}}.\end{aligned}$$