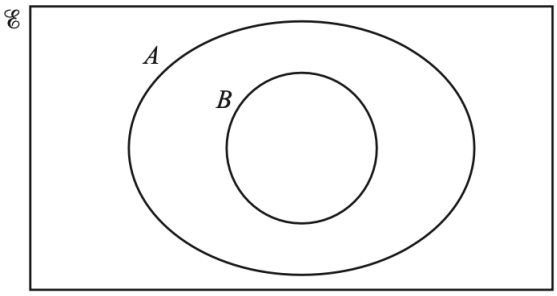
Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2013 June Paper 1 Variant 2: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. The Venn diagram shows the universal set $\mathscr{E}$, the set A, and the set B.



Given that

- $\mathrm{n}(B) = 5$,
- $\mathrm{n}(A') = 10$, and
- $\mathrm{n}(\mathscr{E}) = 26$,

find

(a) $\mathrm{n}(A \cap B)$, (1)

(b) $\mathrm{n}(A)$, (1)

(c) $\mathrm{n}(B' \cap A)$. (1)

2. A 4-digit number is to be formed from the digits 1, 2, 5, 7, 8, and 9.

Each digit may only be used once.

Find the number of different 4-digit numbers that can be formed if

(a) there are no restrictions, (1)

(b) the 4-digit numbers are divisible by 5, (2)

(c) the 4-digit number are divisible by 5 and are greater than 7 000. (2)

3. Show that (3)

$$(1 - \cos\theta - \sin\theta)^2 - 2(1 - \sin\theta)(1 - \cos\theta) \equiv 0.$$

4. Find the set of values of k for which the curve (4)

$$y = 2x^2 + kx + 2k - 6$$

lies above the x-axis for all values of x.

5. The line (6)

$$3x + 4y = 15$$

cuts the curve

$$2xy = 9$$

at the points A and B.

Find the length of the line AB.

6. The normal to the curve (6)

$$y + 2 = 3\tan x,$$

at the point on the curve where $x = \frac{3}{4}\pi$, cuts the y-axis at the point P.

Find the coordinates of P.

7. It is given that

$$\mathrm{f}(x) = 6x^3 - 5x^2 + ax + b$$

has a factor of $(x + 2)$ and leaves a remainder of 27 when divided by $(x - 1)$.

(a) Show that $b = 40$ and find the value of a. (4)

(b) Show that (2)

$$\mathrm{f}(x) = (x + 2)(px^2 + qx + r),$$

where p, q, and r are integers to be found.

(c) Hence solve $\mathrm{f}(x) = 0$. (2)

8. (a) Given that the matrix

$$\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 3 & -5 \end{pmatrix},$$

find

(i) $\mathbf{A}^2$, (2)

(ii) $3\mathbf{A} + 4\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. (2)

(b) (i) Find the inverse matrix of (2)

$$\begin{pmatrix} 6 & 1 \\ -9 & 3 \end{pmatrix}.$$

(ii) Hence solve the equations (3)

$$6x + y = 5,$$
$$-9x + 3y = \tfrac{3}{2}.$$

9. (a) Given that n is a positive integer, find the first 3 terms in the expansion of (2)

$$\left(1 + \tfrac{1}{2}x\right)^n$$

in ascending powers of x.

(b) Given that the coefficient of x^2 in the expansion of (5)

$$(1 - x)\left(1 + \tfrac{1}{2}x\right)^n$$

is $\frac{25}{4}$, find the value of n.

10. (a) (i) Find (2)

$$\int \sqrt{2x - 5}\,\mathrm{d}x.$$

(ii) Hence evaluate (2)

$$\int_3^{15} \sqrt{2x - 5}\,\mathrm{d}x.$$

(b) (i) Find (2)

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^3 \ln x).$$

(ii) Hence find (3)

$$\int x^2 \ln x\,\mathrm{d}x.$$

11. (a) Solve (5)

$$\cos 2x + 2\sec 2x + 3 = 0 \text{ for } 0^\circ \leqslant x \leqslant 360^\circ.$$

(b) Solve (4)

$$2\sin^2(y - \tfrac{1}{6}\pi) = 1 \text{ for } 0 \leqslant y \leqslant \pi.$$

12. A particle P moves in a straight line such that, t s after leaving a point O, its velocity v ms^{-1} is given by

$$v = 36t - 3t^2, \text{ for } t \geqslant 0.$$

(a) Find the value of t when the velocity of P stops increasing. (2)

(b) Find the value of t when P comes instantaneous rest. (2)

(c) Find the distance of P from O when P is at instantaneous rest. (3)

(d) Find the speed of P when P is again at O. (4)