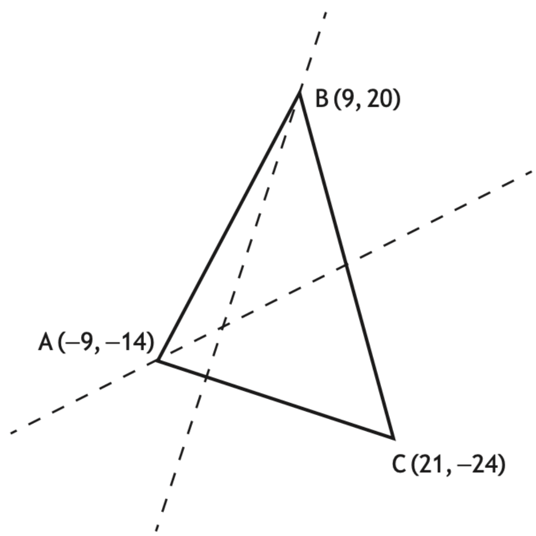
Dr Oliver Mathematics
Mathematics: Higher
2025 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 65.
You must write down all the stages in your working.

1. Triangle ABC has vertices $A(-9,-14)$, $B(9,20)$, and $C(21,-24)$.



   (a) Find the equation of the altitude through B. (3)

   (b) Find the equation of the median through A. (3)

   (c) Determine the point of intersection of the altitude through B and the median through A. (2)

2. Express (3)

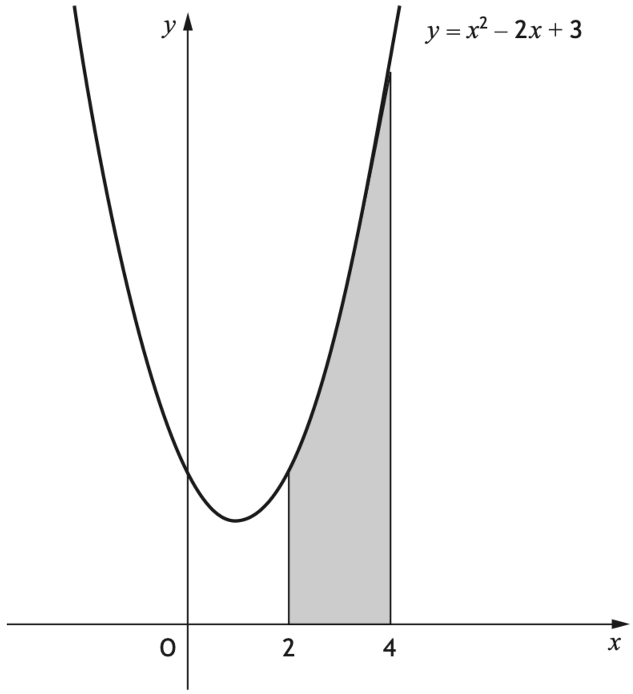
$$2x^2 + 16x + 5$$

   in the form

$$p(x+q)^2 + r.$$

3. The diagram shows the graph of (4)

$$y = x^2 - 2x + 3.$$



Calculate the shaded area.

4. A function, g, is defined by (3)

$$\mathrm{g}(x) = (x-4)^3, \text{ where } x \in \mathbb{R}.$$

Find the inverse function, $\mathrm{g}^{-1}(x)$.

5. (a) Show that the points $A(-3, 2, -1)$, $B(6, -1, 5)$, and $C(12, -3, 9)$ are collinear. (3)

   (b) State the ratio in which B divides AC. (1)

6. (a) Express (4)

$$5\cos x - 9\sin x$$

   in the form

$$k\cos(x+a),$$

   where $k > 0$ and $0 < a < 2\pi$.

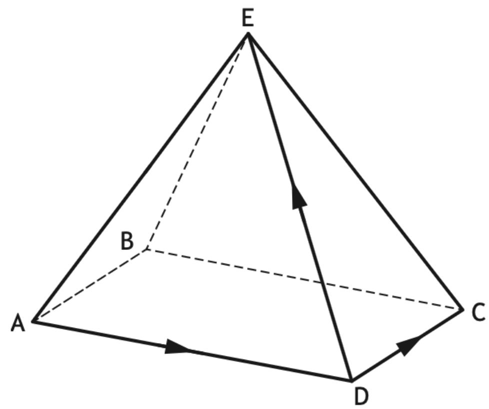
   (b) Hence solve (3)

$$5\cos x - 9\sin x = 7, \text{ for } 0 \leqslant x < 2\pi.$$

7. Find (2)

$$\int (3x+2)^7 \, \mathrm{d}x$$

8. $ABCDE$ is a rectangular-based pyramid as shown. (2)



- $\overrightarrow{AD} = 6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$.
- $\overrightarrow{DC} = 2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.
- $\overrightarrow{DE} = -4\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.

Express $\overrightarrow{BE}$ in terms of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

9. A sequence satisfies the recurrence relation

$$u_{n+1} = mu_n + 4,$$

where m is a constant.

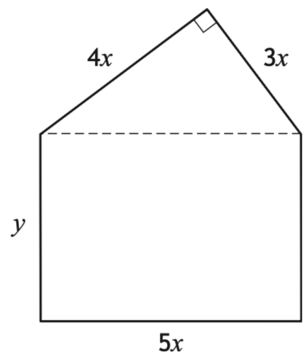
(a) The sequence approaches a limit of 10 as $n \to \infty$. (2)
Determine the value of m.

(b) Given that $u_1 = 19$, calculate the value of u_0. (1)

10. A hotel owner is designing signs showing the room numbers.

- Each sign is a rectangle with a right-angled triangle above it.

- The length and breadth of the rectangle are $5x$ centimetres and y centimetres respectively.
- The shorter sides of the triangle are $3x$ centimetres and $4x$ centimetres.



The area of the sign is 150 square centimetres.

(a) Show that the perimeter, P cm, of the sign is given by (3)

$$P = 9.6x + \frac{60}{x}.$$

Each sign will be lit using a lighting strip placed around its perimeter.

The hotel owner requires the perimeter, P, of the sign to be as small as possible.

(b) Find the minimum value of P. (6)

11. Solve (4)

$$3\sin 2x^\circ + 4\cos x^\circ = 0, \text{ for } x \leqslant x < 360.$$

12. Functions f and g are defined on the set of real numbers by:

- $\mathrm{f}(x) = x^5 + 3$ and
- $\mathrm{g}(x) = 1 - x^3$.

(a) Find an expression for $\mathrm{h}(x)$, where $\mathrm{h}(x) = \mathrm{f}(\mathrm{g}(x))$. (2)

(b) Find $\mathrm{h}'(x)$. (2)

13. A radioactive substance, which has been collected, decays over time.

The mass of the radioactive substance remaining is modelled by

$$M = 150\mathrm{e}^{-0.005\,4t},$$

where M is the mass, in micrograms, t years after the radioactive substance was collected.

(a) Determine the initial mass of the radioactive substance. (1)

(b) Calculate the time taken for the mass of the radioactive substance to decay to 120 micrograms. (4)

14. Circle C_1 has equation

$$(x+5)^2 + (y-6)^2 = 9.$$

(a) State the centre and radius of C_1. (2)

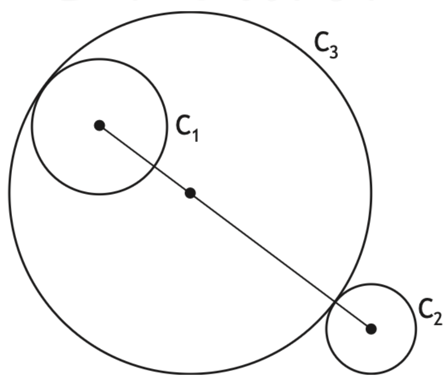
Circle C_2 has equation

$$x^2 + y^2 - 14x + 6y + 54 = 0.$$

(b) State the centre and radius of C_2. (2)

Circles C_1, C_2, and C_3 are touching as shown in the diagram.

The centre of circle C_3 lies on the line joining the centres of C_1 and C_2.



(c) Determine the equation of C_3. (3)