Dr Oliver Mathematics
AQA Mathematics
2016 June Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 105.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Here are the ingredients to make 8 biscuits. (3)

75 g	flour
50 g	sugar
40 g	butter
2	egg yolks

Work out the ingredients to make 20 biscuits.

Solution

Well,

$$\frac{20}{8} \times 75 = \underline{\underline{187.5 \text{ flour}}}$$
$$\frac{20}{8} \times 50 = \underline{\underline{125 \text{ sugar}}}$$
$$\frac{20}{8} \times 40 = \underline{\underline{100 \text{ butter}}}$$
$$\frac{20}{8} \times 2 = \underline{\underline{5 \text{ egg yolks}}}.$$

2. (a) Alice wants to book a holiday for one adult and one child. (5)

Holiday

£720 per adult

£430 per child

Special Offer
15% off

Alice has £1000

Does she have enough money to book this holiday using the special offer?
Tick a box.

Yes ☐ No ☐

You **must** show your working.

Solution
Well,

$$\begin{aligned} \text{she spends} &= (1 - 0.15)[720 + 430] \\ &= 0.85 \times 1\,150 \\ &= 977.50; \end{aligned}$$

so, yes, she does have enough money.

(b) Ben changes £800 to Euros before he goes on holiday. (4)
£1 = 1.25 Euro.

He spends 895 Euros.

He changes the Euros that he has left to Pounds (£).
The exchange rate is now £1 = 1.40 Euro.

How many Pounds does he get back?

Solution

Now,

$$\begin{aligned}\text{Ben gets} &= 1.25 \times £800\\ &= 1\,000 \text{ Euros.}\end{aligned}$$

He then spends 895 Euros:

$$1\,000 - 895 = 105 \text{ Euros}$$

and

$$\begin{aligned}\text{Ben gets back} &= \frac{105 \text{ Euros}}{1.40}\\ &= \underline{\underline{£75}}.\end{aligned}$$

3. This formula converts degrees Celsius (C) to degrees Fahrenheit (F): (3)

$$F = \tfrac{9}{5}C + 32.$$

Use the formula to convert 28°C to °F.
Give your answer to the nearest whole number.

Solution

Well,

$$\begin{aligned}C = 28 &\Rightarrow F = \tfrac{9}{5}(28) + 32\\ &\Rightarrow F = 82.4 \text{ (exact)}\\ &\Rightarrow \underline{\underline{F = 82°\text{F (nearest degree)}}}.\end{aligned}$$

4. (a) The nth term of a sequence is (2)

$$6 - 2n.$$

Work out the first three terms of the sequence.

Solution

Well,

$$\begin{aligned} \text{1st term} &= 6 - 2(1) = 4, \\ \text{2nd term} &= 6 - 2(2) = 2, \\ \text{3rd term} &= 6 - 2(3) = 0; \end{aligned}$$

hence, the first three terms of the sequence are

$$\underline{\underline{4,\ 2,\ \text{and}\ 0}}.$$

(b) Here is the term-to-term rule for a different sequence. (3)

Multiply previous term by 2 and then subtract 3.

The third term in this sequence is 31.

Work out the **first** term.

Solution

Now,

$$\begin{aligned} 31 = 2 \times \text{2nd term} - 3 &\Rightarrow 34 = 2 \times \text{2nd term} \\ &\Rightarrow \text{2nd term} = 17 \end{aligned}$$

and

$$\begin{aligned} 17 = 2 \times \text{1st term} - 3 &\Rightarrow 20 = 2 \times \text{1st term} \\ &\Rightarrow \underline{\underline{\text{1st term} = 10}}. \end{aligned}$$

5. The table shows information about the pay per hour of 40 people.

Pay per hour, x (£)	Frequency
$5 < x \leqslant 15$	14
$15 < x \leqslant 25$	12
$25 < x \leqslant 35$	11
$35 < x \leqslant 45$	2
$45 < x \leqslant 55$	1
	Total = 14

(a) Which group contains the median pay per hour? (1)
Circle your answer.

$$5 < x \leqslant 15 \quad 15 < x \leqslant 25 \quad 25 < x \leqslant 35 \quad 35 < x \leqslant 45 \quad 45 < x \leqslant 55$$

Solution

Pay per hour, x (£)	Frequency	Cumulative Freq	Midpoint×Freq
$5 < x \leqslant 15$	14	14	$10 \times 14 = 140$
$15 < x \leqslant 25$	12	26	$20 \times 12 = 240$
$25 < x \leqslant 35$	11	37	$30 \times 11 = 330$
$35 < x \leqslant 45$	2	39	$40 \times 2 = 80$
$45 < x \leqslant 55$	1	40	$50 \times 1 = 50$
	Total = 14	40	840

The median is at the

$$\tfrac{1}{2}(40 + 1) = 20\tfrac{1}{2}\text{th place}$$

and so

$$\text{median} = \underline{\underline{15 < x \leqslant 25}}.$$

(b) Work out an estimate of the mean pay per hour. (4)

Solution
From (a),

$$\begin{aligned} \text{mean} &= \frac{\sum fx}{\sum f} \\ &\approx \frac{840}{40} \\ &= \underline{\underline{£21}}. \end{aligned}$$

6. A baker makes 130 loaves so that there are (3)

- 6 times as many white loaves as granary loaves and
- half as many brown loaves as white loaves.

How many of each type does he make?

Solution

Let x the number of white loaves. Then

$$\text{granary loaves} = \tfrac{1}{6}x$$

and

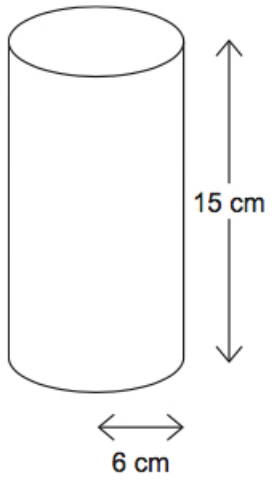
$$\text{brown loaves} = \tfrac{1}{2}x.$$

Now,

$$\begin{aligned} & \text{white loaves} + \text{brown loaves} + \text{granary loaves} = 130 \\ \Rightarrow \quad & x + \tfrac{1}{2}x + \tfrac{1}{6}x = 130 \\ \Rightarrow \quad & \tfrac{5}{3}x = 130 \\ \Rightarrow \quad & x = 78 \\ \Rightarrow \quad & \tfrac{1}{2}x = 39 \\ \Rightarrow \quad & \tfrac{1}{6}x = 13; \end{aligned}$$

hence, he makes 78 white loaves, 39 brown loaves, and 13 granary loaves.

7. (a) The diagram shows a cylinder. (3)



- The radius of the base is 6 cm.
- The height is 15 cm.

Work out the volume.

Solution
Well,

$$\begin{aligned}\text{volume} &= \pi r^2 h \\ &= \pi \times 6^2 \times 15 \\ &= \underline{\underline{540\pi \text{ or } 1\,700 \text{ cm}^3 \text{ (3 sf)}}}.\end{aligned}$$

(b) $1\,000 \text{ cm}^3 = 1$ litre. (4)

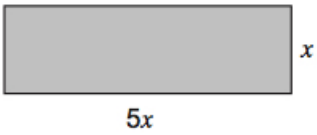
A tank contains $45\,000 \text{ cm}^3$ of water.
The tank leaks at 0.75 litres/minute.

How long does the tank take to empty?

Solution
Now,

$$\begin{aligned}\text{time to empty} &= \frac{45\,000 \text{ cm}^3}{0.75 \text{ litres/minute}} \\ &= \frac{45 \text{ litres}}{0.75 \text{ litres/minute}} \\ &= \underline{\underline{60 \text{ minutes}}}.\end{aligned}$$

8. The diagram shows a rectangle. (5)



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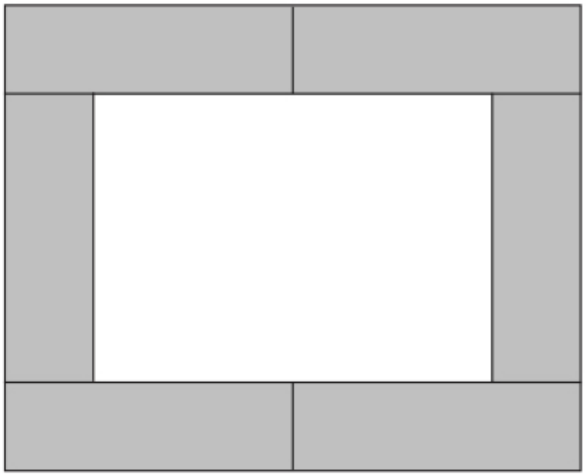
Six of these rectangles are joined to make this shape.



The area of the white rectangle in the middle is 1 440 cm^2.

Work out the area of **one** shaded rectangle.

Solution

Well, the entire area is

$$(5x + 5x) \times (x + 5x + x) = 10x \times 7x = 70x^2$$

and the grey rectangles are

$$6 \times (5x \times x) = 30x^2,$$

and makes the area of the white rectangle

$$70x^2 - 30x^2 = 40x^2.$$

Finally,

$$\begin{aligned} 40x^2 = 1\,440 &\Rightarrow x^2 = 36 \\ &\Rightarrow 5x^2 = \underline{\underline{180 \text{ cm}^2}}. \end{aligned}$$

9. Work out the price of the car before it was reduced. (3)

Solution

Well,

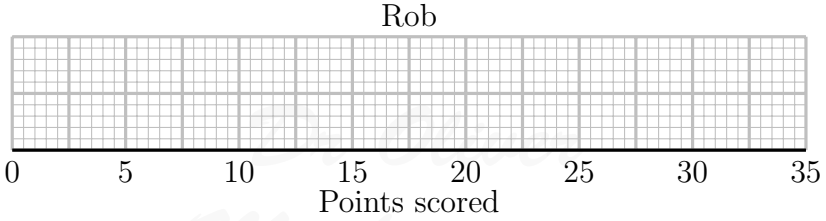
$$\text{new price} = \text{change} \times \text{old price} \Rightarrow 14\,625 = 0.75 \times \text{old price}$$
$$\Rightarrow \text{old price} = \frac{14\,625}{0.75}$$
$$\Rightarrow \text{old price} = \underline{\underline{£19\,500}}.$$

10. Rob played in 15 basketball matches.
The stem-and-leaf diagram shows the number of points he scored in each match.

0	5						
1	2	2	4	5	6	7	8
2	2	2	3	6	6	9	
3	0						

Key: 0|5 represents 5 points.

(a) Draw a box plot to represent the data. (4)

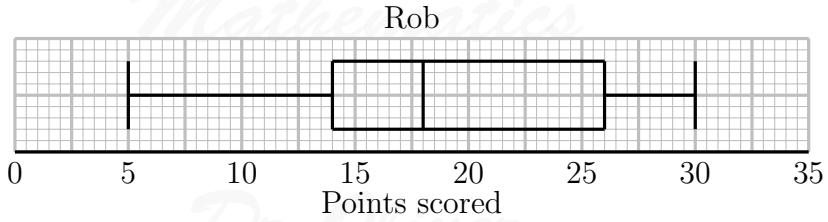


Solution

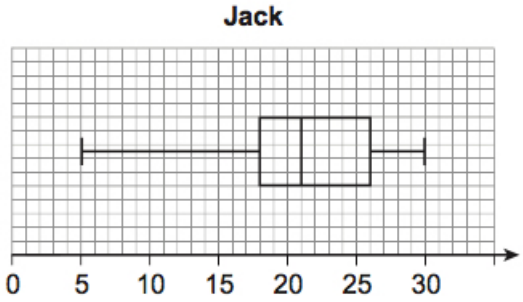
Well,

$$\begin{aligned} \text{LQ} &= \text{4th number} = 14 \\ \text{median} &= \text{8th number} = 18 \\ \text{UQ} &= \text{12th number} = 26, \end{aligned}$$

and the box plot is



(b) This box plot represents the points that Jack scored in 15 basketball matches. (2)



Jack says, "I am better at basketball than Rob."

Give **two** reasons that support his statement.

Solution

Median: Jack's median is 21 and Rob's median is 18 and so Jack has scored more points on average.

IQR: Jack's IQR is $(26 - 18 = 7)$ and Rob's IQR is $(26 - 14 = 12)$ and so Jack has a more consistent than Rob.

Note: Jack and Rob has the same range $(30 - 5 = 25)$.

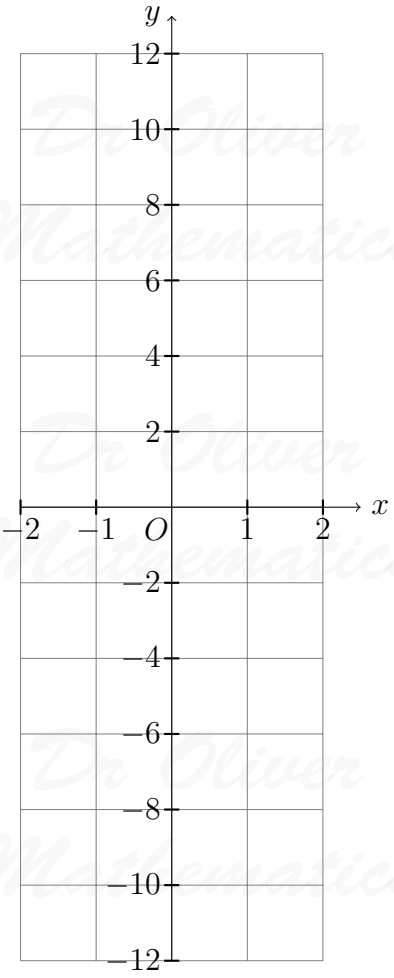
11. Here is a table of values for (2)

$$y = x^3 - 2, \text{ for } x = -2 \text{ to } 2.$$

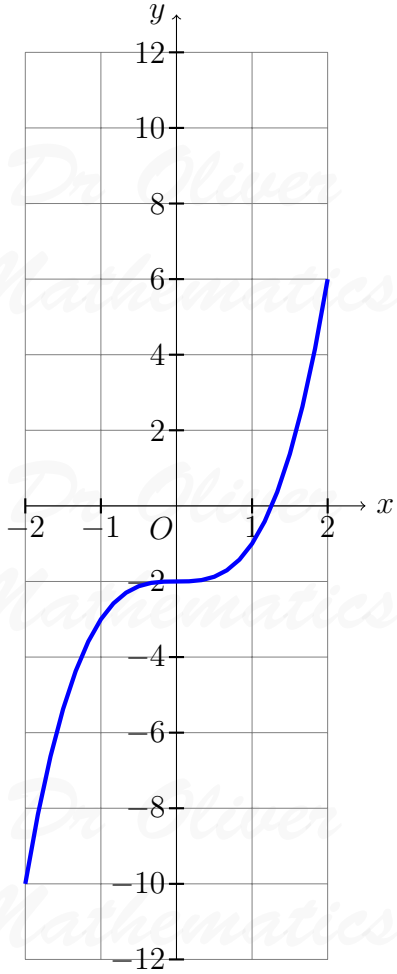
x	-2	-1	0	1	2
y	-10	-3	-2	-1	6

Draw the graph of

$$y = x^3 - 2, \text{ for } x = -2 \text{ to } 2.$$



Solution



12. Two boats leave a port at the same time. (5)

- Boat A travels due West at an average speed of 20 km/h.
- Boat B travels due South at an average speed of 30 km/h.

How far apart are the boats after 2.5 hours?
Give your answer to 2 significant figures.

Solution

Well, after 2.5 hours, boat A is at

$$20 \times 2.5 = 50 \text{ km}$$

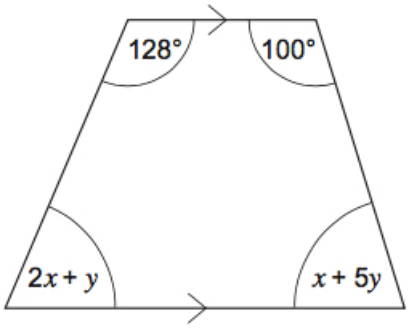
due West and boat B is at

$$30 \times 2.5 = 75 \text{ km}$$

due South. Now,

$$\begin{aligned} \text{distance} &= \sqrt{50^2 + 75^2} \\ &= \sqrt{8\,125} \\ &= 90.138\,781\,89 \text{ (FCD)} \\ &= \underline{\underline{90 \text{ km (2 sf)}}}. \end{aligned}$$

13. The diagram shows a trapezium. (5)



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Work out the values of x and y.

Solution

Interior angles:

$$(2x + y) + 128 = 180 \Rightarrow 2x + y = 52 \quad (1)$$

and angles in a quadrilateral:

$$(2x + y) + (x + 5y) + 128 + 100 = 360 \Rightarrow 3x + 6y = 132 \quad (3).$$

Do $6 \times (1)$:

$$12x + 6y = 312 \quad (3)$$

and do $(3) - (2)$:

$$\begin{aligned} 9x = 180 &\Rightarrow \underline{\underline{x = 20}} \\ &\Rightarrow \underline{\underline{y = 12}} \end{aligned}$$

14. Write down the equation of the straight line that (2)

- passes through the point $(0, 4)$ **and**
- is parallel to the line $y = 5x + 3$.

Solution

$y = 5x + 4$.

15. (3)

- Bags of nails weigh 200 grams each.
- Boxes of screws weigh 140 grams each.
- Both measurements are given to the nearest 10 grams.

Show that 4 bags of nails could weigh the same as 6 boxes of screws.

Solution

Well,

$$195 \leqslant \text{nails} < 205$$

and

$$135 \leqslant \text{screws} < 145.$$

Now,

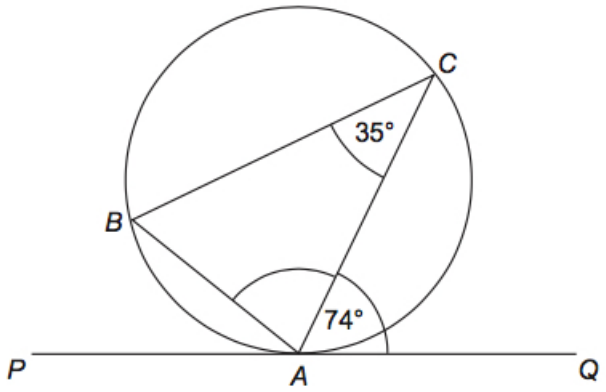
$$4 \times 195 \leqslant 4 \text{ bags of nails} < 4 \times 205 \Rightarrow 780 \leqslant 4 \text{ bags of nails} < 820$$

and

$$6 \times 135 \leqslant 6 \text{ bags of screws} < 6 \times 145 \Rightarrow 810 \leqslant 6 \text{ bags of screws} < 870.$$

There is an intersection of 10 ($810 \leqslant$ measurements < 820) between them: 4 bags of nails could weigh the same as 6 boxes of screws.

16. A, B, and C are points on a circle. (2)
PAQ is a tangent to the circle.



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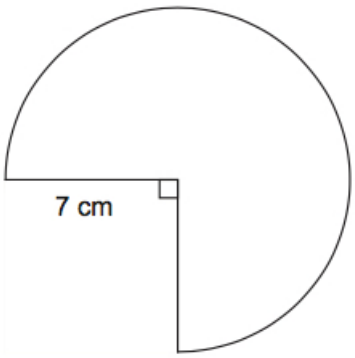
Work out the size of angle CAB.

Solution

Well, $\angle ABC = 74^\circ$ (alternate segment theorem)
$\angle BAC = 180 - (74 + 35) = \underline{\underline{71^\circ}}$ (completing the triangle)

17. The diagram shows a sector of a circle. (3)
The radius is 7 cm.



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Work out the **perimeter** of the shape.

Solution

Well,

$$\begin{aligned} \text{perimeter} &= 7 + 7 + \tfrac{3}{4} \times 2 \times \pi \times 7 \\ &= \underline{\underline{14 + \tfrac{21}{2}\pi \text{ or } 47.0 \text{ cm (3 sf)}}}. \end{aligned}$$

18. The table shows information about the masses of 400 hamsters.

Mass, w (grams)	Frequency
$80 < w \leqslant 100$	100
$100 < w \leqslant 115$	150
$115 < w \leqslant 125$	90
$125 < w \leqslant 150$	60

(a) A sample of size 50, stratified by the groups in the table, is to be taken. (3)

Work out the number of hamsters from each group in the sample.
Write your answers in the table below.

Mass, w (grams)	Number in the sample
$80 < w \leqslant 100$	
$100 < w \leqslant 115$	
$115 < w \leqslant 125$	
$125 < w \leqslant 150$	

Solution

Well, we want

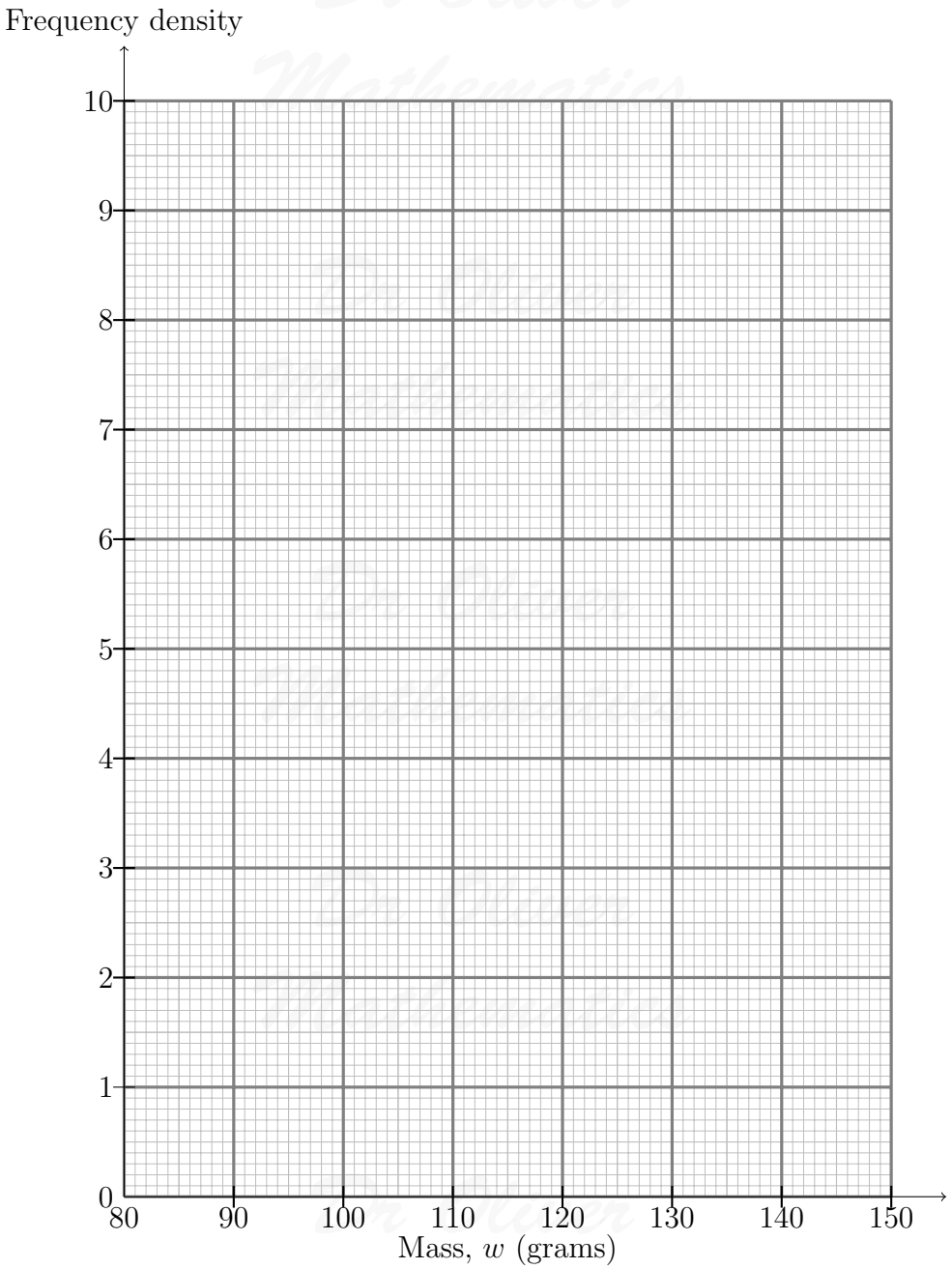
Mass, w (grams)	Number in the sample
$80 < w \leqslant 100$	$\frac{50}{400} \times 100 = 12.5$
$100 < w \leqslant 115$	$\frac{50}{400} \times 150 = 18.75$
$115 < w \leqslant 125$	$\frac{50}{400} \times 90 = 11.25$
$125 < w \leqslant 150$	$\frac{50}{400} \times 60 = 7.5$

Hmm. So we want

Mass, w (grams)	Number in the sample
$80 < w \leqslant 100$	$\underline{\underline{12}}$
$100 < w \leqslant 115$	$\underline{\underline{19}}$
$115 < w \leqslant 125$	$\underline{\underline{11}}$
$125 < w \leqslant 150$	$\underline{\underline{8}}$

(b) Draw a histogram for the data. (4)
You may use the table to help you.

Mass, w (grams)	Frequency
$80 < w \leqslant 100$	100
$100 < w \leqslant 115$	150
$115 < w \leqslant 125$	90
$125 < w \leqslant 150$	60

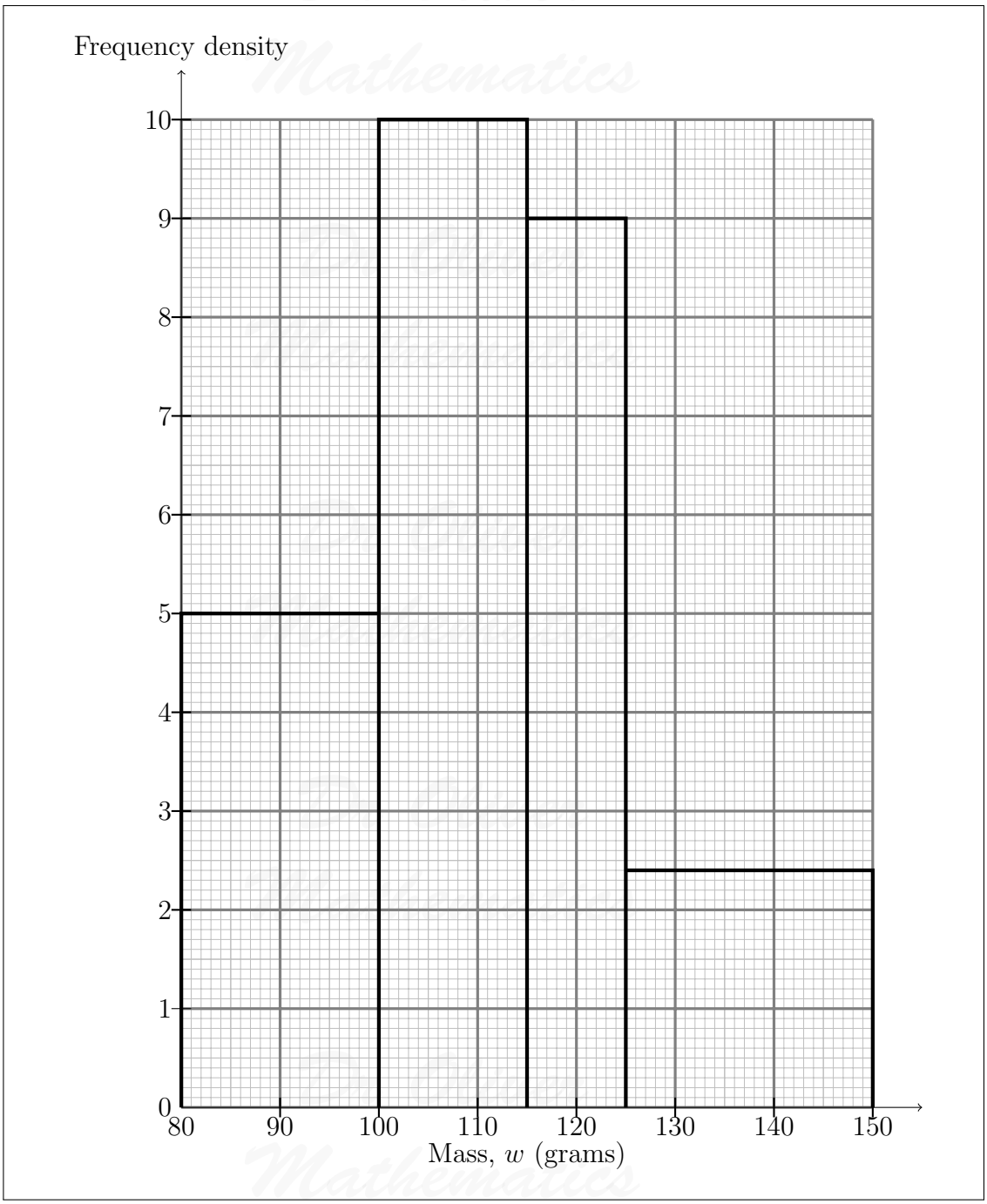


Solution

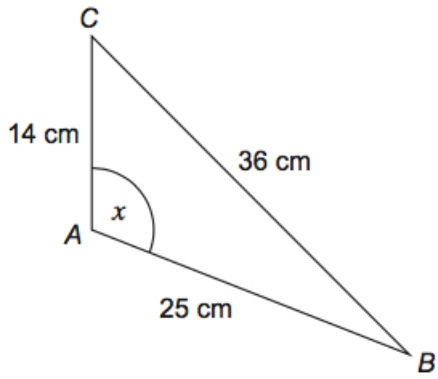
We complete the table

Mass, w (grams)	Frequency	Width	Frequency Density
$80 < w \leqslant 100$	100	20	$\frac{100}{20} = 5$
$100 < w \leqslant 115$	150	15	$\frac{150}{15} = 10$
$115 < w \leqslant 125$	90	10	$\frac{90}{10} = 9$
$125 < w \leqslant 150$	60	25	$\frac{60}{25} = 2.4$

and the histogram looks like this:



19. Work out the size of angle x. (3)



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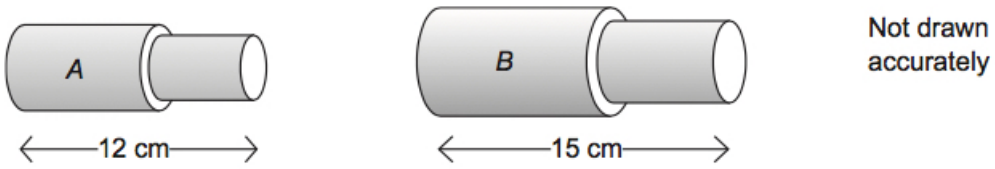
Solution

Well,

$$
\begin{aligned}
& BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos BAC \\
\Rightarrow \quad & 36^2 = 25^2 + 14^2 - 2 \times 25 \times 14 \times \cos BAC \\
\Rightarrow \quad & 1\,296 = 625 + 196 - 700 \cos BAC \\
\Rightarrow \quad & 700 \cos BAC = -475 \\
\Rightarrow \quad & \cos BAC = -\tfrac{19}{28} \\
\Rightarrow \quad & \angle BAC = 132.732\,11 \text{ (FCD)} \\
\Rightarrow \quad & \underline{\underline{\angle BAC = 133^\circ \text{ (3 sf)}}}.
\end{aligned}
$$

20. These two solid shapes are similar. (3)



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The volume of A is $1\,400$ cm^3.

Work out the volume of B.

Solution

Well, the length scale ratio (LSR) is

$$\frac{15}{12} = \tfrac{5}{4}$$

and the volume scale ratio (VSR) is

$$(\tfrac{5}{4})^3 = \tfrac{125}{64}.$$

So, the volume of B is

$$1\,400 \times \tfrac{125}{64} = \underline{\underline{2\,734.375 \text{ cm}^3}}.$$

21. A bag contains 10 counters. (3)
4 of the counters are black and 6 are white.

Two counters are picked at random.

Work out the probability that they are both black.

Solution

Well,

$$\begin{aligned} \mathrm{P}(BB) &= \tfrac{4}{10} \times \tfrac{3}{9} \\ &= \underline{\underline{\tfrac{2}{15}}}. \end{aligned}$$

22. (a) Factorise (2)

$$49c^2 - d^2.$$

Solution

Difference of two squares:

$$\begin{aligned} 49c^2 - d^2 &= (7c)^2 - d^2 \\ &= \underline{\underline{(7c + d)(7c - d)}}. \end{aligned}$$

(b) Simplify (3)

$$\frac{x^2 - 6x}{2x^2 - 7x - 30}.$$

Solution

Now,

$$\left.\begin{array}{ll} \text{add to:} & -7 \\ \text{multiply to:} & (+2) \times (-30) = -60 \end{array}\right\} +5, -12$$

and, e.g.,

$$\begin{aligned} \frac{x^2 - 6x}{2x^2 - 7x - 30} &= \frac{x(x-6)}{2x^2 + 5x - 12x - 30} \\ &= \frac{x(x-6)}{x(2x+5) - 6(2x+5)} \\ &= \frac{x(x-6)}{(x-6)(2x+5)} \\ &= \underline{\underline{\frac{x}{2x+5}}}. \end{aligned}$$

23. You are given that (2)

$$(x + a)^2 - 7 \equiv x^2 + 10x + b.$$

Work out the values of a and b.

Solution

Well,

$\times$	x	$+a$
x	x^2	$+ax$
$+a$	$+ax$	$+a^2$

and

$$\begin{aligned} (x + a)^2 - 7 &\equiv (x^2 + 2ax + a^2) - 7 \\ &\equiv x^2 + 2ax + (a^2 - 7). \end{aligned}$$

Comparing the x-term:

$$2a = 10 \Rightarrow \underline{\underline{a = 5}}.$$

Comparing the constant term:

$$\begin{aligned} b &= 5^2 - 7 \\ &= 25 - 7 \\ &= \underline{\underline{18}}. \end{aligned}$$

24. Solve the equation (6)

$$\frac{6}{x+3} + \frac{1}{2x+3} = 3.$$

Give your answers to 2 decimal places.

Solution

Multiply by $(2x+5)(x+3)$:

$$\begin{aligned} & \frac{6}{x+3} + \frac{1}{2x+5} = 3 \\ \Rightarrow \quad & 6(2x+5) + (x+3) = 3(2x+5)(x+3) \end{aligned}$$

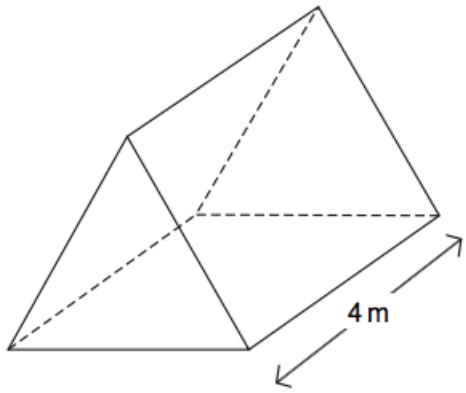
$\times$	$2x$	$+5$
x	$2x^2$	$+5x$
$+3$	$+6x$	$+15$

$$\begin{aligned} \Rightarrow \quad & 12x + 30 + x + 3 = 3(2x^2 + 11x + 15) \\ \Rightarrow \quad & 13x + 33 = 6x^2 + 33x + 45 \\ \Rightarrow \quad & 6x^2 + 20x + 12 = 0 \\ \Rightarrow \quad & 2(3x^2 + 10x + 6) = 0 \end{aligned}$$

quadratic formula:

$$\begin{aligned} \Rightarrow \quad & x = \frac{-10 \pm \sqrt{(-10)^2 - 4(3)(6)}}{2(3)} \\ \Rightarrow \quad & x = \frac{-10 \pm \sqrt{28}}{6} \\ \Rightarrow \quad & x = -2.548\,583\,77,\ -0.784\,749\,563 \text{ (FCD)} \\ \Rightarrow \quad & x = \underline{\underline{-2.55,\ -0.78 \text{ (2 dp)}}}. \end{aligned}$$

25. A tent is in the shape of a triangular prism. (5)



- The length of the tent is 4 metres.
- The volume is 8 m^3.
- The cross-section of the tent is an **equilateral** triangle.

Shaun is 1.95 metres tall.

Can he stand at the highest part of the tent without having to bend over?
You **must** show your working.

Solution

Well,

$$\text{volume} = \tfrac{1}{2}lbh,$$

where l is length, h is the height, and b is the base. Now,

$$\begin{aligned}\text{volume} = \text{cross-sectional area} \times \text{length} &\Rightarrow 8 = \text{CSA} \times 4\\ &\Rightarrow \text{CSA} = 2.\end{aligned}$$

But we know that it is an equilateral triangle:

$$\begin{aligned}\text{area} = \tfrac{1}{2}ab\sin C &\Rightarrow \tfrac{1}{2} \times b^2 \times \sin 60^\circ = 2\\ &\Rightarrow \tfrac{\sqrt{3}}{4}b^2 = 2\\ &\Rightarrow b^2 = \tfrac{8}{3}\sqrt{3}\\ &\Rightarrow b = \sqrt{\tfrac{8}{3}\sqrt{3}}.\end{aligned}$$

Hence,

$$\sin = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin 60^\circ = \frac{h}{\sqrt{\frac{8}{3}\sqrt{3}}}$$
$$\Rightarrow h = \sqrt{\tfrac{8}{3}\sqrt{3}} \sin 60^\circ$$
$$\Rightarrow h = 1.861\,209\,718 \text{ (FCD)};$$

hence, the answer is no.