Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2011 November Paper 1 Variant 1: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. (a) Sets A and B are such that

$$\text{n}(A) = 15 \text{ and } \text{n}(B) = 7.$$

Find the greatest and least possible values of

(i) $\text{n}(A \cap B)$, (2)

(ii) $\text{n}(A \cup B)$ (2)

(b) On a Venn diagram draw 3 sets P, Q, and R such that (2)

$$P \cup Q = \varnothing \text{ and } P \cup R = P.$$

2. The function f is such that (6)

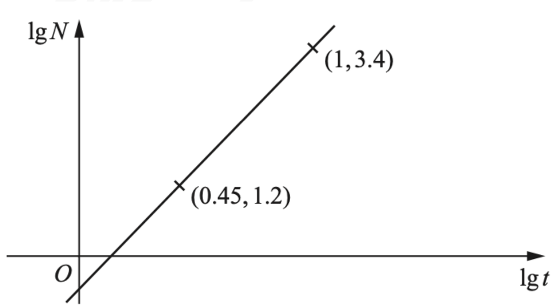
$$\text{f}(x) = 4x^3 - 8x^2 + ax + b,$$

where a and b are constants.

It is given that $(2x - 1)$ is a factor of $\text{f}(x)$ and that when $\text{f}(x)$ is divided by $(x + 2)$ the remainder is 20.

Find the remainder when $\text{f}(x)$ is divided by $(x - 1)$.

3. Variables t and N are such that when $\log_{10} N$ is plotted against $\log_{10} t$, a straight line graph passing through the points $(0.45, 1.2)$ and $(1, 3.4)$ is obtained.



(a) Express the equation of the straight line graph in the form (4)

$$\log_{10} N = m \log_{10} t + \log_{10} c,$$

where m and c are constants to be found.

(b) Hence express N in terms of t. (1)

4. Six-digit numbers are to be formed using the digits 3, 4, 5, 6, 7, and 9. Each digit may only be used once in any number.

   (a) Find how many different six-digit numbers can be formed (1)

   Find how many of these six-digit numbers are

   (b) even, (1)

   (c) greater than 500 000, (1)

   (d) even and greater than 500 000. (3)

5. A particle moves in a straight line such that its displacement, x m, from a fixed point O at time t s, is given by

$$x = 3 + \sin 2t, \text{ where } t \geqslant 0.$$

   (a) Find the velocity of the particle when $t = 0$. (2)

   (b) Find the value of t when the particle is first at rest. (2)

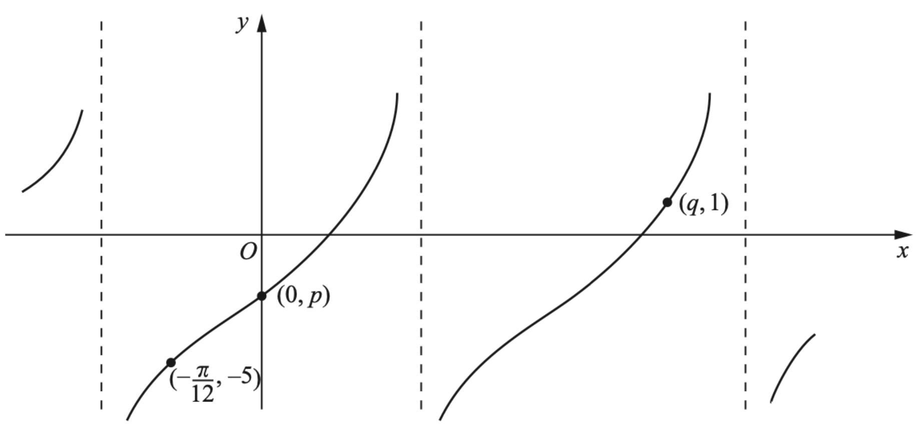
   (c) Find the distance travelled by the particle before it first comes to rest. (2)

   (d) Find the acceleration of the particle when $t = \frac{3}{4}\pi$. (2)

6. (a) The diagram shows part of the graph (4)

$$y = p + 3 \tan 3x$$

   passing through the points $(-\frac{1}{12}\pi, -5)$, $(0, p)$, and $(q, 1)$.



   Find the value of p and of q.

(b) It is given that (4)

$$f(x) = a\cos(bx) + c,$$

where a, b, and c are integers.

The maximum value of f is 11, the minimum value of f is 3, and the period of f is 72°.

Find the value of a, of b, and of c.

7. The coefficient of x^2 in the expansion of

$$(1 + \tfrac{1}{5}x)^n,$$

where n is a positive integer, is $\frac{3}{5}$.

(a) Find the value of n. (4)

(b) Using this value of n, find the term independent of x in the expansion of (4)

$$(1 + \tfrac{1}{5}x)^n \left(2 - \frac{3}{x}\right)^2.$$

8. (a) Find (6)

$$\int (e^x + 1)^2 \, dx$$

and hence evaluate

$$\int_0^2 (e^x + 1)^2 \, dx.$$

(b) A curve is such that (5)

$$\frac{dy}{dx} = (4x + 1)^{-\frac{1}{2}}.$$

Given that the curve passes through the point with coordinates $(2, 4.5)$, find the equation of the curve.

9. (a) Solve (5)

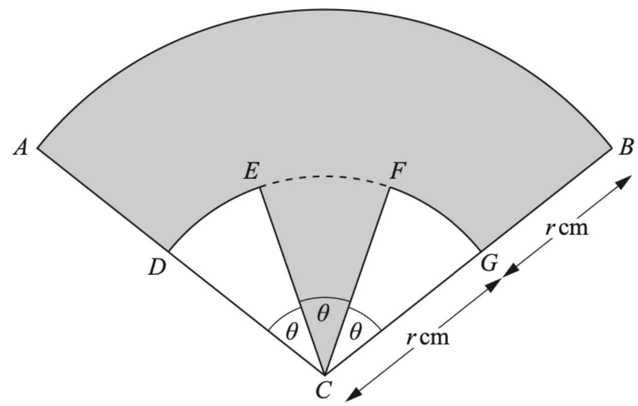
$$1 + \cot^2 x = 8 \sin x \text{ for } 0^\circ \leqslant x \leqslant 360^\circ.$$

(b) Solve (5)

$$4\sin(2y - 0.3) + 5\cos(2y - 0.3) = 0 \text{ for } 0 \leqslant y \leqslant \pi \text{ radians}.$$

EITHER

10. The figure shows a sector ABC of a circle centre C, radius $2r$ cm, where angle ACB is 3θ radians.



- The points D, E, F, and G lie on an arc of a circle centre C, radius r cm.
- The points D and G are the midpoints of CA and CB respectively.
- Angles DCE and FCG are each θ radians.
- The area of the shaded region is 5 cm^2.

(a) By first expressing θ in terms of r, show that the perimeter, P cm, of the shaded region is given by (6)

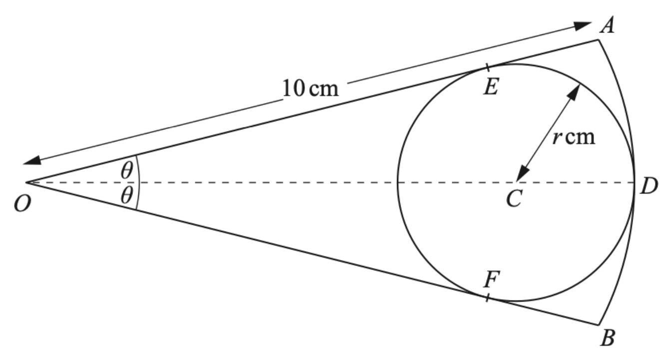
$$P = 4r + \frac{8}{r}.$$

(b) Given that r can vary, show that the stationary value of P can be written in the form $k\sqrt{2}$, where k is a constant to be found. (4)

(c) Determine the nature of this stationary value and find the value of θ for which it occurs. (2)

OR

11. The figure shows a sector OAB of a circle, centre O, radius 10 cm.



- Angle $AOB = 2\theta$ radians where $0 < \theta < \frac{1}{2}\pi$.
- A circle centre C, radius r cm, touches the arc AB at the point D.
- The lines OA and OB are tangents to the circle at the points E and F respectively.

(a) Write down, in terms of r, the length of OC. (1)

(b) Hence show that (2)

$$r = \frac{10 \sin \theta}{1 + \sin \theta}.$$

(c) Given that θ can vary, find $\dfrac{dr}{d\theta}$ when $r = \frac{10}{3}$. (6)

(d) Given that r is increasing at 2 cm s^{-1}, find the rate at which θ is increasing when $\theta = \frac{1}{6}\pi$. (3)