Dr Oliver Mathematics
AQA Further Maths Level 2
June 2012 Paper 1
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.
You are **not** permitted to use a scientific or graphical calculator in this paper.

1.
$$f(x) = 2x^2 + 7$$

for all values of x.

(a) What is the value of $f(-1)$? (1)

Solution

$$\begin{aligned} f(-1) &= 2[(-1)^2] + 7 \\ &= 2 + 7 \\ &= \underline{\underline{9}}. \end{aligned}$$

(b) What is the range of $f(x)$? (1)

Solution

$\underline{\underline{x \in \mathbb{R},\ f(x) \geqslant 7}}$.

2. (2)

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \text{ and } \mathbf{B} \begin{pmatrix} 5 \\ 4 \end{pmatrix}.$$

Work out the matrix **AB**.

Solution

$$\begin{aligned} \mathbf{AB} &= \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} 10 \\ 17 \end{pmatrix}}}. \end{aligned}$$

3. Work out the greatest integer value of x that satisfies the inequality (2)

$$3x + 10 < 1.$$

Solution

$$\begin{aligned} 3x + 10 < 1 &\Rightarrow 3x < -9 \\ &\Rightarrow x < -3. \end{aligned}$$

As x is an integer, $\underline{\underline{x = -4}}$.

4. (a) Factorise fully (3)

$$2x^2 - 2x - 40.$$

Solution

$$2x^2 - 2x - 40 = 2(x^2 - x - 20)$$

$$\left.\begin{array}{ll} \text{add to:} & -1 \\ \text{multiply to:} & -20 \end{array}\right\} -5,\ +4$$

$$= \underline{\underline{2(x-5)(x+4)}}.$$

(b) Factorise fully (3)

$$(x+y)^2 + (x+y)(2x+5y).$$

Solution

$$\begin{aligned} (x+y)^2 + (x+y)(2x+5y) &= (x+y)\left[(x+y) + (2x+5y)\right] \\ &= (x+y)(3x+6y) \\ &= \underline{\underline{3(x+y)(x+2y)}}. \end{aligned}$$

5. Simplify (2)

$$(2cd^4)^3.$$

Solution

$$\underline{\underline{(2cd^4)^3 = \underline{8c^3d^{12}}}}.$$

6. Solve the simultaneous equations (4)

$$\begin{aligned} 2y &= 3x + 4 \\ 2x &= -3y - 7 \end{aligned}$$

Do **not** use trial and improvement.

Solution

$$\begin{aligned} 2y = 3x + 4 &\Rightarrow -3x + 2y = 4 \quad (1) \\ 2x = -3y - 7 &\Rightarrow 2x + 3y = -7 \quad (2). \end{aligned}$$

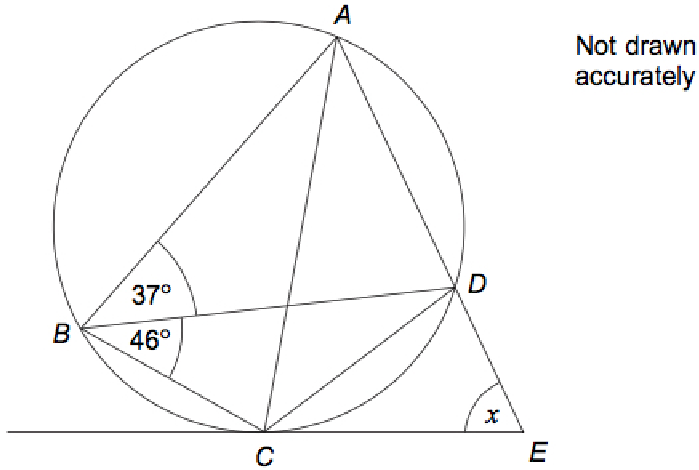
E.g., $2 \times (1)$ and $3 \times (2)$:

$$\begin{aligned} -6x + 4y &= 8 \quad (3) \\ 6x + 9y &= -21 \quad (4). \end{aligned}$$

Add:

$$\begin{aligned} 13y = -13 &\Rightarrow \underline{\underline{y = -1}} \\ &\Rightarrow 2(-1) = 3x + 4 \\ &\Rightarrow -2 = 3x + 4 \\ &\Rightarrow 3x = -6 \\ &\Rightarrow \underline{\underline{x = -2}}. \end{aligned}$$

7. The diagram shows a cyclic quadrilateral $ABCD$. (3)
ADE is a straight line.
CE is a tangent to the circle.



Work out the size of angle x.

Solution

$\angle DCE = 46^\circ$ (Alternate Segment Theorem)
$\angle ADC = 180 - (37 + 46) = 180 - 83 = 97^\circ$ (cyclic angles)
$\angle CDE = 180 - 97 = 83^\circ$ (angles on a straight line)
$\angle x = 180 - (83 + 46) = 180 - 129 = \underline{\underline{51^\circ}}$ (angles in a triangle)

8. A curve has equation

$$y = x^3 + 5x^2 + 1.$$

(a) When $x = -1$, show that the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ is -7. (2)

Solution

$$y = x^3 + 5x^2 + 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 10x$$

and

$$\begin{aligned} x = -1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 3[(-1)^2] + 10(-1) \\ \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 3 - 10 \\ \Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = -7}}, \end{aligned}$$

as required.

(b) Work out the equation of the tangent to the curve (4)

$$y = x^3 + 5x^2 + 1.$$

at the point where $x = -1$.

Solution

$$\begin{aligned} x = -1 \Rightarrow y &= (-1)^3 + 5[(-1)^2] + 1 \\ \Rightarrow y &= -1 + 5 + 1 \\ \Rightarrow y &= 5 \end{aligned}$$

and the equation of the tangent is

$$\begin{aligned} y - 5 = -7(x + 1) &\Rightarrow y - 5 = -7x - 7 \\ &\Rightarrow \underline{\underline{y = -7x - 2}}. \end{aligned}$$

9. Write this ratio in its simplest form: (3)

$$\sqrt{12} : \sqrt{48} : \sqrt{300}.$$

Solution

$$\begin{aligned} \sqrt{12} : \sqrt{48} : \sqrt{300} &= \sqrt{4 \times 3} : \sqrt{16 \times 3} : \sqrt{100 \times 3} \\ &= \sqrt{4} \times \sqrt{3} : \sqrt{16} \times \sqrt{3} : \sqrt{100} \times \sqrt{3} \\ &= 2\sqrt{3} : 4\sqrt{3} : 10\sqrt{3} \\ &= \underline{\underline{1 : 2 : 5}}. \end{aligned}$$

10. The nth term of the linear sequence (4)

$$2 \quad 7 \quad 12 \quad 17 \quad \ldots$$

is $5n - 3$.

A new sequence is formed by squaring each term of the linear sequence and adding 1.

Prove algebraically that all the terms in the new sequence are multiples of 5.

Solution

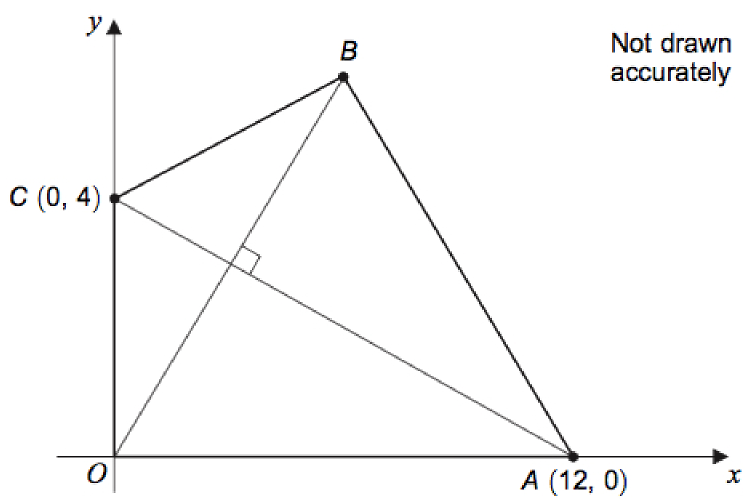
$\times$	$5n$	-3
$5n$	$25n^2$	$-15n$
-3	$-15n$	$+9$

The new sequence is

$$\begin{aligned}(5n-3)^2+1 &= (25n^2-30n+9)+1\\ &= 25n^2-30n+10\\ &= 5(5n^2-6n+2)\\ &= 5\times\text{some integer;}\end{aligned}$$

hence, the new sequence are multiples of 5.

11. $OABC$ is a kite.



(a) Work out the equation of AC. (2)

Solution
The gradient of AC is

$$\frac{4-0}{0-12} = -\tfrac{1}{3}$$

and the the equation of AC is

$$y - 0 = -\tfrac{1}{3}(x - 12) \Rightarrow \underline{\underline{y = -\tfrac{1}{3}x + 4}}.$$

(b) Work out the coordinates of B. (6)

Solution

Let's call the point where AC intersects with OB E. Then

$$m_{OB} = -\frac{1}{-\frac{1}{3}} = 3$$

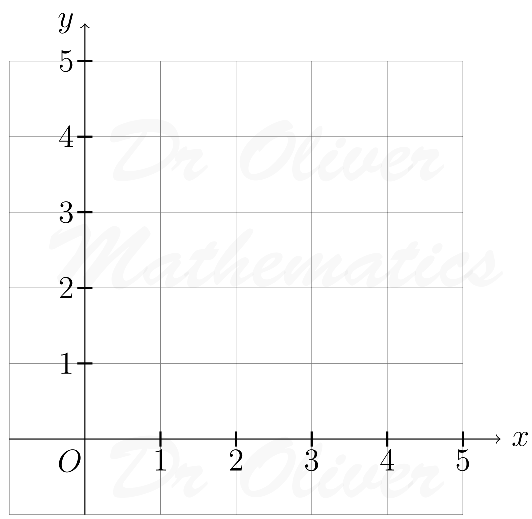
and, since $O(0, 0)$, the equation of the line through OB is $y = 3x$. Now,

$$\begin{aligned} 3x_E = -\tfrac{1}{3}x_E + 4 &\Rightarrow \tfrac{10}{3}x_E = 4 \\ &\Rightarrow x_E = 1.2 \\ &\Rightarrow x_B = 2.4 \\ &\Rightarrow y_B = 7.2. \end{aligned}$$

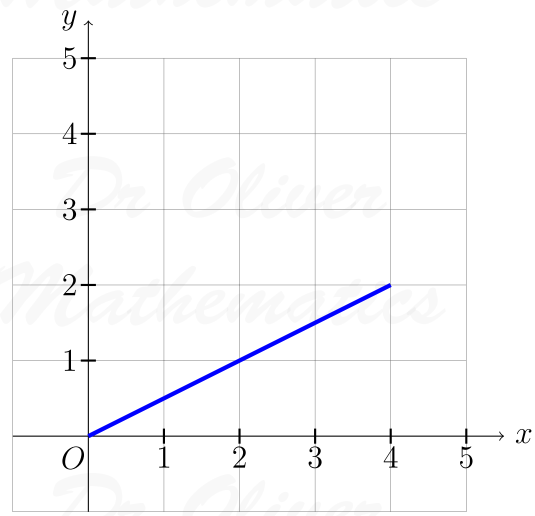
Hence, the coordinates of $\underline{\underline{B(2.4, 7.2)}}$.

12. A graph passes through $(0, 0)$.
The rate of change of y with respect to x is always $\frac{1}{2}$.

(a) Draw the graph of y for values of x from 0 to 4. (1)

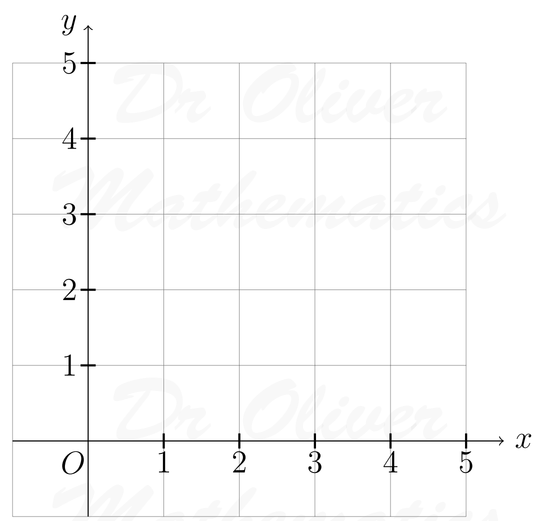


Solution

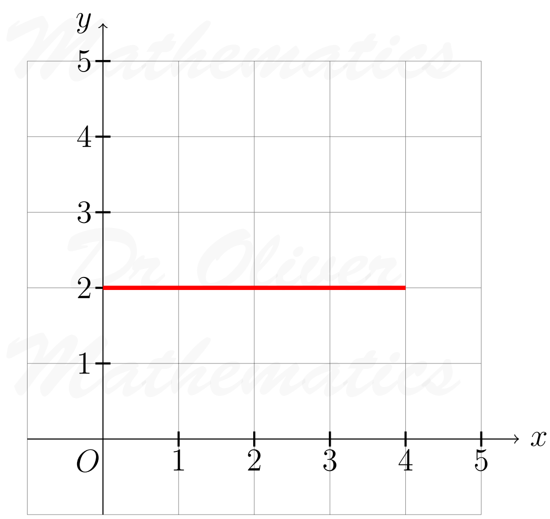


A graph passes through $(1, 2)$.
The rate of change of y with respect to x is always 0.

(b) Draw the graph of y for values of x from 0 to 4. (1)



Solution



$$y = 2x^3 + ax,$$

where a is a constant.

The value of $\dfrac{dy}{dx}$ when $x = 2$ is twice the value of $\dfrac{dy}{dx}$ when $x = -1$.

(c) Work out the value of a. (5)

Solution

$$y = 2x^3 + ax \Rightarrow \frac{dy}{dx} = 6x^2 + a.$$

Now,

$$\begin{aligned} 6(2^2) + a = 2\left\{6[(-1)^2 + a\right\} &\Rightarrow 24 + a = 2(6 + a) \\ &\Rightarrow 24 + a = 12 + 2a \\ &\Rightarrow \underline{\underline{a = 12}}. \end{aligned}$$

13. Simplify (5)

$$\frac{x^2 + 4x - 12}{x^2 - 25} \div \frac{x + 6}{x^2 - 5x}.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & +4 \\ \text{multiply to:} & -12 \end{array}\right\} -2,\ +6$$

So

$$x^2 + 4x - 12 = (x + 6)(x - 2).$$

$$\left.\begin{array}{ll}\text{add to:} & 0 \\ \text{multiply to:} & -25\end{array}\right\} -5,\ +5$$

So

$$x^2 - 25 = (x - 5)(x + 5).$$

Finally,

$$\begin{aligned}\frac{x^2 + 4x - 12}{x^2 - 25} \div \frac{x + 6}{x^2 - 5x} &= \frac{x^2 + 4x - 12}{x^2 - 25} \times \frac{x^2 - 5x}{x + 6} \\ &= \frac{(x + 6)(x - 2)}{(x - 5)(x + 5)} \times \frac{x(x - 5)}{x + 6} \\ &= \underline{\underline{\frac{x(x - 2)}{x + 5}}}.\end{aligned}$$

14. (5)

$$x^{\frac{3}{2}} = 8 \text{ where } x > 0$$

and

$$y^{-2} = \tfrac{25}{4} \text{ where } y > 0.$$

Work out the value of

$$\frac{x}{y}.$$

Solution

$$\begin{aligned}x^{\frac{3}{2}} = 8 \Rightarrow x &= 8^{\frac{2}{3}} \\ \Rightarrow x &= (8^{\frac{1}{3}})^2 \\ \Rightarrow x &= 2^2 \\ \Rightarrow x &= 4\end{aligned}$$

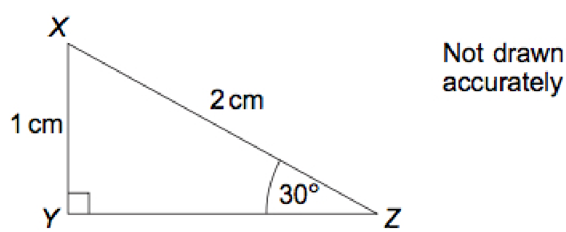
and

$$y^{-2} = \tfrac{25}{4} \Rightarrow y^2 = \tfrac{4}{25}$$
$$\Rightarrow y = \sqrt{\tfrac{4}{25}}$$
$$\Rightarrow y = \tfrac{\sqrt{4}}{\sqrt{25}}$$
$$\Rightarrow y = \tfrac{2}{5}.$$

Hence,

$$\frac{x}{y} = \frac{4}{\frac{2}{5}}$$
$$= \underline{\underline{10}}$$

15. XYZ is a right-angled triangle.



(a) Use triangle XYZ to show that (2)

$$\sin 60^\circ = \frac{\sqrt{3}}{2}.$$

Solution

$$XY^2 + YZ^2 = XZ^2 \Rightarrow 1^2 + YZ^2 = 2^2$$
$$\Rightarrow YZ^2 = 3$$
$$\Rightarrow YZ = \sqrt{3}$$

and

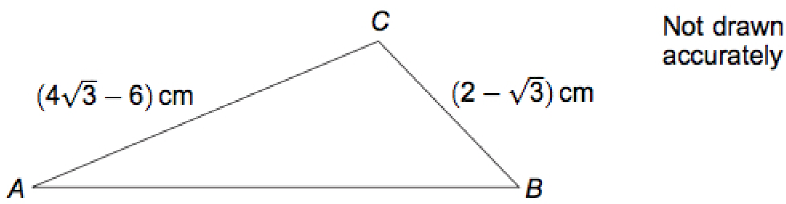
$$\angle XYZ = 180 - 90 - 30$$
$$= 60^\circ.$$

Hence,

$$\sin = \frac{\text{opp}}{\text{hyp}} \Rightarrow \underline{\underline{\sin 60^\circ = \frac{\sqrt{3}}{2}}},$$

as required.

Triangle ABC has an obtuse angle at C.



(b) Given that $\sin A = \frac{1}{4}$, use triangle ABC to show that angle $B = 60^\circ$. (6)

Solution

$$\frac{\sin B}{AC} = \frac{\sin A}{BC} \Rightarrow \frac{\sin B}{4\sqrt{3} - 6} = \frac{\frac{1}{4}}{2 - \sqrt{3}}$$

$$\Rightarrow \sin B = \frac{4\sqrt{3} - 6}{4(2 - \sqrt{3})}$$

$$\Rightarrow \sin B = \frac{(4\sqrt{3} - 6)(2 + \sqrt{3})}{4(2 - \sqrt{3})(2 + \sqrt{3})}$$

$\times$	$4\sqrt{3}$	-6
2	$8\sqrt{3}$	-12
$+\sqrt{3}$	$+12$	$-6\sqrt{3}$

$\times$	2	$-\sqrt{3}$
2	4	$-2\sqrt{3}$
$+\sqrt{3}$	$+2\sqrt{3}$	3

$$\begin{aligned}
&\Rightarrow \sin B = \frac{2\sqrt{3}}{4} \\
&\Rightarrow \sin B = \frac{\sqrt{3}}{2} \\
&\Rightarrow \underline{\underline{B = 60^\circ}},
\end{aligned}$$

as $\angle ACB$ is greater than 90°.

16. Prove that (3)

$$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}.$$

Solution

$$\begin{aligned}
\tan\theta + \frac{1}{\tan\theta} &\equiv \frac{\sin\theta}{\cos\theta} + \frac{1}{\frac{\sin\theta}{\cos\theta}} \\
&\equiv \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \\
&\equiv \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} \\
&\equiv \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} \\
&\equiv \underline{\underline{\frac{1}{\sin\theta\cos\theta}}},
\end{aligned}$$

as required.