Dr Oliver Mathematics
Mathematics: Higher
2022 Paper 1: Non-Calculator
1 hour 15 minutes

The total number of marks available is 55.
You must write down all the stages in your working.

1. Determine the equation of the line perpendicular to (3)

$$5x + 2y = 7,$$

passing through $(-1, 6)$.

2. Evaluate (3)

$$2 \log_3 6 - \log_3 4.$$

3. A function, h, is defined by (3)

$$\mathrm{h}(x) = 4 + \tfrac{1}{3}x,$$

where $x \in \mathbb{R}$.

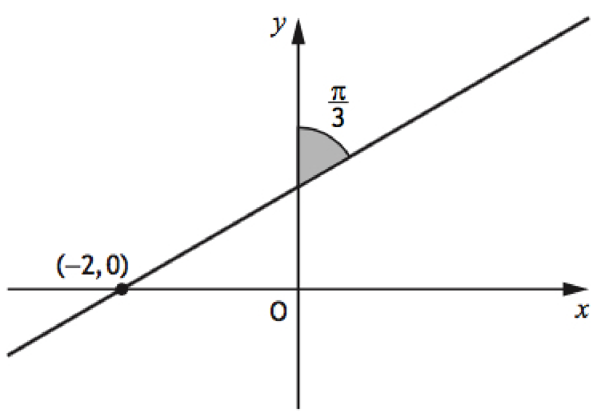
Find the inverse function, $\mathrm{h}^{-1}(x)$.

4. Differentiate (3)

$$y = \sqrt{x^3} - 2x^{-1},$$

where $x > 0$.

5. A line makes an angle of $\frac{1}{3}\pi$ radians with the y-axis, and passes through the point $(-2, 0)$ as shown below. (3)

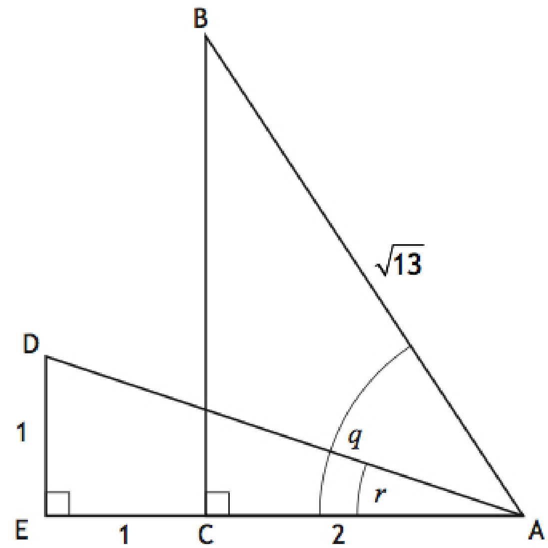


Determine the equation of the line.

6. Evaluate (4)

$$\int_{-5}^{2} (10-3x)^{-\frac{1}{2}}\,\mathrm{d}x.$$

7. Triangles ABC and ADE are both right angled.
Angle $BAC = q$ and angle $DAE = r$,as shown in the diagram.



(a) Determine the value of:

(i) $\sin r$, (1)

(ii) $\sin q$. (1)

(b) Hence determine the value of $\sin(q-r)$. (3)

8. Solve (4)

$$\log_6 x + \log_6(x+5) = 2,$$

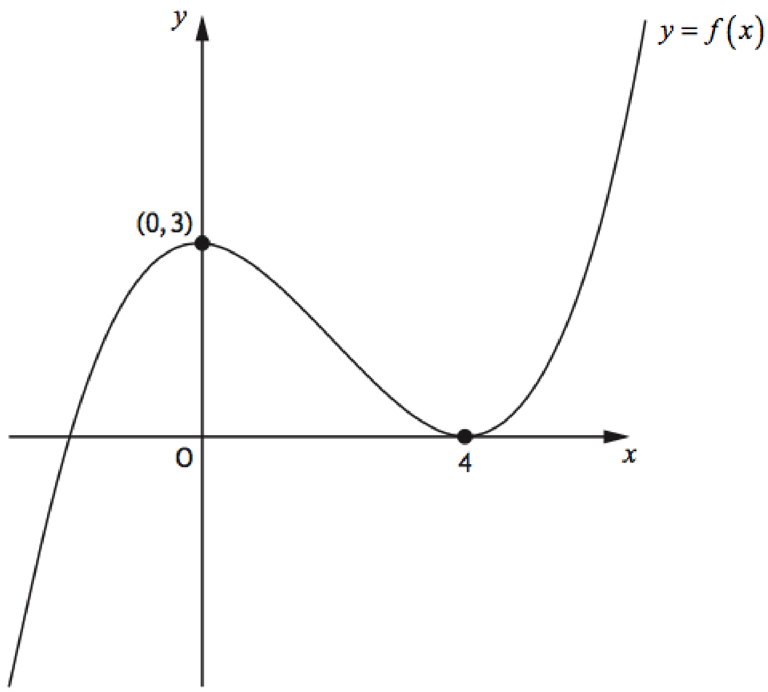
where $x > 0$.

9. Solve the equation (5)

$$\cos 2x^\circ = 5\cos x^\circ - 3,$$

for $0 \leqslant x < 360$.

10. The diagram shows the graph of a cubic function with equation $y = \mathrm{f}(x)$.
The curve has stationary points at $(0,3)$ and $(4,0)$.



(a) Sketch the graph of (3)

$$y = 2\,\mathrm{f}(x) + 1.$$

(b) State the coordinates of the stationary points on the graph of $y = \mathrm{f}(\frac{1}{2}x)$. (1)

11. Express (3)

$$2x^2 + 12x + 23$$

in the form

$$p(x + q)^2 + r.$$

12. Given that (3)

$$\mathrm{f}(x) = 4\sin(3x - \tfrac{1}{3}\pi),$$

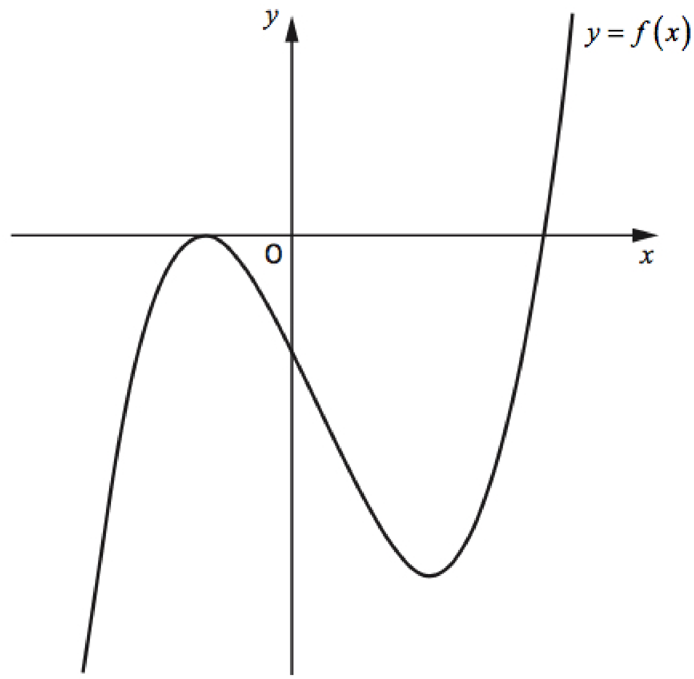
evaluate $\mathrm{f}'(\frac{1}{6}\pi)$.

13. (a) (i) Show that $(x + 2)$ is a factor of (2)

$$\mathrm{f}(x) = x^3 - 2x^2 - 20x - 24.$$

(ii) Hence, or otherwise, solve $\mathrm{f}(x) = 0$. (3)

The diagram shows the graph of $y = \mathrm{f}(x)$.



The graph of $y = \mathrm{f}(x - k)$, $k > 0$, has a stationary point at $(1, 0)$.

(b) State the value of k. (1)

14. C_1 is the circle with equation

$$(x - 7)^2 + (y + 5)^2 = 100.$$

(a) (i) State the centre and radius of C_1. (2)

(ii) Hence, or otherwise, show that the point $P(-2, 7)$ lies outside C_1. (2)

C_2 is a circle with centre P and radius r.

(b) Determine the value(s) of r for which circles C_1 and C_2 have exactly one point of intersection. (2)