# Dr Oliver Mathematics
# Advance Level Further Mathematics
# Further Mathematics 2: Calculator
# 1 hour 30 minutes

The total number of marks available is 75.
You must write down all the stages in your working.

1. (a) Express (1)

$$\frac{1}{(r+3)(r+4)}$$

in partial fractions.

(b) Hence, using the method of differences, show that (5)

$$\sum_{r=1}^{n} \frac{1}{(r+3)(r+4)} = \frac{n}{a(n+a)},$$

where $a$ is a constant to be found.

(c) Find the exact value of (2)

$$\sum_{r=15}^{30} \frac{1}{(r+3)(r+4)}.$$

2. A transformation from the $z$-plane to the $w$-plane is given by (4)

$$w = \frac{1 - \mathrm{i}z}{z}, \; z \neq 0.$$

The transformation maps points on the real axis in the $z$-plane onto the line $l$ in the $w$-plane.
Find an equation of the line $l$.

3. (a) By writing (4)

$$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4},$$

show that

(i) $\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$,

(ii) $\cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$

(b) Hence find the exact values of $z$ for which (5)

$$z^4 = 4\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right).$$

Give your answers in the form $z = a + \mathrm{i}b$ where $a, b \in \mathbb{Z}$.

4. Use algebra to find the set of values of $x$ for which (7)

$$|x^2 - 2| > 4x.$$

5.

$$y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 3x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y^2 = 0.$$

Given that at $x = 0$, $y = 2$, and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$,

(a) show that, at $x = 0$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \frac{3}{2}$. (6)

(b) Find a series solution for $y$ up to and including the term in $x^3$. (3)

6. (a) Find the general solution of the differential equation (8)

$$6\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} - 6y = x - 6x^2.$$

(b) Find the particular solution for which $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2}$ when $x = 0$. (5)

7. The curve $C$ shown in Figure 1 has polar equation

$$r = 2 + \sqrt{3}\cos\theta,\ 0 \leqslant \theta < 2\pi.$$

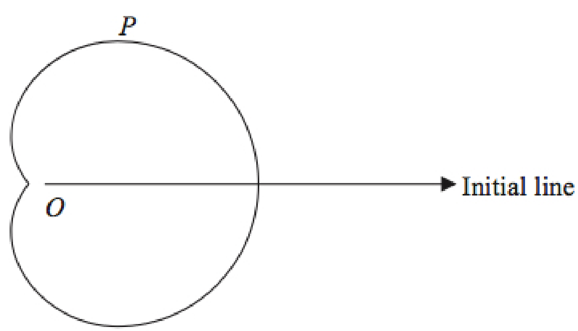


Figure 1: $r = 2 + \sqrt{3}\cos\theta$

The tangent to $C$ at the point $P$ is parallel to the initial line.

(a) Show that (6)

$$OP = \tfrac{1}{2}(3 + \sqrt{7}).$$

(b) Find the exact area enclosed by the curve $C$. (6)

8. (a) Using the substitution $t = x^2$, or otherwise, find (6)

$$\int 2x^5 e^{-x^2}\, dx.$$

(b) Hence find the general solution of the differential equation (4)

$$x\frac{dy}{dx} + 4y = 2x^2 e^{-x^2},$$

giving your answer in the form $y = f(x)$.

Given that $y = 0$ when $x = 1$,

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(x)$. (3)