Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2007 November Paper 1: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Given that (4)

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix},$$

find the value of each of the constants m and n for which

$$\mathbf{A}^2 + m\mathbf{A} = n\mathbf{I},$$

where $\mathbf{I}$ is the identity matrix.

2. Show that (4)

$$\frac{1}{1-\cos\theta} - \frac{1}{1+\cos\theta} \equiv 2\operatorname{cosec}\theta\cot\theta.$$

3. Given that

$$p = \frac{\sqrt{3}+1}{\sqrt{3}-1},$$

express in its simplest surd form,

(a) p, (3)

(b) $p - \dfrac{1}{p}$. (2)

4. A badminton team of 4 men and 4 women is to be selected from 9 men and 6 women.

(a) Find the total number of ways in which the team can be selected if there are no restrictions on the selection. (3)

Two of the men are twins.

(b) Find the number of ways in which the team can be selected if exactly one of the twins is in the team. (3)

5. In this question, $\mathbf{i}$ is a unit vector due east, and $\mathbf{j}$ is a unit vector due north.

A plane flies from P to Q where

$$\overrightarrow{PQ} = (960\mathbf{i} + 400\mathbf{j}) \text{ km}.$$

A constant wind is blowing with velocity

$$(-60\mathbf{i} + 60\mathbf{j}) \text{ km h}^{-1}.$$

Given that the plane takes 4 hours to travel from P to Q, find

(a) the velocity, in still air, of the plane, giving your answer in the form $(a\mathbf{i} + b\mathbf{j})$ km h^{-1}. (4)

(b) the bearing, to the nearest degree, on which the plane must be directed. (2)

6. A curve is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6}{\sqrt{4x+1}},$$

and $(6, 20)$ is a point on the curve.

(a) Find the equation of the curve. (4)

A line with gradient $-\frac{1}{2}$ is a normal to the curve.

(b) Find the coordinates of the points at which this normal meets the coordinate axes. (4)

7. (a) Use the substitution $u = 2^x$ to solve the equation (5)

$$2^{2x} = 2^{x+2} + 5.$$

(b) Solve the equation (4)

$$2\log_9 3 + \log_5(7y - 3) = \log_2 8.$$

8. (a) The remainder when the expression (4)

$$x^3 - 11x^2 + kx - 30$$

is divided by $(x - 1)$ is 4 times the remainder when this expression is divided by $(x - 2)$.

Find the value of the constant k.

(b) Solve the equation (5)

$$x^3 - 4x^2 - 8x + 8 = 0,$$

expressing non-integer solutions in the form $a \pm \sqrt{b}$, where a and b are integers.

9. The table shows experimental values of two variables, x and y.

x	2	4	6	8	10
y	14.4	10.8	11.2	12.6	14.4

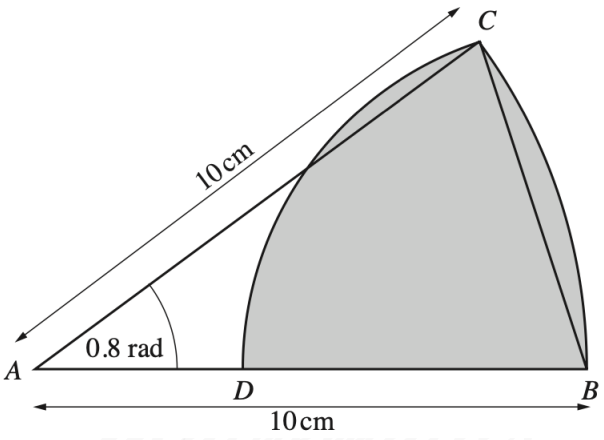
(a) Using graph paper, plot xy against x^2. (2)

(b) Use the graph of xy against x^2 to express y in terms of x. (4)

(c) Find the value of y for which (3)

$$y = \frac{83}{x}.$$

10. The diagram shows a sector ABC of the circle, centre A and radius 10 cm, in which angle $BAC = 0.8$ radians.



The arc CD of a circle has centre B and the point D lies on AB.

(a) Show that the length of the straight line BC is 7.79 cm, correct to 2 decimal places. (2)

(b) Find the perimeter of the shaded region. (4)

(c) Find the area of the shaded region. (4)

EITHER

11. A curve has the equation

$$y = x\mathrm{e}^{2x}.$$

(a) Obtain expressions for (5)

$$\frac{\mathrm{d}y}{\mathrm{d}x} \text{ and } \frac{\mathrm{d}^2y}{\mathrm{d}x^2}.$$

(b) Show that the y-coordinate of the stationary point of the curve is $-\frac{1}{2\mathrm{e}}$. (3)

(c) Determine the nature of this stationary point. (2)

OR

12. (a) Show that (3)

$$\frac{\mathrm{d}y}{\mathrm{d}x}\left(\frac{\ln x}{x^2}\right) = \frac{1-2\ln x}{x^3}.$$

(b) Show that the y-coordinate of the stationary point of the curve (3)

$$y = \frac{\ln x}{x^2}$$

is $\frac{1}{2\mathrm{e}}$.

(c) Use the result from part (a) to find (4)

$$\int \left(\frac{\ln x}{x^3}\right) \mathrm{d}x.$$