Dr Oliver Mathematics
GCSE Mathematics
2020 Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Write 84 as a product of its prime factors. (2)

Solution

$$\begin{array}{r|l} & 84 \\ \hline 2 & 42 \\ \hline 2 & 21 \\ \hline 3 & 7 \\ \hline 7 & 1 \\ \hline \end{array}$$

So
$$84 = 2 \times 2 \times 3 \times 7 = \underline{\underline{2^2 \times 3 \times 7}}.$$

(b) Find the lowest common multiple (LCM) of 60 and 84. (2)

Solution

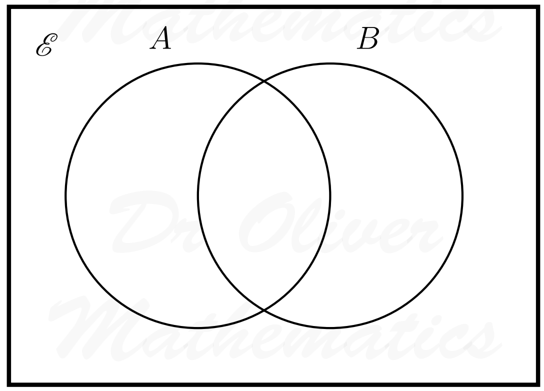
$$\begin{array}{r|l} & 60 \\ \hline 2 & 30 \\ \hline 2 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

So
$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5.$$

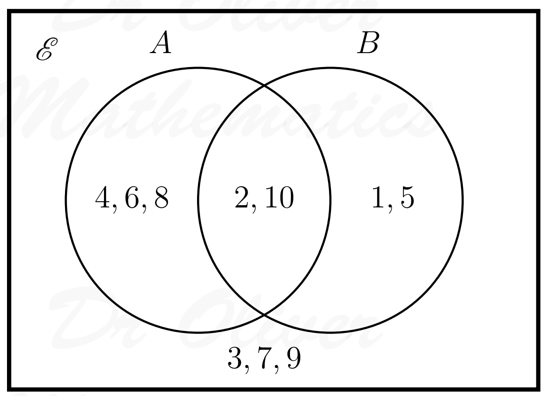
Hence,
$$\text{LCM}(60, 84) = 2^2 \times 3 \times 5 \times 7 = \underline{\underline{420}}.$$

2. $\mathscr{E} = \{1,2,3,4,5,6,7,8,9,10.$
$A = \{\text{even numbers}\}$.
$B = \{\text{factors of } 10\}$.

(a) Complete the Venn diagram for this information. (3)



Solution



A number is chosen at random from the universal set, $\mathscr{E}$.

(b) Find the probability that this number is in the set $A \cap B$. (2)

Solution

$$\begin{aligned} \mathrm{P}(A \cap B) &= \frac{2}{10} \\ &= \underline{\underline{\tfrac{1}{5}}}. \end{aligned}$$

3. Carlo puts tins into small boxes and into large boxes. (5)

He puts 6 tins into each small box.
He puts 20 tins into each large box.

Carlo puts a total of 3 000 tins into the boxes so that

$$\text{number of tins in small boxes} : \text{number of tins in large boxes} = 2 : 3.$$

Carlo says that less than 30% of the boxes filled with tins are large boxes.

Is Carlo correct?
You must show all your working.

Solution

Well,

$$\frac{3\,000}{2+3} = \frac{3\,000}{5} = 600.$$

That means

$$2 \times 600 = 1\,200$$

in small boxes and

$$3 \times 600 = 1\,800$$

in large boxes. He puts 6 tins into each small box:

$$\frac{1\,200}{6} = 200.$$

He puts 20 tins into each large box:

$$\frac{1\,800}{20} = 90.$$

So, the percentage of the boxes filled with tins are large boxes is

$$\left(\frac{90}{200+90}\right) \times 100\% = 31.034\ldots\%.$$

Hence, Carlo is <u>incorrect</u>.

4. (a) Complete the table of values for (2)

$$y = 5 - x^3.$$

x	-2	-1	0	1	2
y		6			

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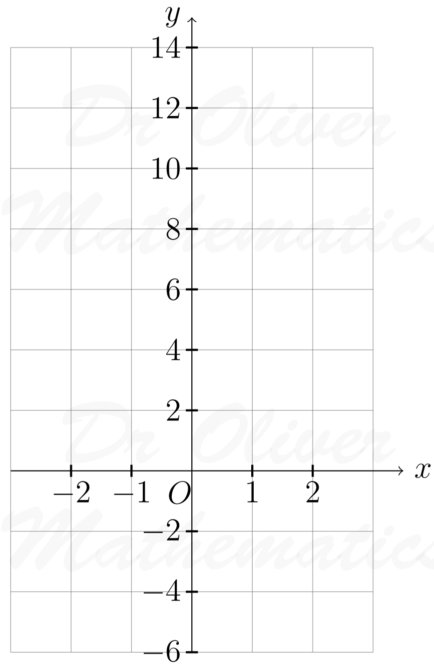
Solution

x	-2	-1	0	1	2
y	$\underline{\underline{13}}$	6	$\underline{\underline{5}}$	$\underline{\underline{4}}$	$\underline{\underline{-3}}$

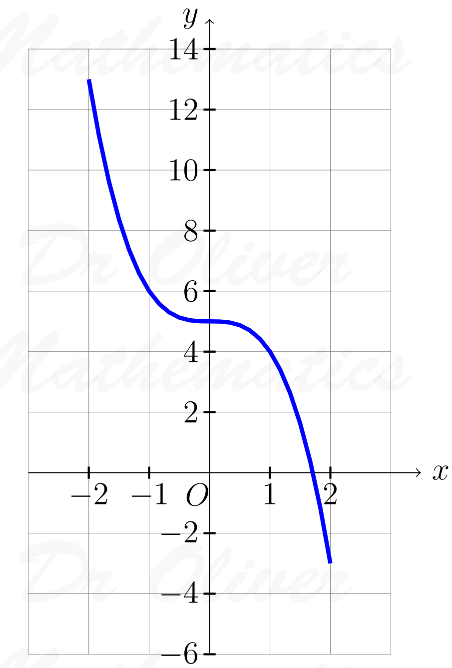
(b) On the grid below, draw the graph of (2)

$$y = 5 - x^3$$

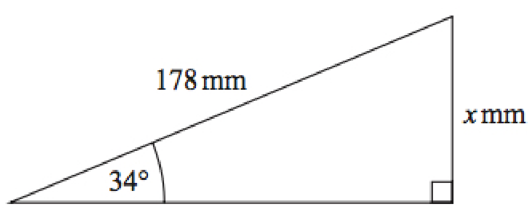
for values of x from -2 to 2.



Solution



5. Work out the value of x. (2)



Give your answer correct to 1 decimal place.

Solution

$$\begin{aligned} \sin = \frac{\text{opp}}{\text{hyp}} &\Rightarrow \sin 34^\circ = \frac{x}{178} \\ &\Rightarrow x = 178 \sin 34^\circ \\ &\Rightarrow x = 99.536\,336\,82 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 99.5 \text{ mm (1 dp)}}}. \end{aligned}$$

6. $$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}.$$ (2)

Find

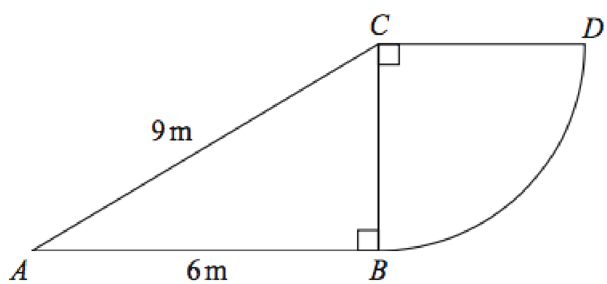
$$2\mathbf{a} - 3\mathbf{b}$$

as a column vector.

Solution

$$\begin{aligned} 2\mathbf{a} - 3\mathbf{b} &= 2\begin{pmatrix} 3 \\ 4 \end{pmatrix} - 3\begin{pmatrix} 5 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ 8 \end{pmatrix} - \begin{pmatrix} 15 \\ -6 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} -9 \\ 14 \end{pmatrix}}}. \end{aligned}$$

7. The diagram shows a right-angled triangle and a quarter circle. (4)



The right-angled triangle ABC has angle $ABC = 90°$.
The quarter circle has centre C and radius CB.

Work out the area of the quarter circle.
Give your answer correct to 3 significant figures.
You must show all your working.

Solution

Pythagoras' theorem:

$$\begin{aligned} AC^2 = AB^2 + BC^2 &\Rightarrow 9^2 = 6^2 + BC^2 \\ &\Rightarrow BC^2 = 45 \end{aligned}$$

and

$$\begin{aligned} \text{area} &= \tfrac{1}{4} \times \pi \times 45 \\ &= 35.342\,917\,35 \text{ (FCD)} \\ &= \underline{\underline{35.3 \text{ m}^2 \text{ (3 sf)}}}. \end{aligned}$$

8. Tariq buys a laptop.

He gets a discount of 5% off the normal price.
Tariq pays £551 for the laptop.

(a) Work out the normal price of the laptop. (2)

Solution

The normal price is

$$\frac{551}{0.95} = \underline{\underline{£580}}.$$

Joan invests £6 000 in a savings account.
The savings account pays compound interest at a rate of

- 2.4% for the first year and
- 1.7% for each extra year.

(b) Work out the value of Joan's investment at the end of 3 years. (3)

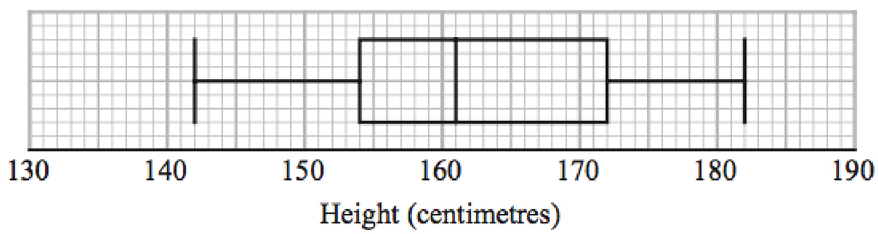
Solution

$$\begin{aligned} \text{Value} &= 6\,000 \times 1.024 \times (1.017)^2 \\ &= 6\,353.671\,616 \text{ (FCD)} \\ &= \underline{\underline{£6\,353.67 \text{ (nearest penny)}}}. \end{aligned}$$

9. Aisha recorded the heights, in centimetres, of some girls. (2)
She used her results to work out the information in this table.

Least height	142 cm
Lower quartile	154 cm
Interquartile range	17 cm
Median	162 cm
Range	40 cm

Aisha drew this box plot for the information in the table.
The box plot is **not** fully correct.



Write down the two things Aisha should do to make the box plot fully correct.

Solution

The median is incorrect: 162, not 161.
The IQR is incorrect: 17, not $172 - 154 = 18$.

10. (a) Simplify (1)

$$\left(\frac{1}{m^2}\right)^0.$$

Solution

$$\left(\frac{1}{m^2}\right)^0 = \underline{\underline{1}}.$$

(b) Simplify (1)

$$\frac{8(x-4)}{(x-4)^2}.$$

Solution

$$\frac{8(x-4)}{(x-4)^2} = \underline{\underline{\frac{8}{x-4}}}.$$

(c) Simplify (2)

$$(3n^4w^2)^3.$$

Solution

$$(3n^4w^2)^3 = \underline{\underline{27n^{12}w^6}}.$$

11. Jack is in a restaurant. (2)
There are 5 starters, 8 main courses and some desserts on the menu.

Jack is going to choose one starter, one main course, and one dessert.
He says there are 240 ways that he can choose his starter, his main course, and his dessert.

Could Jack be correct?
You must show how you get your answer.

Solution

$$\frac{240}{5 \times 8} = 6.$$

So, provided they do $\underline{\underline{\text{6 desserts}}}$, Jack will be correct.

12. The graph gives information about the volume, v litres, of petrol in the tank of Jim's car after it has travelled a distance of d kilometres.

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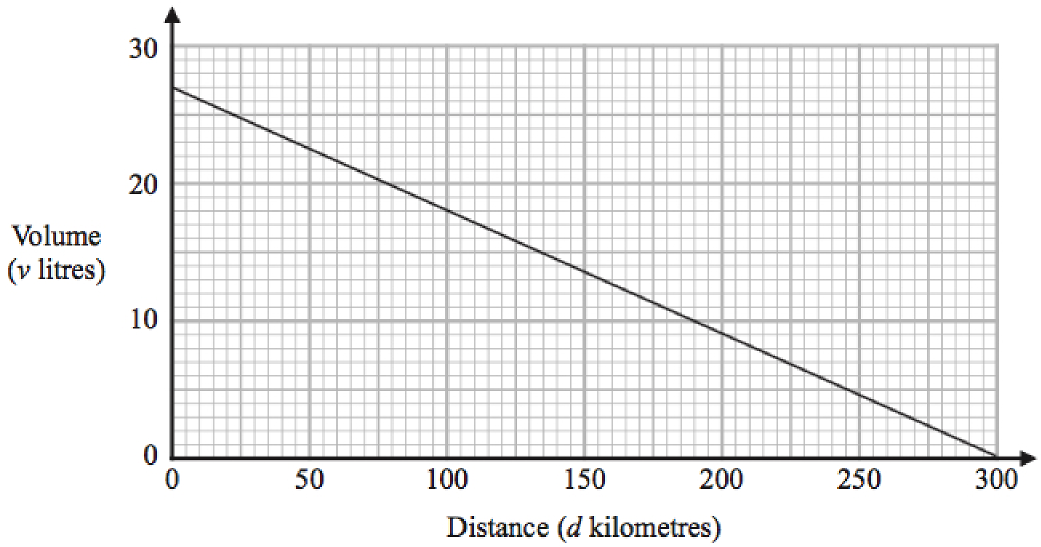

(a) Find the gradient of the graph. (2)

Solution

We will choose $(0, 27)$ and $(300, 0)$:

$$\begin{aligned}\text{Gradient} &= \frac{27 - 0}{0 - 300} \\ &= \underline{\underline{-0.09}}.\end{aligned}$$

(b) Interpret what the gradient of the graph represents. (1)

Solution

E.g., for every $\underline{\underline{\text{9 litres}}}$, you can travel $\underline{\underline{\text{100 kilometres}}}$.

13. Here is triangle ABC. (3)

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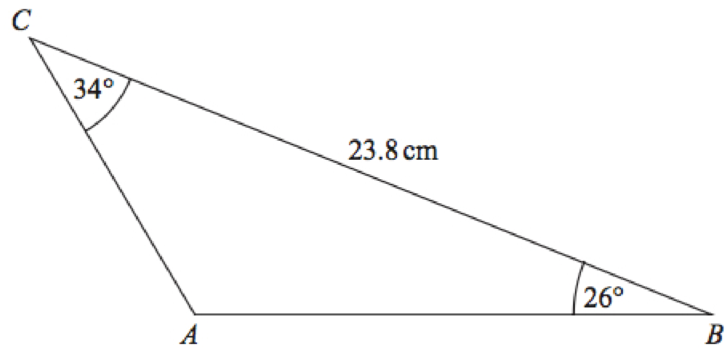


Work out the length of AB.
Give your answer correct to 1 decimal place.

Solution

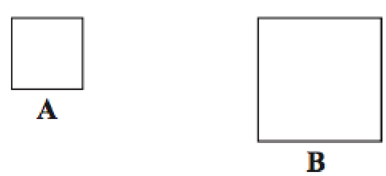
Well,

$$\angle CAB = 180 - (34 + 26) = 120^\circ$$

and we use the sine rule:

$$\begin{aligned} \frac{AB}{\sin ACB} = \frac{BC}{\sin CAB} &\Rightarrow \frac{AB}{\sin 34^\circ} = \frac{23.8}{\sin 120^\circ} \\ &\Rightarrow AB = \frac{23.8 \sin 34^\circ}{\sin 120^\circ} \\ &\Rightarrow AB = 15.367\,668\,25 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{AB = 15.4 \text{ cm (1 dp)}}}. \end{aligned}$$

14. Here are two squares, **A** and **B**. (4)



The length of each side of square **B** is 4 cm greater than the length of each side of square **A**.
The area of square **B** is 70 cm^2 greater than the area of square **A**.

Find the area of square **B**.
Give your answer correct to 3 significant figures.
You must show all your working.

Solution

Let the side of the square **A** be x cm. Then the side of the square **B** be $(x+4)$ cm. Now,

$$\begin{aligned}(x+4)^2 - x^2 = 70 &\Rightarrow (x^2 + 8x + 16) - x^2 = 70 \\ &\Rightarrow 8x = 54 \\ &\Rightarrow x = 6.75\end{aligned}$$

so the area of square **B** is

$$\begin{aligned}(4 + 6.75)^2 &= 115.562\,5 \text{ (exact!)} \\ &= \underline{\underline{116 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

15. Describe fully the single transformation that maps triangle **A** onto triangle **B**. (2)

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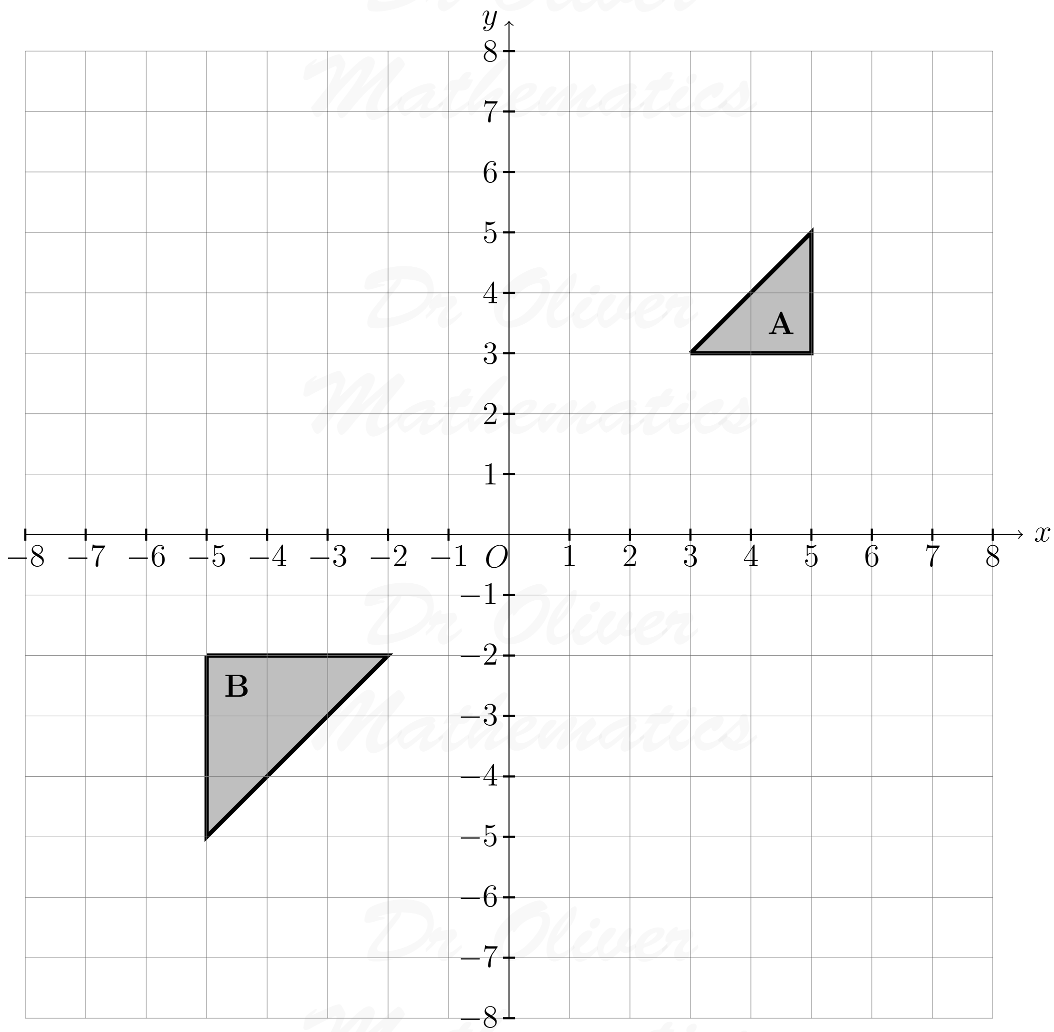
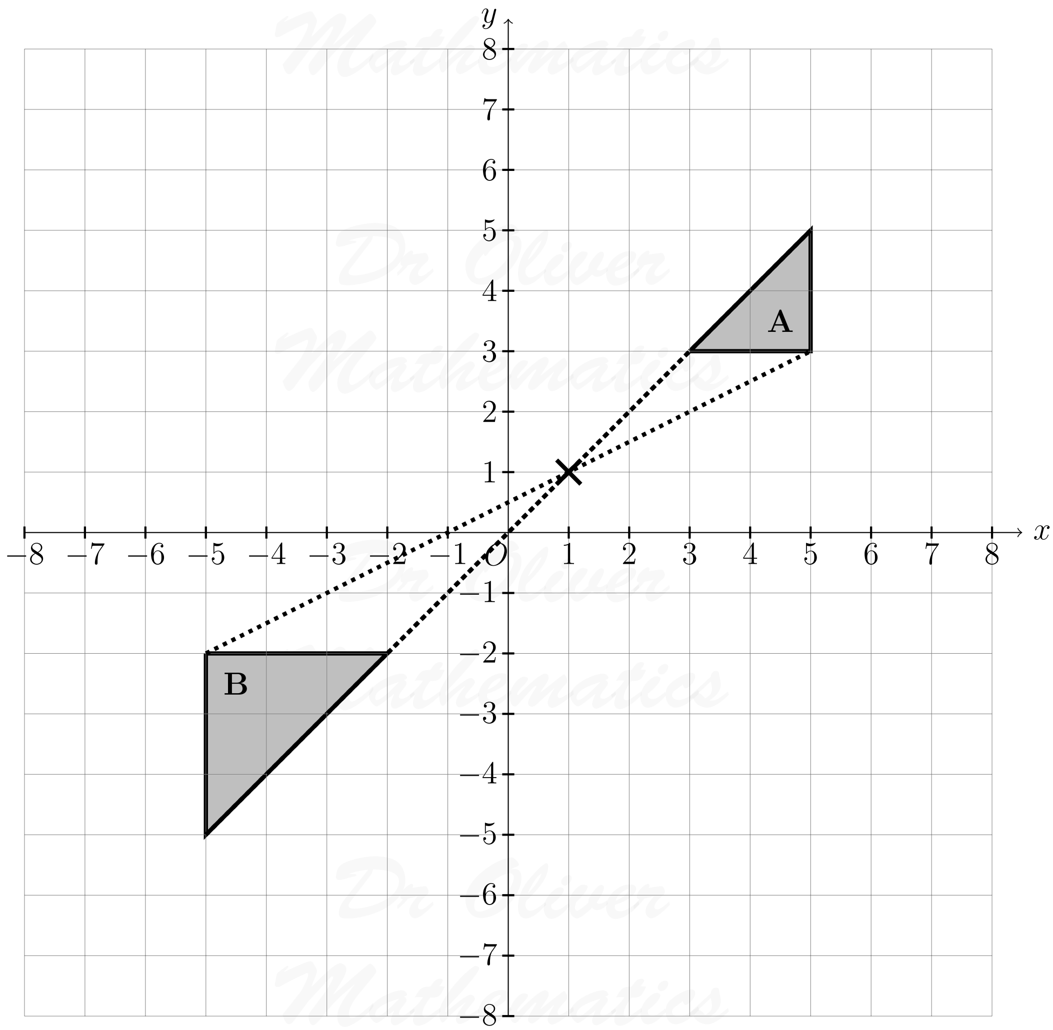


Solution



Enlargement, scale factor $-\frac{3}{2}$, centre $(1, 1)$.

16. Here are the first five terms of a quadratic sequence: (3)

$$10 \qquad 21 \qquad 38 \qquad 61 \qquad 90.$$

Find an expression, in terms of n, for the nth term of this sequence.

Solution

Let the

$$n\text{th term} = an^2 + bn + c.$$

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We only need the second line of differences (why?):

$$
\begin{array}{ccccc}
10 & & 21 & & 38 \\
 & 11 & & 17 & \\
 & & 6 & &
\end{array}
$$

$$
\begin{array}{ccccc}
a+b+c & & 4a+2b+c & & 9a+3b+c \\
 & 3a+b & & 5a+b & \\
 & & 2a & &
\end{array}
$$

We compare terms:

$$2a = 6 \Rightarrow a = 3,$$

$$\begin{aligned} 3a + b = 6 &\Rightarrow 3 \times 3 + b = 11 \\ &\Rightarrow b = 2, \end{aligned}$$

and

$$\begin{aligned} a + b + c = 10 &\Rightarrow 3 + 2 + c = 10 \\ &\Rightarrow c = 5; \end{aligned}$$

hence,

$$n\text{th term} = \underline{\underline{3n^2 + 2n + 5}}.$$

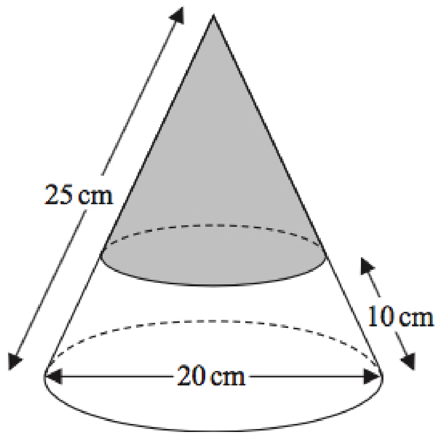
17. Write down the coordinates of the turning point on the graph of (1)

$$y = (x + 12)^2 - 7.$$

Solution

$\underline{\underline{(-12, -7)}}$.

18. The diagram represents a solid cone. (4)

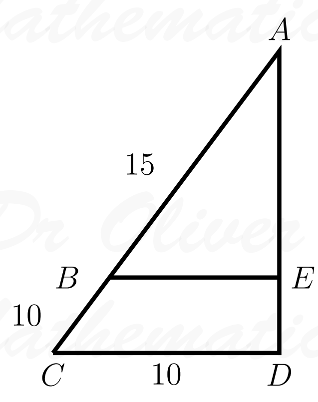


The cone has a base diameter of 20 cm and a slant height of 25 cm.

A circle is drawn around the surface of the cone at a slant height of 10 cm above the base.
The curved surface of the cone above the circle is painted grey.

Work out the area of the curved surface of the cone that is not painted grey.
Give your answer as a multiple of π.
You must show all your working.

Solution



Similar figures:

$$\frac{BE}{BA} = \frac{CD}{CA} \Rightarrow \frac{BE}{15} = \frac{10}{10+15}$$
$$\Rightarrow BE = \frac{10 \times 15}{25}$$
$$\Rightarrow BE = 6 \text{ cm.}$$

Hence,

$$\begin{aligned} \text{not painted grey} &= \text{whole cone} - \text{painted grey} \\ &= (\pi \times 25 \times 10) - (\pi \times 15 \times 6) \\ &= 250\pi - 90\pi \\ &= \underline{\underline{160\pi}}. \end{aligned}$$

19. A hot air balloon is descending. (4)
The height of the balloon n minutes after it starts to descend is h_n metres.

The height of the balloon $(n+1)$ minutes after it starts to descend, h_{n+1} metres, is given by

$$h_{n+1} = K \times h_n + 20,$$

where K is a constant.

The balloon starts to descend from a height of 1 200 metres at 09.15.
At 09.16 the height of the balloon is 1 040 metres.

Work out the height of the balloon at 09.18.

Solution
Well,

$$h_0 = 1\,200$$

and

$$\begin{aligned} h_1 = Kh_0 + 20 &\Rightarrow 1\,040 = K1\,200 + 20 \\ &\Rightarrow 1\,020 = K1\,200 \\ &\Rightarrow K = 0.85; \end{aligned}$$

so,

$$h_{n+1} = 0.85h_n + 20.$$

Finally,

$$\begin{aligned} h_2 &= 0.85(1\,040) + 20 \\ &= 904 \\ h_3 &= 0.85(904) + 20 \\ &= \underline{\underline{788.4 \text{ m}}}. \end{aligned}$$

20. There are only red sweets and yellow sweets in a bag.

There are n red sweets in the bag.
There are 8 yellow sweets in the bag.

Sajid is going to take at random a sweet from the bag and eat it.

He says that the probability that the sweet will be red is $\frac{7}{10}$.

(a) Show why the probability cannot be $\frac{7}{10}$. (3)

Solution

The probability that the sweet will be red is

$$\frac{n}{n+8}.$$

Can this be equal to $\frac{7}{10}$? Well,

$$\begin{aligned} \frac{n}{n+8} = \tfrac{7}{10} &\Rightarrow 10n = 7(n+8) \\ &\Rightarrow 10n = 7n + 56 \\ &\Rightarrow 3n = 56 \\ &\Rightarrow n = 18\tfrac{2}{3}. \end{aligned}$$

A fraction of a sweet! Hence, the probability *cannot* be $\underline{\underline{\frac{7}{10}}}$.

After Sajid has taken the first sweet from the bag and eaten it, he is going to take at random a second sweet from the bag.

Given that the probability that both the sweets he takes will be red is $\frac{3}{5}$,

(b) work out the number of red sweets in the bag. (5)
You must show all your working.

Solution

$$\mathrm{P}(RR) = \tfrac{3}{5} \Rightarrow \frac{n}{n+8} \times \frac{n-1}{n+7} = \tfrac{3}{5}$$
$$\Rightarrow 5n(n-1) = 3(n+7)(n+8)$$

$\times$	n	$+7$
n	n^2	$+7n$
$+8$	$+8n$	$+56$

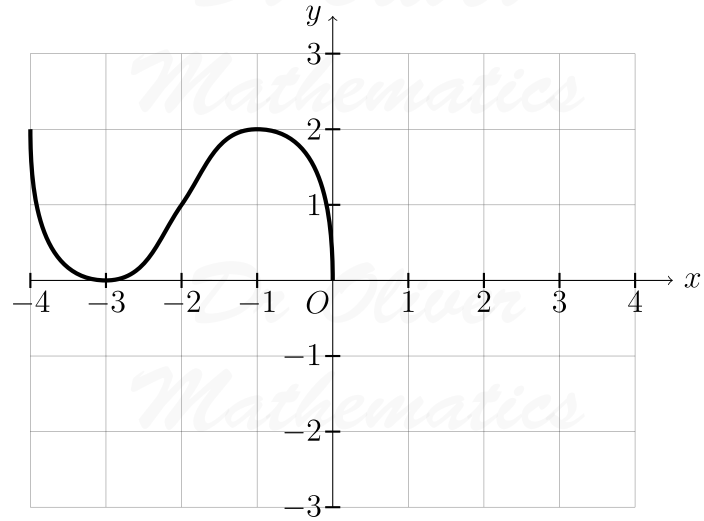
$$\begin{aligned}
&\Rightarrow 5n(n-1) = 3(n^2 + 15n + 56) \\
&\Rightarrow 5n^2 - 5n = 3n^2 + 45n + 168 \\
&\Rightarrow 2n^2 - 50n - 168 = 0 \\
&\Rightarrow 2(n^2 - 25n - 84) = 0
\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -25 \\ \text{multiply to:} & -84\end{array}\right\} -28,\ +3$$

$$\begin{aligned}
&\Rightarrow 2(n-28)(n+3) = 0 \\
&\Rightarrow n = 28 \text{ or } n = -3;
\end{aligned}$$

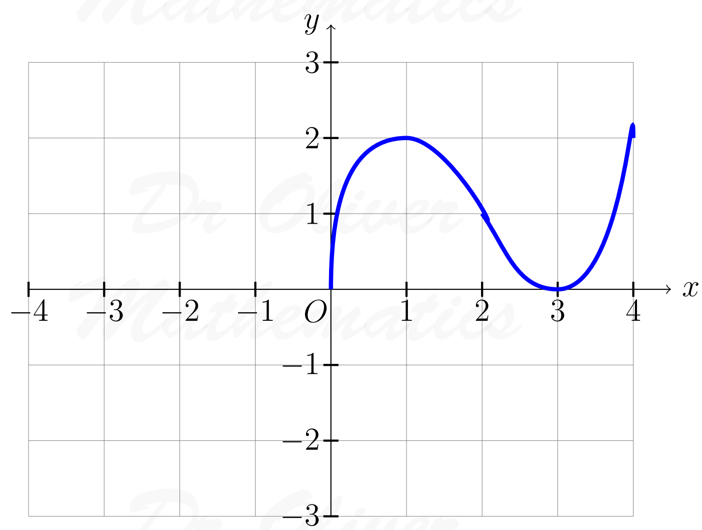
$n \neq -3$ and, hence, $\underline{\underline{n = 28}}$.

21. The graph of the curve with equation $y = \mathrm{f}(x)$ is shown on the grid below.



(a) Sketch the graph of the curve with equation $y = \mathrm{f}(-x)$. (2)

Solution



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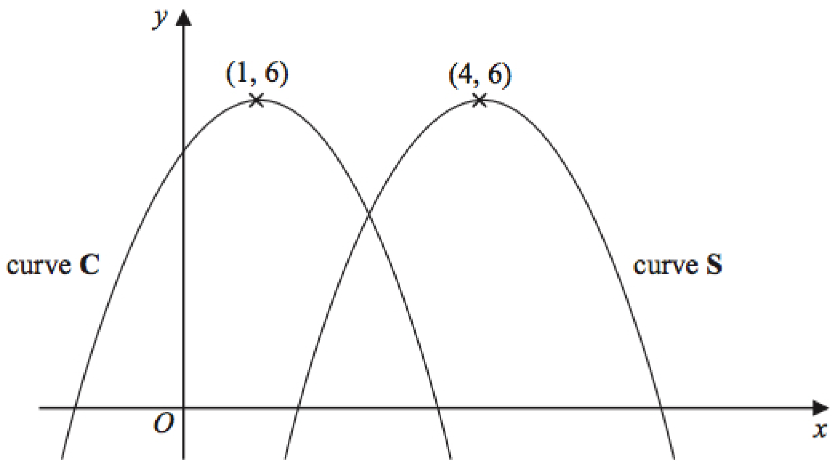
The curve **C** with equation

$$y = 5 + 2x - x^2$$

is transformed by a translation to give the curve **S** such that the point (1, 6) on **C** is mapped to the point $(4, 6)$ on **S**.



(b) Find an equation for **S**. (2)

Solution

E.g.,

$$\underline{\underline{y = 5 + 2(x-3) - (x-3)^2}}.$$

22. **C** is a circle with centre the origin. (5)
A tangent to **C** passes through the points $(-20, 0)$ and $(0, 10)$.

Work out an equation of **C**.
You must show all your working.

Solution

$$\begin{aligned} m_{\text{tangent}} &= \frac{10-0}{0-(-20)} \\ &= \tfrac{1}{2} \end{aligned}$$

which gives

$$y = \tfrac{1}{2}x + c,$$

for some constant c. Now,

$$x = 0, y = 10 \Rightarrow 10 = 0 + c \Rightarrow c = 10$$

and the equation of tangent is

$$y = \tfrac{1}{2}x + 10.$$

Next, the $m_{normal} = -2$ which means that the equation of the normal is

$$y = -2x + d,$$

for some constant d. But it goes through the origin! So

$$y = -2x$$

and

$$\begin{aligned} \tfrac{1}{2}x + 10 = -2x &\Rightarrow \tfrac{5}{2}x = -10 \\ &\Rightarrow x = -4 \\ &\Rightarrow y = 8; \end{aligned}$$

Hence, the circle has equation

$$x^2 + y^2 = (-4)^2 + 8^2 \Rightarrow \underline{\underline{x^2 + y^2 = 80}}.$$

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