Dr Oliver Mathematics
Mathematics
Factor Theorems and Remainder Theorems
Past Examination Questions

This booklet consists of 24 questions across a variety of examination topics.
The total number of marks available is 170.
(Please read *Synthetic Division* if you have not done so.)

1.
$$f(x) \equiv x^3 - 2x^2 + ax + b,$$

where a and b are constants.

When $f(x)$ is divided by $(x - 2)$, the remainder is 1.

When $f(x)$ is divided by $(x + 1)$, the remainder is 28.

(a) Find the value of a and the value of b. (6)

Solution

$$\begin{aligned} f(2) = 1 &\Rightarrow 1 = 8 - 8 + 2a + b \\ &\Rightarrow 2a + b = 1 \\ f(-1) = 28 &\Rightarrow 28 = -1 - 2 - a + b \\ &\Rightarrow -a + b = 31. \end{aligned}$$

Now,

$$\begin{aligned} (2a + b) - (-a + b) = 1 - 31 &\Rightarrow 3a = -30 \\ &\Rightarrow \underline{\underline{a = -10}} \\ &\Rightarrow 10 + b = 31 \\ &\Rightarrow \underline{\underline{b = 21}}. \end{aligned}$$

(b) Show that $(x - 3)$ is a factor of $f(x)$. (2)

Solution

3	1	−2	−10	21
	↓	3	3	−21
	1	1	−7	0

Thus,

$$\frac{x^3 - 2x^2 - 10x + 21}{x - 3} \equiv \frac{(x-3)(x^2 + x - 7)}{x - 3} \equiv x^2 + x - 7,$$

and

$$\underline{\underline{(x - 3) \text{ is a factor of } f(x)}}.$$

2. (a) Use the factor theorem to show that $(x + 4)$ is a factor of $2x^3 + x^2 - 25x + 12$. (2)

Solution

$$f(-4) = -128 + 16 + 100 + 12 = 0$$

and so

$$\underline{\underline{(x + 4) \text{ is a factor of } 2x^3 + x^2 - 25x + 12}}.$$

(b) Factorise $2x^3 + x^2 - 25x + 12$. (4)

Solution

−4	2	1	−25	12
	↓	−8	28	−12
	2	−7	3	0

Finally,

$$2x^3 + x^2 - 25x + 12 \equiv (x + 4)(2x^2 - 7x + 3) \equiv \underline{\underline{(x + 4)(2x - 1)(x - 3)}}.$$

3.

$$f(x) \equiv 2x^3 + x^2 - 5x + c,$$

where c is a constant. Given that $\mathrm{f}(1) = 0$,

(a) find the value of c, (2)

Solution

$$\begin{aligned} \mathrm{f}(1) = 0 &\Rightarrow 2 + 1 - 5 + c = 0 \\ &\Rightarrow \underline{\underline{c = 2}}. \end{aligned}$$

(b) factorise $\mathrm{f}(x)$ completely, (4)

Solution

$$\begin{array}{c|cccc} 1 & 2 & 1 & -5 & 2 \\ & \downarrow & 2 & 3 & -2 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

Finally,

$$\begin{aligned} 2x^3 + x^2 - 5x + 2 &\equiv (x-1)(2x^2 + 3x - 2) \\ &\equiv \underline{\underline{(x-1)(2x-1)(x+2)}}. \end{aligned}$$

(c) find the remainder when $\mathrm{f}(x)$ is divided by $(2x - 3)$. (2)

Solution

$$\begin{array}{c|cccc} \frac{3}{2} & 2 & 1 & -5 & 2 \\ & \downarrow & 3 & 6 & \frac{3}{2} \\ \hline & 2 & 4 & 1 & \frac{7}{2} \end{array}$$

and

$$\text{the remainder is } \underline{\underline{\tfrac{7}{2}}}.$$

4.

$$\mathrm{f}(x) \equiv 2x^3 + 3x^2 - 29x - 60.$$

(a) Find the remainder when $\mathrm{f}(x)$ is divided by $(x + 2)$. (2)

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Solution

$$\begin{aligned} f(-2) &= -16 + 12 + 58 - 60 \\ &= \underline{\underline{-6}}. \end{aligned}$$

(b) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$. (2)

Solution

$$f(-3) = -54 + 27 + 87 - 60 = 0,$$

and

$$\underline{\underline{(x + 3) \text{ is a factor of } f(x)}}.$$

(c) Factorise $f(x)$ completely. (4)

Solution

-3	2	3	-29	-60
	$\downarrow$	-6	9	60
	2	-3	-20	0

Finally,

$$\begin{aligned} 2x^3 + 3x^2 - 29x - 60 &\equiv (x + 3)(2x^2 - 3x - 20) \\ &\equiv \underline{\underline{(x + 3)(2x + 5)(x - 4)}}. \end{aligned}$$

5.

$$f(x) \equiv x^3 + 4x^2 + x - 6.$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$. (2)

Solution

$$f(-2) = -8 + 16 - 2 - 6 = 0,$$

and

$$\underline{\underline{(x + 2) \text{ is a factor of } f(x)}}.$$

(b) Factorise $\mathrm{f}(x)$ completely. (4)

Solution

-2	1	4	1	-6
	$\downarrow$	-2	-4	6
	1	2	-3	0

Finally,

$$\begin{aligned} x^3 + 4x^2 + x - 6 &\equiv (x+2)(x^2+2x-3) \\ &\equiv (x+2)(x+3)(x-1). \end{aligned}$$

(c) Write down all the solutions to the equation (1)

$$x^3 + 4x^2 + x - 6 = 0.$$

Solution

$$x^3 + 4x^2 + x - 6 = 0 \Rightarrow x = -3,\ x = -2,\ \text{or } x = 1.$$

6.

$$\mathrm{f}(x) \equiv 3x^3 - 5x^2 - 16x + 12.$$

(a) Find the remainder when $\mathrm{f}(x)$ is divided by $(x - 2)$. (2)

Solution

$$\mathrm{f}(2) = 24 - 20 - 32 + 12 = -16.$$

Given that $(x + 2)$ is a factor of $\mathrm{f}(x)$,

(b) factorise $\mathrm{f}(x)$ completely. (4)

Solution

-2	3	-5	-16	12
	$\downarrow$	-6	22	-12
	3	-11	6	0

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Finally,

$$\begin{aligned} 3x^3 - 5x^2 - 16x + 12 &\equiv (x+2)(3x^2 - 11x + 6) \\ &\equiv \underline{\underline{(x+2)(3x-2)(x-3)}}. \end{aligned}$$

7. Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(a) (i) $x - 3$, (3)

Solution
Let

$$\text{f}(x) \equiv x^3 - 2x^2 - 4x + 8.$$

Then

$$\text{f}(3) = 27 - 18 - 12 + 8 = \underline{\underline{5}}.$$

(ii) $x + 2$.

Solution

$$\text{f}(-2) = -8 - 8 + 8 + 8 = \underline{\underline{0}}.$$

(b) Hence, or otherwise, find all the solutions to the equation (4)

$$x^3 - 2x^2 - 4x + 8 = 0.$$

Solution

-2	1	-2	-4	8
	$\downarrow$	-2	8	-8
	1	-4	4	0

Now,

$$\begin{aligned} x^3 - 2x^2 - 4x + 8 &\equiv (x+2)(x^2 - 4x + 4) \\ &\equiv (x+2)(x-2)^2, \end{aligned}$$

and the solutions are

$$\underline{\underline{-2, 2 \text{ (twice)}}}.$$

8.

$$\mathrm{f}(x) \equiv 2x^3 - 3x^2 - 39x + 20.$$

(a) Use the factor theorem to show that $(x + 4)$ is a factor of $\mathrm{f}(x)$. (2)

Solution

$$\mathrm{f}(-4) = -128 - 48 + 156 + 20 = 0$$

and

$$\underline{\underline{(x + 4) \text{ is a factor of } \mathrm{f}(x)}}.$$

(b) Factorise $\mathrm{f}(x)$ completely. (4)

Solution

-4	2	-3	-39	20
	$\downarrow$	-8	44	-20
	2	-11	5	0

Finally,

$$\begin{aligned} 2x^3 - 3x^2 - 39x + 20 &\equiv (x + 4)(2x^2 - 11x + 5) \\ &\equiv \underline{\underline{(x + 4)(2x - 1)(x - 5)}}. \end{aligned}$$

9.

$$\mathrm{f}(x) \equiv x^4 + 5x^3 + ax + b,$$

where a and b are constants.

The remainder when $\mathrm{f}(x)$ divided by $(x - 2)$ is equal to the remainder when $\mathrm{f}(x)$ is divided by $(x + 1)$.

(a) Find the value of a. (5)

Solution

$$f(2) = 16 + 40 + 2a + b = 2a + b + 56$$

and

$$f(-1) = 1 - 5 - a + b = -a + b - 4.$$

Now,

$$\begin{aligned}(2a + b + 56) - (-a + b - 4) = 0 &\Rightarrow 3a + 60 = 0\\ &\Rightarrow 3a = -60\\ &\Rightarrow \underline{\underline{a = -20}}.\end{aligned}$$

Given that $(x + 3)$ is a factor of $f(x)$,

(b) find the value of b. (3)

Solution

-3	1	5	0	-20	b
	$\downarrow$	-3	-6	18	6
	1	2	-6	-2	$b + 6$

Hence,

$$b + 6 = 0 \Rightarrow \underline{\underline{b = -6}}.$$

10.

$$f(x) \equiv (3x - 2)(x - k) - 8,$$

where k is a constant.

(a) Write down the value of $f(k)$. (1)

Solution

$$f(k) = (3k - 2)(k - k) - 8 = \underline{\underline{-8}}.$$

When $f(x)$ is divided by $(x - 2)$ the remainder is 4.

(b) Find the value of k. (2)

Solution

$$
\begin{aligned}
\mathrm{f}(2) = 4 &\Rightarrow 4(2-k) - 8 = 4 \\
&\Rightarrow 8 - 4k = 12 \\
&\Rightarrow -4k = 4 \\
&\Rightarrow \underline{\underline{k = -1}}.
\end{aligned}
$$

(c) Factorise $\mathrm{f}(x)$ completely. (3)

Solution

$$
\begin{aligned}
(3x-2)(x+1) - 8 &= (3x^2 + x - 2) - 8 \\
&= 3x^2 + x - 10 \\
&= \underline{\underline{(3x-5)(x+2)}}.
\end{aligned}
$$

11.

$$\mathrm{f}(x) \equiv 2x^3 + ax^2 + bx - 6,$$

where a and b are constants.

When $\mathrm{f}(x)$ is divided by $(2x-1)$, the remainder is -5.

When $\mathrm{f}(x)$ is divided by $(x+2)$, there is no remainder.

(a) Find the value of a and the value of b. (6)

Solution

$$
\begin{aligned}
\mathrm{f}(\tfrac{1}{2}) = -5 &\Rightarrow -5 = \tfrac{1}{4} + \tfrac{1}{4}a + \tfrac{1}{2}b - 6 \\
&\Rightarrow \tfrac{1}{4}a + \tfrac{1}{2}b = \tfrac{3}{4} \\
\mathrm{f}(-2) = 0 &\Rightarrow 0 = -16 + 4a - 2b - 6 \\
&\Rightarrow 4a - 2b = 22.
\end{aligned}
$$

Now,

$$\begin{aligned} \tfrac{1}{4}a + \tfrac{1}{2}b &= \tfrac{3}{4} \\ 4a - 2b &= 22 \end{aligned} \Rightarrow \begin{aligned} a + 2b &= 3 \\ 4a - 2b &= 22 \end{aligned}$$

$$\Rightarrow 5a = 25$$
$$\Rightarrow \underline{\underline{a = 5}}$$
$$\Rightarrow 20 - 2b = 22$$
$$\Rightarrow 2b = -2$$
$$\Rightarrow \underline{\underline{b = -1}}.$$

(b) Factorise $\mathrm{f}(x)$ completely. (3)

Solution

-2	2	5	-1	-6
	$\downarrow$	-4	-2	6
	2	1	-3	0

Finally,

$$\begin{aligned} 2x^3 + 5x^2 - x - 6 &\equiv (x+2)(2x^2 + x - 3) \\ &\equiv \underline{\underline{(x+2)(2x+3)(x-1)}}. \end{aligned}$$

12.

$$\mathrm{f}(x) \equiv 3x^3 - 5x^2 - 58x + 40.$$

(a) Find the remainder when $\mathrm{f}(x)$ is divided by $(x - 3)$. (2)

Solution

$$\begin{aligned} \mathrm{f}(3) &= 81 - 45 - 174 + 40 \\ &= \underline{\underline{-98}}. \end{aligned}$$

Given that $(x - 5)$ is a factor of $\mathrm{f}(x)$,

(b) find all the solutions of $\mathrm{f}(x) = 0$. (5)

Solution

5	3	−5	−58	40
	↓	15	50	−40
	3	10	−8	0

Finally,

$$\begin{aligned} 3x^3 - 5x^2 - 58x + 40 = 0 &\equiv (x-5)(3x^2+10x-8) = 0 \\ &\equiv (x-5)(3x-2)(x+4) = 0 \\ &\equiv \underline{\underline{x = -4, \tfrac{2}{3}, \text{ or } 5}}. \end{aligned}$$

13.

$$\mathrm{f}(x) \equiv x^4 + x^3 + 2x^2 + ax + b,$$

where a and b are constants.

When $\mathrm{f}(x)$ is divided by $(x-1)$, the remainder is 7.

(a) Show that $a + b = 3$. (2)

Solution

$$\begin{aligned} \mathrm{f}(1) = 7 &\Rightarrow 1 + 1 + 2 + a + b = 7 \\ &\Rightarrow \underline{\underline{a + b = 3}}. \end{aligned}$$

When $f(x)$ is divided by $(x+2)$, the remainder is -8.

(b) Find the value of a and the value of b. (5)

Solution

$$\begin{aligned} \mathrm{f}(-2) = -8 &\Rightarrow 16 - 8 + 8 - 2a + b = -8 \\ &\Rightarrow -2a + b = -24. \end{aligned}$$

Subtracting:

$$(a+b)-(-2a+b)=3-(-24) \Rightarrow 3a=27$$
$$\Rightarrow \underline{\underline{a=9}}$$
$$\Rightarrow \underline{\underline{b=-6}}.$$

14.

$$f(x) \equiv 2x^3 - 7x^2 - 5x + 4.$$

(a) Find the remainder when $f(x)$ is divided by $(x-1)$. (2)

Solution

$$\begin{aligned} f(1) &= 2-7-5+4 \\ &= \underline{\underline{-6}}. \end{aligned}$$

(b) Use the factor theorem to show that $(x+1)$ is a factor of $f(x)$. (2)

Solution

$$f(-1) = -2-7+5+4 = 0$$

and

$$\underline{\underline{(x+1) \text{ is a factor of } f(x)}}.$$

(c) Factorise $f(x)$ completely. (4)

Solution

−1	2	−7	−5	4
	↓	−2	9	−4
	2	−9	4	0

Finally,

$$\begin{aligned} 2x^3 - 7x^2 - 5x + 4 &\equiv (x+1)(2x^2 - 9x + 4) \\ &\equiv \underline{\underline{(x+1)(2x-1)(x-4)}}. \end{aligned}$$

15.

$$f(x) \equiv x^3 + ax^2 + bx + 3,$$

where a and b are constants.

Given that when $f(x)$ is divided by $(x + 2)$ the remainder is 7,

(a) show that $2a - b = 6$. (2)

Solution

$$\begin{aligned} f(-2) = 7 &\Rightarrow -8 + 4a - 2b + 3 = 7 \\ &\Rightarrow 4a - 2b = 12 \\ &\Rightarrow \underline{\underline{2a - b = 6}}, \end{aligned}$$

as required.

Given also that when $f(x)$ is divided by $(x - 1)$ the remainder is 4,

(b) find the value of a and the value of b. (4)

Solution

$$\begin{aligned} f(1) = 1 &\Rightarrow 1 + a + b + 3 = 4 \\ &\Rightarrow a + b = 0. \end{aligned}$$

Add:

$$\begin{aligned} (2a - b) + (a + b) = 6 - 0 &\Rightarrow 3a = 6 \\ &\Rightarrow \underline{\underline{a = 2}} \\ &\Rightarrow \underline{\underline{b = -2}}. \end{aligned}$$

16.

$$f(x) \equiv 2x^3 - 7x^2 - 10x + 24.$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$. (2)

Solution

$$f(-2) = -16 - 28 + 20 + 2 = 0$$

and

$$\underline{\underline{(x + 2) \text{ is a factor of } f(x)}}.$$

(b) Factorise $\mathrm{f}(x)$ completely. (4)

Solution

-2	2	-7	-10	24
	$\downarrow$	-4	22	-24
	2	-11	12	0

Finally,

$$\begin{aligned} 2x^3 - 7x^2 - 10x + 24 &\equiv (x+2)(2x^2 - 11x + 12) \\ &\equiv \underline{\underline{(x+2)(2x-3)(x-4)}}. \end{aligned}$$

17.

$$\mathrm{f}(x) \equiv ax^3 + bx^2 - 4x - 3,$$

where a and b are constants.

Given that $(x-1)$ is a factor of $\mathrm{f}(x)$,

(a) show that $a + b = 7$. (2)

Solution

$$\begin{aligned} \mathrm{f}(1) = 0 &\Rightarrow a + b - 4 - 3 = 0 \\ &\Rightarrow \underline{\underline{a + b = 7}}, \end{aligned}$$

as required.

Given also that, when $\mathrm{f}(x)$ is divided by $(x+2)$, the remainder is 9,

(b) find the value of a and the value of b, showing each step in your working. (4)

Solution

$$\begin{aligned} \mathrm{f}(-2) = 9 &\Rightarrow -8a + 4b + 8 - 3 = 9 \\ &\Rightarrow -8a + 4b = 4. \end{aligned}$$

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Now,

$$\begin{aligned} b = 7 - a \Rightarrow & -8a + 4(7-a) = 4 \\ \Rightarrow & -8a + 28 - 4a = 4 \\ \Rightarrow & \; 12a = 24 \\ \Rightarrow & \; \underline{\underline{a = 2}} \\ \Rightarrow & \; \underline{\underline{b = 5}}. \end{aligned}$$

18.

$$f(x) \equiv 2x^3 - 5x^2 + ax + 18,$$

where a is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) show that $a = -9$, (2)

Solution

$$\begin{aligned} f(3) = 0 \Rightarrow & \; 54 - 45 + 3a + 18 = 0 \\ \Rightarrow & \; 3a = -27 \\ \Rightarrow & \; \underline{\underline{a = -9}}. \end{aligned}$$

(b) factorise $f(x)$ completely. (4)

Solution

3	2	-5	-9	18
	$\downarrow$	6	3	-18
	2	1	-6	0

Finally,

$$\begin{aligned} 2x^3 - 5x^2 - 9x + 18 & \equiv (x-3)(2x^2 + x - 6) \\ & \equiv \underline{\underline{(x-3)(2x-3)(x+2)}}. \end{aligned}$$

19.

$$f(x) \equiv ax^3 - 11x^2 + bx + 4,$$

where a and b are constants.

When $f(x)$ is divided by $(x-3)$ the remainder is 55.

When $f(x)$ is divided by $(x+1)$ the remainder is -9.

(a) Find the value of a and the value of b. (5)

Solution

$$\begin{aligned} f(3) = 55 &\Rightarrow 27a - 99 + 3b + 4 = 55 \\ &\Rightarrow 27a + 3b = 150 \\ f(-1) = -9 &\Rightarrow -a - 11 - b + 4 = -9 \\ &\Rightarrow b = 2 - a. \end{aligned}$$

Now,

$$\begin{aligned} b = 2 - a &\Rightarrow 27a + 3(2-a) = 150 \\ &\Rightarrow 27a + 6 - 3a = 150 \\ &\Rightarrow 24a = 144 \\ &\Rightarrow \underline{\underline{a = 6}} \\ &\Rightarrow \underline{\underline{b = -4}}. \end{aligned}$$

Given that $(3x+2)$ is a factor of $f(x)$,

(b) factorise $f(x)$ completely. (4)

Solution

$-\frac{2}{3}$	6	-11	-4	4
	$\downarrow$	-4	10	-4
	6	-15	6	0

Finally,

$$\begin{aligned} 6x^3 - 11x^2 - 4x + 4 &\equiv (x + \tfrac{2}{3})(6x^2 - 15x + 6) \\ &\equiv (3x + 2)(2x^2 - 5x + 2) \\ &\equiv \underline{\underline{(x - 3)(2x - 1)(x - 2)}}. \end{aligned}$$

20.

$$f(x) \equiv 2x^3 - 7x^2 + 4x + 4.$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$. (2)

Solution

$$f(2) = 16 - 28 + 8 + 4 = 0$$

and

$$\underline{\underline{(x - 2) \text{ is a factor of } f(x)}}.$$

(b) Factorise $f(x)$ completely. (4)

Solution

2	2	−7	4	4
	↓	4	−6	−4
	2	−3	−2	0

Finally,

$$\begin{aligned} 2x^3 - 7x^2 + 4x + 4 &\equiv (x - 2)(2x^2 - 3x - 2) \\ &\equiv \underline{\underline{(x - 2)(2x + 1)(x - 2)}}. \end{aligned}$$

21.

$$f(x) \equiv -4x^3 + ax^2 + 9x - 18,$$

where a is a constant.

Given that $(x - 2)$ is a factor of $f(x)$,

(a) find the value of a, (2)

Solution

$$\begin{aligned} f(2) = 0 &\Rightarrow -32 + 4a + 18 - 18 = 0 \\ &\Rightarrow 4a = 32 \\ &\Rightarrow \underline{\underline{a = 8}}. \end{aligned}$$

(b) factorise $f(x)$ completely, (3)

Solution

$$\begin{array}{c|cccc} 2 & -4 & 8 & 9 & -18 \\ & \downarrow & -8 & 0 & 18 \\ \hline & -4 & 0 & 9 & 0 \end{array}$$

Finally,

$$\begin{aligned} 4x^3 + 8x^2 + 9x - 18 &\equiv (x-2)(9-4x^2) \\ &\equiv \underline{\underline{(x-2)(3+2x)(3-2x)}}. \end{aligned}$$

(c) find the remainder when $f(x)$ is divided by $(2x - 1)$. (2)

Solution

$$\begin{array}{c|cccc} \frac{1}{2} & -4 & 8 & 9 & -18 \\ & \downarrow & -2 & 3 & 6 \\ \hline & -4 & 6 & 12 & -12 \end{array}$$

and

the remainder is $\underline{\underline{-12}}$.

22.

$$f(x) \equiv 6x^3 + 3x^2 + Ax + B,$$

where A and B are constants.

Given that when $f(x)$ is divided by $(x + 1)$ the remainder is 45,

(a) show that $B - A = 48$. (2)

Solution

$$\begin{aligned} f(-1) = 48 &\Rightarrow -6 + 3 - A + B = 45 \\ &\Rightarrow \underline{\underline{B - A = 48}}. \end{aligned}$$

Given also that $(2x + 1)$ is a factor of $f(x)$,

(b) find the value of A and the value of B. (4)

Solution

$$\begin{aligned} f(-\tfrac{1}{2}) = 0 &\Rightarrow -\tfrac{3}{4} + \tfrac{3}{4} - \tfrac{1}{2}A + B = 0 \\ &\Rightarrow B = \tfrac{1}{2}A \\ &\Rightarrow \tfrac{1}{2}A - A = 48 \\ &\Rightarrow -\tfrac{1}{2}A = 48 \\ &\Rightarrow \underline{\underline{A = -96}} \\ &\Rightarrow \underline{\underline{B = -48}}. \end{aligned}$$

(c) Factorise $f(x)$ fully. (3)

Solution

$-\frac{1}{2}$	6	3	-96	-48
	$\downarrow$	-3	0	48
	6	0	-96	0

Finally,

$$\begin{aligned} 6x^3 + 3x^2 - 96x - 48 &\equiv (x + \tfrac{1}{2})(6x^2 - 96) \\ &\equiv (2x + 1)(3x^2 - 48) \\ &\equiv 3(2x + 1)(x^2 - 16) \\ &\equiv \underline{\underline{3(2x + 1)(x - 4)(x + 4)}}. \end{aligned}$$

23.

$$f(x) \equiv 6x^3 + 13x^2 - 4.$$

(a) Use the remainder theorem to find the remainder when $f(x)$ is divided by $(2x+3)$. (2)

Solution

$$\begin{aligned} f(-\tfrac{3}{2}) &= -20\tfrac{1}{4} + 29\tfrac{1}{4} - 4 \\ &= \underline{\underline{5}}. \end{aligned}$$

(b) Use the factor theorem to show that $(x+2)$ is a factor of $f(x)$. (2)

Solution

$$f(-2) = -48 + 52 - 4 = 0$$

and

$$\underline{\underline{(x+2) \text{ is a factor of } f(x)}}.$$

(c) Factorise $f(x)$ completely. (4)

Solution

-2	6	13	0	-4
	$\downarrow$	-12	-2	4
	6	1	-2	0

Finally,

$$\begin{aligned} 6x^3 + 13x^2 - 4 &\equiv (x+2)(6x^2 + x - 2) \\ &\equiv \underline{\underline{(x+2)(3x+2)(2x-1)}}. \end{aligned}$$

24.

$$f(x) = -6x^3 - 7x^2 + 40x + 21.$$

(a) Use the factor theorem to show that $(x+3)$ is a factor of $f(x)$. (2)

Solution

$$f(-3) = 162 - 63 - 120 + 21 = 0$$

and

$$\underline{\underline{(x+3) \text{ is a factor of } f(x)}}.$$

(b) Factorise $f(x)$ completely. (4)

Solution

-3	-6	-7	40	21
	$\downarrow$	18	-33	-21
	-6	11	7	0

Finally,

$$\begin{aligned} 6x^3 + 13x^2 - 4 &\equiv (x+3)(7 + 11x - 6x^2) \\ &\equiv \underline{\underline{(x+3)(7-3x)(1+2x)}}. \end{aligned}$$

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