

Dr Oliver Mathematics
Mathematics: National Qualifications N5
2022 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 50.

You must write down all the stages in your working.

1. Expand and simplify

$$(3x - 2)(2x^2 + 5x - 1).$$

(3)

Solution

\times	$2x^2$	$+5x$	-1
$3x$	$6x^3$	$+15x^2$	$-3x$
-2	$-4x^2$	$-10x$	$+2$

Hence,

$$(3x - 2)(2x^2 + 5x - 1) = \underline{\underline{6x^3 + 11x^2 - 13x + 2.}}$$

2. A company's annual profit at the end of 2021 was £215 000.
The profit is expected to increase by 3% each year.

(3)

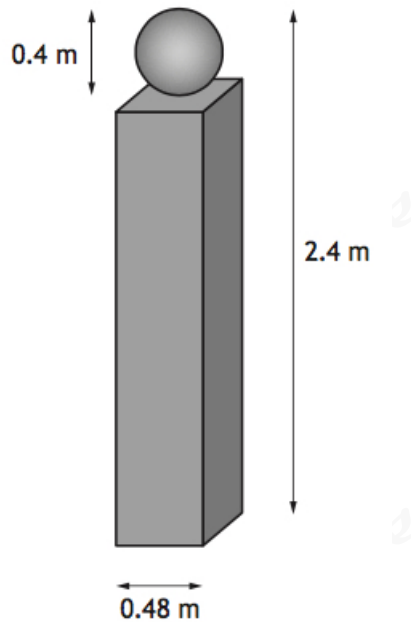
Calculate the company's expected annual profit by the end of 2025.

Give your answer correct to the nearest thousand pounds.

Solution

$$\begin{aligned} \text{Expected annual profit} &= 215\,000 \times (1.03)^4 \\ &= 241\,984.3942 \text{ (FCD)} \\ &= \underline{\underline{\pounds 242\,000}} \text{ (nearest thousand pounds).} \end{aligned}$$

3. A concrete gatepost is made in the shape of a cuboid with a sphere on top. (3)
The sphere has diameter 0.4 metres.
The cuboid has a square base of length 0.48 metres.
The total height of the gatepost is 2.4 metres.



Calculate the volume of concrete needed to make a gatepost.

Solution

$$\begin{aligned}\text{Volume} &= \text{sphere} + \text{cuboid} \\ &= \left(\frac{4}{3} \times \pi \times 0.2^3\right) + (0.48 \times 0.48 \times 2) \\ &= 0.033\ 510\ 321\ 64 + 0.460\ 8 \text{ (FCD)} \\ &= 0.494\ 310\ 321\ 6 \text{ (FCD)} \\ &= \underline{\underline{0.494\ \text{m}^3\ (3\ \text{sf})}}.\end{aligned}$$

4. Moira buys 4 mangoes and 3 apples at a fruit shop. (1)
The total cost is £4.25.
(a) Write down an equation to illustrate this information.

Solution

Let m be the cost of a mango and let a be the cost of an apple. Then

$$\underline{\underline{4m + 3a = 4.25}} \quad (1).$$

Sami buys 5 mangoes and 2 apples in the same fruit shop.
The total cost is £4.70.

- (b) Write down an equation to illustrate this information. (1)

Solution

$$\underline{\underline{5m + 2a = 4.70}} \quad (2).$$

- (c) Calculate, algebraically, the cost of a mango and the cost of an apple. (4)

Solution

E.g.,

$$5 \times (1) : 20m + 15a = 21.25 \quad (3)$$

$$4 \times (2) : 20m + 8a = 18.80 \quad (4)$$

Do (3) – (4):

$$\begin{aligned} 7a &= 2.45 \Rightarrow a = 0.35 \\ &\Rightarrow 4m + 3(0.35) = 4.25 \\ &\Rightarrow 4m + 1.05 = 4.25 \\ &\Rightarrow 4m = 3.20 \\ &\Rightarrow m = 0.80; \end{aligned}$$

hence, a mango costs 80 p and an apple costs 35 p.

5. A school netball team recorded the number of sit-ups each player completed in a minute. The numbers for the seven players were:

29 27 24 31 22 19 30

- (a) Calculate the mean and standard deviation of the numbers of sit-ups. (4)

Solution

x	x^2
29	841
27	729
24	576
31	961
22	484
19	361
30	900
<hr/>	
$\sum x = 182$	$\sum x^2 = 4852$

$$\begin{aligned}\text{Mean} &= \frac{\sum x}{n} \\ &= \frac{182}{7} \\ &= \underline{\underline{26}}\end{aligned}$$

and

$$\begin{aligned}\text{standard deviation} &= \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} \\ &= \sqrt{\frac{4852 - \frac{182^2}{7}}{6}} \\ &= 4.472135955 \text{ (FCD)} \\ &= \underline{\underline{4.47}} \text{ (3 sf)}.\end{aligned}$$

Some players in the school's hockey team also recorded the number of sit-ups they completed in a minute.

Their numbers gave a mean of 29 and a standard deviation of 3.2.

- (b) Make two valid comments comparing the numbers of sit-ups of the players in the netball team and the hockey team. (2)

Solution

Average

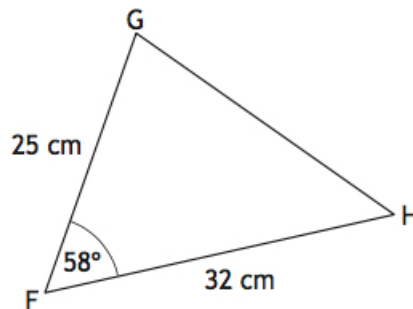
Since the mean for the netball's team is lower than the mean for the hockey's team, the netball's team did worse on average.

Spread

Since the standard deviation for the hockey's team is smaller than the standard deviation for the netball's team, the numbers of sit-ups were more consistent for the hockey's team.

6. The diagram shows triangle FGH .

(2)



- $FG = 25$ centimetres.
- $FH = 32$ centimetres.
- Angle $GFH = 58^\circ$.

Calculate the area of triangle FGH .

Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 25 \times 32 \times \sin 58^\circ \\ &= 339.219\ 238\ 5 \text{ (FCD)} \\ &= \underline{\underline{339 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

7. Solve the equation

$$4x^2 + 2x - 7 = 0.$$

(4)

Give your answers correct to 2 significant figures.

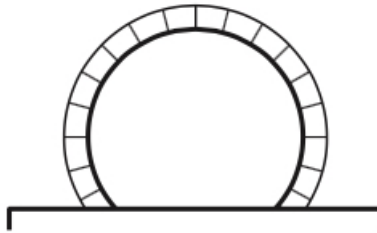
Solution

$a = 4$, $b = 2$, and $c = -7$:

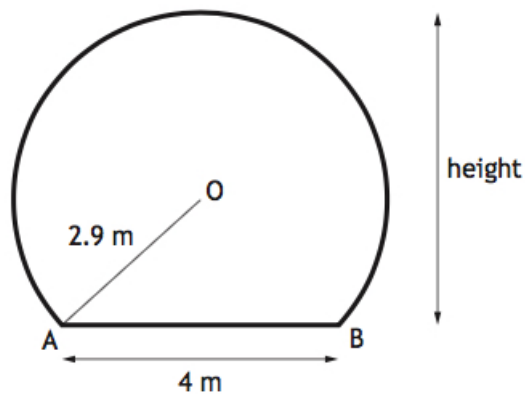
$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-2 \pm \sqrt{2^2 - 4 \times 4 \times (-7)}}{2 \times 4} \\
 &= \frac{-2 \pm \sqrt{116}}{8} \\
 &= -1.596\ 291\ 202, 1.096\ 291\ 202 \text{ (FCD)} \\
 &= \underline{\underline{-1.6, 1.1 \text{ (2 sf)}}}.
 \end{aligned}$$

8. A train tunnel has a circular cross-section with a horizontal floor.

(4)



A diagram of the cross-section is shown below.

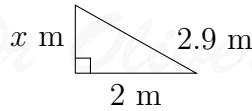


- The centre of the circle is O .
- Chord AB is 4 metres.

- The radius OA is 2.9 metres.

Calculate the height of the tunnel.

Solution



Now,

$$\begin{aligned} x &= \sqrt{2.9^2 - 2^2} \\ &= 2.1 \end{aligned}$$

and

$$\begin{aligned} \text{height} &= 2.1 + 2.9 \\ &= \underline{\underline{5 \text{ m}}}. \end{aligned}$$

9. Solve the equation

$$3 \sin x^\circ + 4 = 6,$$

(3)

for $0 \leq x \leq 360$.

Solution

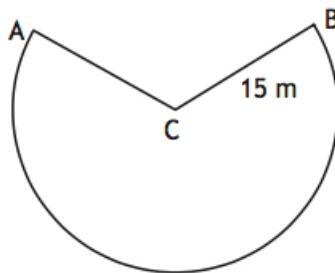
$$\begin{aligned} 3 \sin x^\circ + 4 = 6 &\Rightarrow 3 \sin x^\circ = 2 \\ &\Rightarrow \sin x^\circ = \frac{2}{3} \\ &\Rightarrow x = 41.8103149, 138.1896851 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 41.8, 138.2 \text{ (1 dp)}}}. \end{aligned}$$

10. An attraction at a theme park has a carriage attached to an arm.

(3)



The arm swings from A to B along the arc of a circle, centre C , as shown in the diagram below.



- The length of the arm, CB , is 15 metres.
- The length of the major arc, AB , is 69.4 metres.

Calculate the size of the reflex angle ACB .

Solution

If the circle was complete,

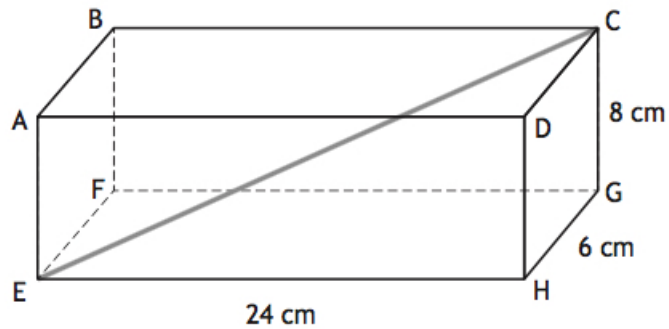
$$\begin{aligned} \text{circumference} &= 2 \times \pi \times 15 \\ &= 30\pi. \end{aligned}$$

Hence,

$$\begin{aligned} \frac{\text{reflex angle}}{360} &= \frac{69.4}{30\pi} \Rightarrow \text{reflex angle} = \frac{69.4 \times 360}{30\pi} \\ &\Rightarrow \text{reflex angle} = 265.088\ 473\ 2 \text{ (FCD)} \\ &\Rightarrow \text{reflex angle} = \underline{\underline{265^\circ \text{ (3 sf)}}}. \end{aligned}$$

11. The diagram shows a cuboid, $ABCDEFGH$.

(3)



- The length of the cuboid, EH , is 24 centimetres.
- The breadth of the cuboid, HG , is 6 centimetres.
- The height of the cuboid, CG , is 8 centimetres.

Calculate the length of EC , the space diagonal of the cuboid.

Solution

$$\begin{aligned} EC^2 &= EH^2 + HG^2 + CG^2 \\ &= 24^2 + 6^2 + 8^2 \\ &= 576 + 36 + 64 \\ &= 676 \\ EC &= \underline{26 \text{ cm.}} \end{aligned}$$

12. Simplify

(3)

$$\frac{2ab + 6a}{b^2 - 9}$$

Solution

$$\frac{2ab + 6a}{b^2 - 9} = \frac{2a(b + 3)}{b^2 - 9}$$

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array} \right\} \begin{array}{l} 0 \\ -9 \end{array} \quad -3, +3$$

$$= \frac{2a(b+3)}{(b+3)(b-3)}$$

$$= \frac{2a\cancel{(b+3)}}{\cancel{(b+3)}(b-3)}$$

$$= \frac{2a}{\underline{\underline{b-3}}}$$

13. Simplify

(2)

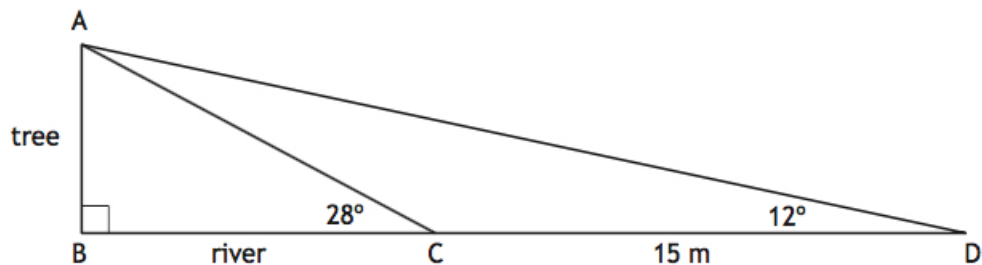
$$\frac{\sin x^\circ + 2 \cos x^\circ}{\cos x^\circ}$$

Solution

$$\begin{aligned} \frac{\sin x^\circ + 2 \cos x^\circ}{\cos x^\circ} &= \frac{\sin x^\circ}{\cos x^\circ} + \frac{2 \cos x^\circ}{\cos x^\circ} \\ &= \underline{\underline{\tan x^\circ + 2}}. \end{aligned}$$

14. The width of a river is represented by BC in the diagram below. AB represents a tree on the river bank.

(5)



- From C , the angle of elevation to A is 28° .
- From D , the angle of elevation to A is 12° .
- The distance from C to D is 15 metres.

- BCD is a straight line.

Calculate BC , the width of the river.

Solution

$\angle ACD = 180 - 28 = 152^\circ$ (supplementary angles)

$\angle CAD = 180 - (152 + 12) = 16^\circ$ (completing the triangle)

Now,

$$\begin{aligned} \frac{AC}{\sin CDA} &= \frac{CD}{\sin CAD} \Rightarrow \frac{AC}{\sin 12^\circ} = \frac{15}{\sin 16^\circ} \\ &\Rightarrow AC = \frac{15 \sin 12^\circ}{\sin 16^\circ}. \end{aligned}$$

Finally,

$$\begin{aligned} \cos &= \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos 28^\circ = \frac{BC}{\frac{15 \sin 12^\circ}{\sin 16^\circ}} \\ &\Rightarrow BC = \frac{15 \sin 12^\circ \cos 28^\circ}{\sin 16^\circ} \\ &= 9.990\,035\,262 \text{ (FCD)} \\ &= \underline{\underline{10 \text{ m (2 sf)}}}. \end{aligned}$$