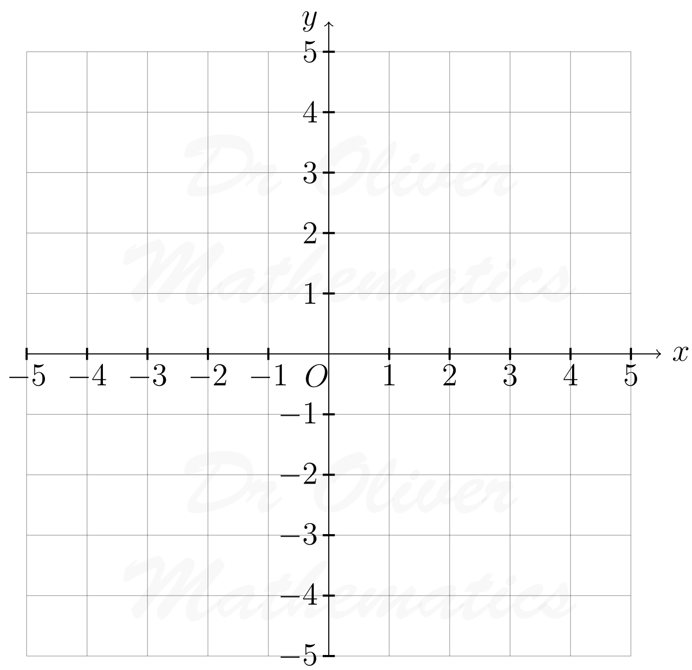
Dr Oliver Mathematics
AQA Further Maths Level 2
June 2017 Paper 1
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.
You are **not** permitted to use a scientific or graphical calculator in this paper.

1. On the grid below, draw a straight line through $(2, 1)$ with gradient $\frac{3}{4}$. (2)



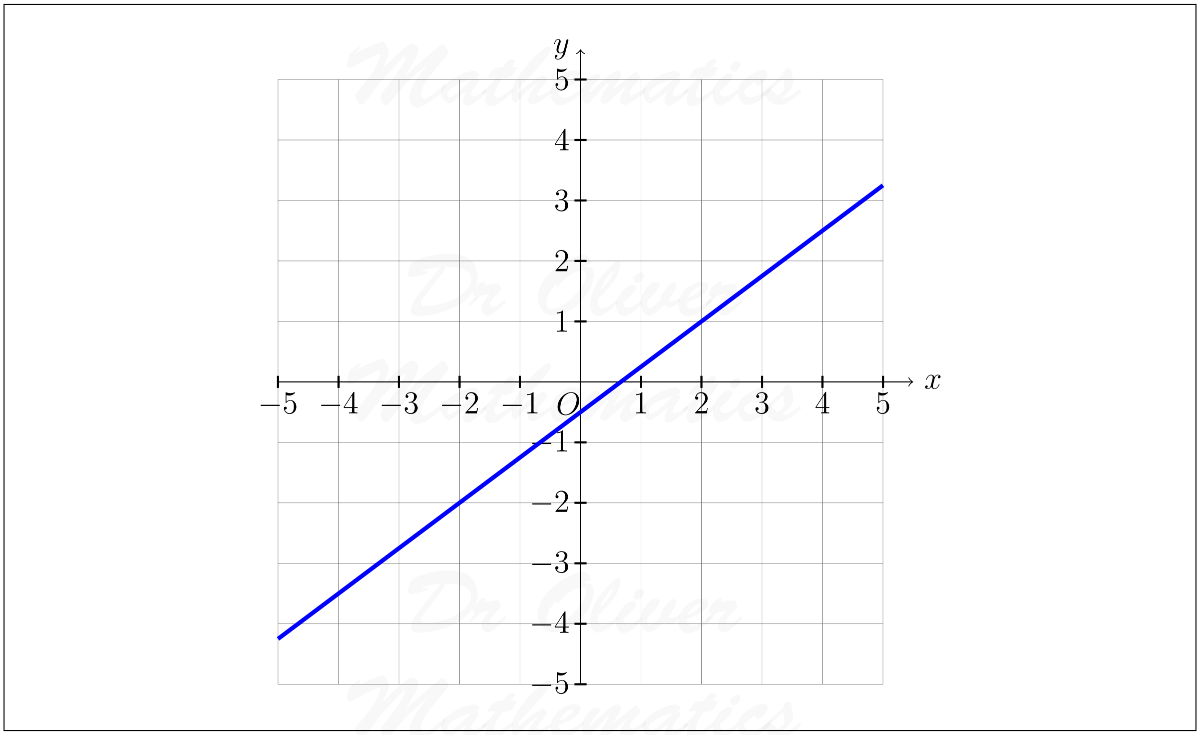
Solution

Well,

$$y - 1 = \tfrac{3}{4}(x - 2) \Rightarrow y - 1 = \tfrac{3}{4}x - \tfrac{3}{2}$$
$$\Rightarrow y = \tfrac{3}{4}x - \tfrac{1}{2}$$

and

$$x = -5 \Rightarrow y = -4\tfrac{1}{4}$$
$$x = 5 \Rightarrow y = 3\tfrac{1}{4}.$$



2. A curve has equation (3)

$$y = ax^2 + 3x,$$

where a is a constant.
When $x = -1$, the gradient of the curve is -5.

Work out the value of a.

Solution

$$y = ax^2 + 3x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + 3$$

and

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = -5 &\Rightarrow 2a(-1) + 3 = -5\\ &\Rightarrow -2a = -8\\ &\Rightarrow \underline{\underline{a = 4}}.\end{aligned}$$

3. (a) On the axes below, sketch the graph of (3)

$$y = x^2 + 7x - 18.$$

Label all points of intersection with the axes.
You do **not** need to work out the coordinates of any stationary points.

Solution

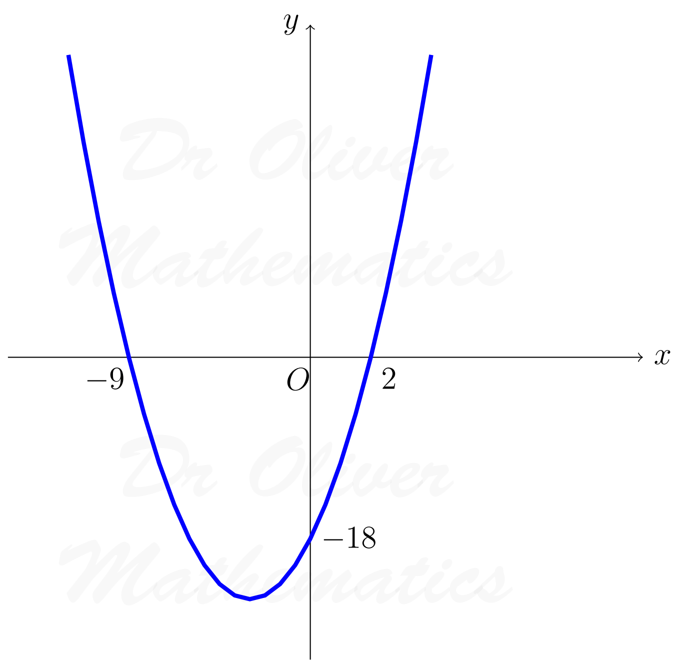
$$\left.\begin{array}{ll}\text{add to:} & +7\\ \text{multiply to:} & -18\end{array}\right\}+9,\,-2$$

Now,

$$x^2+7x-18=(x+9)(x-2)$$

and

$$x=0\Rightarrow y=-18.$$



(b) Work out the equation of the line of symmetry of the graph of (1)

$$y=x^2+7x-18.$$

Solution

$$\frac{-9+2}{2}=\frac{-7}{2}=-3\tfrac{1}{2}$$

and so the equation of the line of symmetry is

$$\underline{\underline{x=-3\tfrac{1}{2}}}.$$

4. A straight line passes through the points $(-4, 7)$, $(6, -5)$, and $(8, t)$. (3)

Use an algebraic method to work out the value of t.
You **must** show your working.

Solution

$$\begin{aligned}\frac{8-(-4)}{t-7} = \frac{6-(-4)}{-5-7} &\Rightarrow \frac{12}{t-7} = \frac{10}{-12} \\ &\Rightarrow -144 = 10(t-7) \\ &\Rightarrow -14.4 = t - 7 \\ &\Rightarrow \underline{\underline{t = -7.4}}.\end{aligned}$$

5. (3)

$$(x+4)(x^2 - kx - 5)$$

is expanded and simplified.

The coefficient of the x^2 term is twice the coefficient of the x term.

Work out the value of k.

Solution

$\times$	x^2	$-kx$	-5
x	x^3	$-kx^2$	$-5x$
$+4$	$+4x^2$	$-4kx$	-20

Now,

$$\begin{aligned}4 - k = 2(-4k - 5) &\Rightarrow 4 - k = -8k - 10 \\ &\Rightarrow 7k = -14 \\ &\Rightarrow \underline{\underline{k = -2}}.\end{aligned}$$

6. Factorise fully (3)

$$(x+6)^4 + (x+6)^3(3x+4).$$

Do **not** attempt to expand the brackets.

Solution

$$\begin{aligned}(x+6)^4+(x+6)^3(3x+4) &\Rightarrow (x+6)^3[(x+6)+(3x+4)]\\ &\Rightarrow (x+6)^3(4x+10)\\ &\Rightarrow \underline{\underline{2(x+6)^3(2x+5)}}.\end{aligned}$$

7. The function f is given by

$$\mathrm{f}(x)=\sqrt{2x-5}.$$

(a) Which of these inequalities is a possible domain for $\mathrm{f}(x)$? Circle the inequality. (1)

$$x \geqslant 0 \qquad x \geqslant \tfrac{2}{5} \qquad x \geqslant 2 \qquad x \geqslant \tfrac{5}{2}$$

Solution

$$\begin{aligned}\sqrt{2x-5} \geqslant 0 &\Rightarrow 2x-5 \geqslant 0\\ &\Rightarrow 2x \geqslant 5\\ &\Rightarrow \underline{\underline{x \geqslant \tfrac{5}{2}}}.\end{aligned}$$

(b) Work out x when $\mathrm{f}(x)=1.2$. (2)

Solution

$$\begin{aligned}\mathrm{f}(x)=1.2 &\Rightarrow \sqrt{2x-5}=1.2\\ &\Rightarrow 2x-5=1.2^2\\ &\Rightarrow 2x-5=1.44\\ &\Rightarrow 2x=6.44\\ &\Rightarrow \underline{\underline{x=3.22}}.\end{aligned}$$

(c) Work out the value of $\mathrm{f}(2\tfrac{5}{8})$. Give your answer as a fraction in its simplest form. (3)

Solution

$$\begin{aligned} f(2\tfrac{5}{8}) &= \sqrt{2(2\tfrac{5}{8}) - 5} \\ &= \sqrt{5\tfrac{1}{4} - 5} \\ &= \sqrt{\tfrac{1}{4}} \\ &= \underline{\underline{\tfrac{1}{2}}}. \end{aligned}$$

8. The first four terms of a quadratic sequence are (4)

$$10 \qquad 33 \qquad 64 \qquad 103 \qquad \ldots$$

Work out an expression for the nth term.

Solution

Let nth term be

$$an^2 + bn + c.$$

Write down the sequence:	10		33		64		103
First line of differences:		23		31		39	
Second line of differences:			8		8		

Sequence:	$a + b + c$		$4a + 2b + c$		$9a + 3b + c$
First line:		$3a + b$		$5a + b$	
Second line:			$2a$		

We compare terms:

$$2a = 8 \Rightarrow a = 4,$$

$$\begin{aligned} 3a + b = 23 &\Rightarrow 3 \times 4 + b = 23 \\ &\Rightarrow 12 + b = 23 \\ &\Rightarrow b = 11, \end{aligned}$$

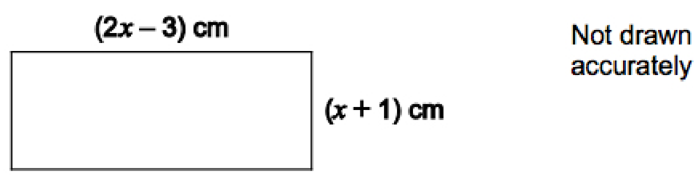
and

$$\color{red}{a+b+c=10} \Rightarrow 4+11+c=10$$
$$\Rightarrow c=-5;$$

hence,

$$n\text{th term} = \underline{\underline{4n^2+11n-5}}.$$

9. Here is a rectangle.



(a) Show that the area of the rectangle is (1)

$$(2x^2-x-3)\text{ cm}^2.$$

Solution

$\times$	$2x$	-3
x	$2x^2$	$-3x$
$+1$	$+2x$	-3

Hence,

$$\begin{aligned}\text{area} &= (2x-3)(x+1)\\ &= \underline{\underline{(2x^2-x-3)\text{ cm}^2}}.\end{aligned}$$

The area of the rectangle is greater than 7 cm^2.

(b) Work out the range of possible values of x. (4)
Give your answer as an inequality.

Solution

$$\begin{aligned}\text{Area} > 7 &\Rightarrow 2x^2 - x - 3 > 7\\ &\Rightarrow 2x^2 - x - 10 > 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -1\\ \text{multiply to:} & (+2)\times(-10) = -20\end{array}\right\} -5,\ +4$$

$$\begin{aligned}&\Rightarrow 2x^2 - 5x + 4x - 10 > 0\\ &\Rightarrow x(2x-5) + 2(2x-5) > 0\\ &\Rightarrow (x+2)(2x-5) > 0\\ &\Rightarrow x < -2 \text{ or } x > 2\tfrac{1}{2};\end{aligned}$$

now, $x < -2$ is not a solution (why?) and we are left with $\underline{\underline{x > 2\frac{1}{2}}}$.

10. Circle C_1 has centre L and equation (4)

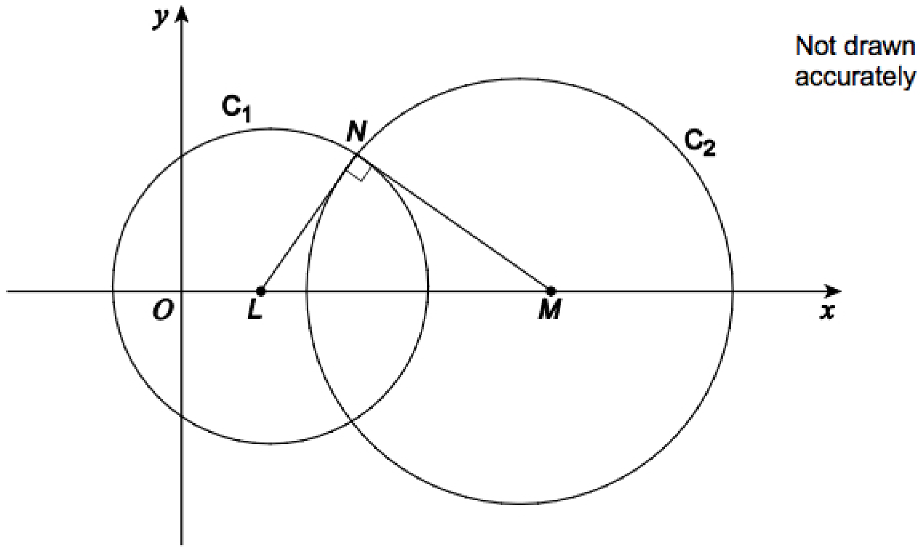
$$(x-3)^2 + y^2 = 36.$$

Circle C_2 has centre M and equation

$$(x-h)^2 + y^2 = 64,$$

where h is a constant.

The circles intersect at N.
LN is perpendicular to MN.



Work out the value of h.

Solution

Well, $L(3,0)$ and the radius of C_1 is $\sqrt{36}=6$.
$M(h,0)$ and the radius of C_2 is $\sqrt{64}=8$.
Now, we have a right-angled triangle in $\triangle LMN$:

$$\begin{aligned} LM &= \sqrt{LN^2+NM^2} \\ &= \sqrt{6^2+8^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \\ &= 10; \end{aligned}$$

hence, $h = 3+10 = \underline{\underline{13 \text{ cm}}}$.

11. Simplify fully (4)

$$\frac{x}{x-3}+\frac{6}{(x-3)(x-5)}.$$

Solution

$$\frac{x}{x-3} + \frac{6}{(x-3)(x-5)} = \frac{x(x-5)}{(x-3)(x-5)} + \frac{6}{(x-3)(x-5)}$$
$$= \frac{(x^2-5x)+6}{(x-3)(x-5)}$$

$$\left.\begin{array}{ll} \text{add to:} & -5 \\ \text{multiply to:} & +6 \end{array}\right\} -3, -2$$

$$= \frac{(x-3)(x-2)}{(x-3)(x-5)}$$
$$= \underline{\underline{\frac{x-2}{x-5}}}.$$

12. The transformation matrix $\mathbf{M}$ represents a 90° clockwise rotation about the origin.

(a) Write down the matrix $\mathbf{M}$. (1)

Solution

$$\mathbf{M} = \underline{\underline{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}}.$$

(b) Describe fully the **single** transformation represented by $\mathbf{M}^2$. (2)

Solution

$\mathbf{M}^2$ represents a $\underline{\underline{\text{180° (anti-)clockwise rotation about the origin}}}$.

(c) Write down the matrix for the **single** transformation represented by $\mathbf{M}^2$. (1)

Solution

$$\mathbf{M}^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
$$= \underline{\underline{\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}}}.$$

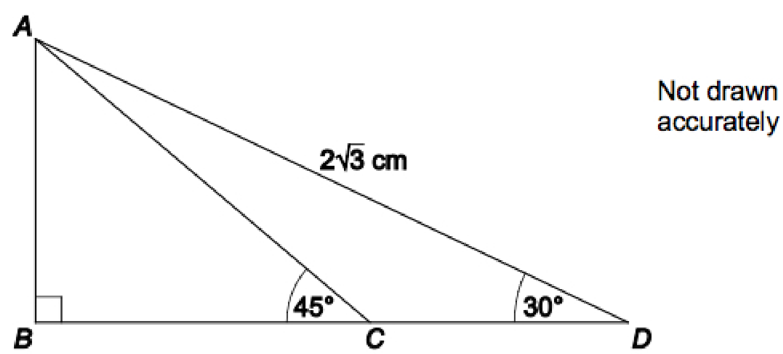
13. Solve (3)

$$x^{-\frac{1}{4}} = 0.2.$$

Solution

$$\begin{aligned} x^{-\frac{1}{4}} = 0.2 &\Rightarrow x^{\frac{1}{4}} = 5 \\ &\Rightarrow x = 5^4 \\ &\Rightarrow \underline{\underline{x = 625}}. \end{aligned}$$

14. In the diagram, BCD is a straight line. (4)
$AD = 2\sqrt{3}$ cm.



Work out the exact length of CD.
Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.

Solution

$$\begin{aligned} \cos = \frac{\text{adj}}{\text{hyp}} &\Rightarrow \cos 30^\circ = \frac{BD}{2\sqrt{3}} \\ &\Rightarrow BD = 2\sqrt{3}\cos 30^\circ \\ &\Rightarrow BD = 2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) \\ &\Rightarrow BD = 3 \text{ cm}. \end{aligned}$$

Now,

$$\begin{aligned} AB^2 + BD^2 = AD^2 &\Rightarrow AB^2 + 3^2 = (2\sqrt{3})^2 \\ &\Rightarrow AB^2 + 9 = 12 \\ &\Rightarrow AB^2 = 3 \\ &\Rightarrow AB = \sqrt{3} \\ &\Rightarrow BC = \sqrt{3}, \end{aligned}$$

as the triangle ABC is isosceles. Finally,

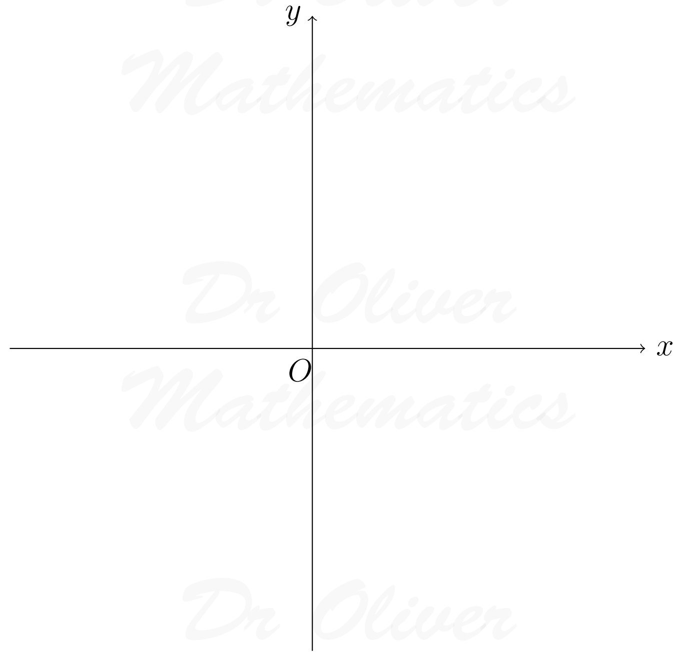
$$CD = BD - BC = \underline{\underline{(3 - \sqrt{3})\text{ cm}}};$$

hence, $\underline{\underline{a = 3}}$ and $\underline{\underline{b = -3}}$.

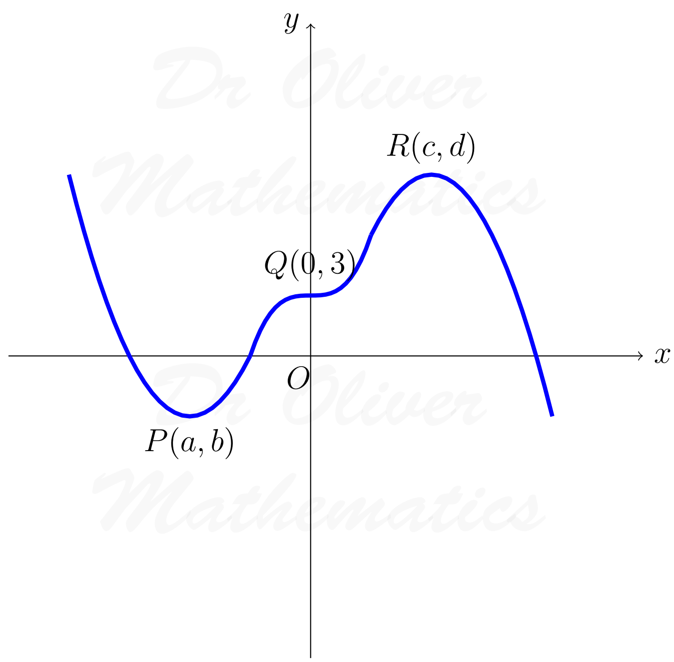
15. The continuous curve $y = \mathrm{f}(x)$ has exactly three stationary points. (4)
The three stationary points are

- a minimum point P at (a, b) where $a < 0$ and $b < 0$,
- a point of inflection Q at $(0, 3)$, and
- a maximum point R at (c, d) where $c > 0$ and $d > 3$.

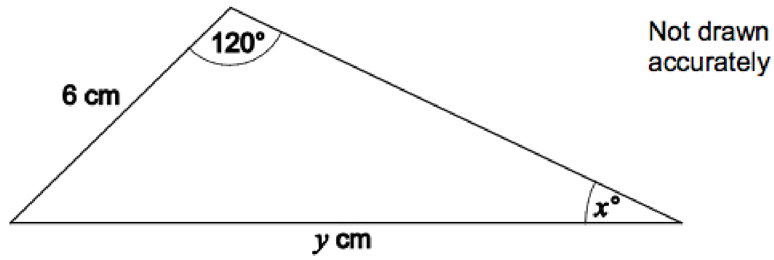
The curve cuts the x-axis at three distinct points.
On the axes below, sketch the curve.
Label the points P, Q, and R on your sketch.



Solution



16. Here is a triangle. (4)



$\sin x^\circ = \dfrac{1}{\sqrt{12}}$.

Work out the value of y.

Solution

Sine rule:

$$\begin{aligned}\frac{y}{\sin 120^\circ} = \frac{6}{\sin x^\circ} &\Rightarrow \frac{y}{\sin 120^\circ} = \frac{6}{\frac{1}{\sqrt{12}}} \\ &\Rightarrow \frac{y}{\frac{\sqrt{3}}{2}} = 6\sqrt{12} \\ &\Rightarrow y = \tfrac{\sqrt{3}}{2} \times 6 \times \sqrt{4 \times 3} \\ &\Rightarrow y = \tfrac{\sqrt{3}}{2} \times 6 \times 2\sqrt{3} \\ &\Rightarrow \underline{\underline{y = 18}}.\end{aligned}$$

17. (a) Factorise (2)

$$2x^2 + 7x + 5.$$

Solution

$$\left.\begin{array}{ll}\text{add to:} & +7 \\ \text{multiply to:} & (+2) \times (+5) = +10\end{array}\right\} +2,\ +5$$

$$\begin{aligned}2x^2 + 7x + 5 &= 2x^2 + 2x + 5x + 5 \\ &= 2x(x+1) + 5(x+1) \\ &= \underline{\underline{(2x+5)(x+1)}}.\end{aligned}$$

(b) Hence, or otherwise, work out the value of θ between 0° and 360° for which (3)

$$2\sin^2\theta + 7\sin\theta + 5 = 0.$$

Solution

$$\begin{aligned} 2\sin^2\theta + 7\sin\theta + 5 = 0 &\Rightarrow (2\sin\theta + 5)(\sin\theta + 1) = 0 \\ &\Rightarrow \sin\theta = -\tfrac{5}{2} \text{ or } \sin\theta = -1. \end{aligned}$$

Now, $\sin\theta \neq -\frac{5}{2}$ as $-1 \leqslant \sin\theta \leqslant 1$ and that leaves

$$\sin\theta = -1 \Rightarrow \underline{\underline{\theta = 270}}.$$

18. Simplify fully (5)

$$\frac{24 - \sqrt{300}}{4\sqrt{3} - 5}.$$

Give your answer in the form $a\sqrt{b}$, where a and b are integers.

Solution

Well,

$$\begin{aligned} \sqrt{300} &= \sqrt{100 \times 3} \\ &= \sqrt{100} \times \sqrt{3} \\ &= 10\sqrt{3} \end{aligned}$$

and

$$\begin{aligned} \frac{24 - \sqrt{300}}{4\sqrt{3} - 5} &= \frac{24 - 10\sqrt{3}}{4\sqrt{3} - 5} \\ &= \frac{24 - 10\sqrt{3}}{4\sqrt{3} - 5} \times \frac{4\sqrt{3} + 5}{4\sqrt{3} + 5} \end{aligned}$$

$\times$	24	$-10\sqrt{3}$
$4\sqrt{3}$	$96\sqrt{3}$	-120
$+5$	$+120$	$-50\sqrt{3}$

$$
\begin{aligned}
&= \frac{46\sqrt{3}}{(4\sqrt{3})^2 - 5^2} \\
&= \frac{46\sqrt{3}}{48 - 25} \\
&= \frac{46\sqrt{3}}{23} \\
&= \underline{\underline{2\sqrt{3}}};
\end{aligned}
$$

hence, $\underline{\underline{a = 2}}$ and $\underline{\underline{b = 3}}$.