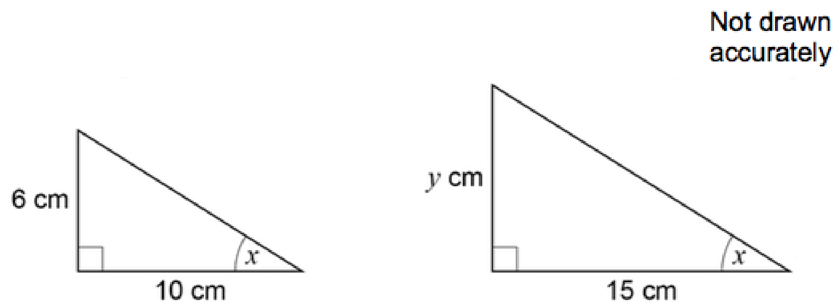
Dr Oliver Mathematics
AQA GCSE Mathematics
2019 June Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Here are two right-angled triangles. (1)



Circle the value of y.

$$11 \quad 7.5 \quad 9 \quad 4$$

Solution

Similar triangles:

$$\frac{6}{10} = \frac{y}{15} \Rightarrow y = \frac{3}{5} \times 15$$
$$\Rightarrow y = 3 \times 3$$
$$\Rightarrow y = 9$$

so

$$11 \quad 7.5 \quad \underline{\underline{9}} \quad 4$$

2. Work out the value of (1)

$$(1\tfrac{2}{3})^2.$$

Circle your answer.

$$1\tfrac{4}{9} \quad 3\tfrac{1}{3} \quad 2\tfrac{4}{9} \quad 2\tfrac{7}{9}$$

Solution

$$\begin{aligned}(1\tfrac{2}{3})^2 &= (\tfrac{5}{3})^2 \\ &= \tfrac{5^2}{3^2} \\ &= \tfrac{25}{9} \\ &= 2\tfrac{7}{9}\end{aligned}$$

so

$$1\tfrac{4}{9} \quad 3\tfrac{1}{3} \quad 2\tfrac{4}{9} \quad \underline{\underline{2\tfrac{7}{9}}}$$

3. Work out the arc length, in metres, of a semicircle of radius 6 metres. (1)

Circle your answer.

$$3\pi \quad 6\pi \quad 12\pi \quad 18\pi$$

Solution

$$\begin{aligned}\text{Arc length} &= \tfrac{1}{2} \times 2 \times \pi \times 6 \\ &= 6\pi\end{aligned}$$

so

$$3\pi \quad \underline{\underline{6\pi}} \quad 12\pi \quad 18\pi$$

4. Circle the fraction that is equivalent to 4.625. (1)

$$\frac{39}{8} \quad \frac{37}{8} \quad \frac{185}{4} \quad \frac{17}{4}$$

Solution

Well,

$$\begin{aligned} 4.625 &= 4\tfrac{5}{8} \\ &= \tfrac{32+5}{8} \\ &= \tfrac{37}{8} \end{aligned}$$

so

$$\frac{39}{8} \quad \underline{\underline{\frac{37}{8}}} \quad \frac{185}{4} \quad \frac{17}{4}$$

5. (a) Write 0.000 97 in standard form. (1)

Solution

$$0.000\,97 = \underline{\underline{9.7 \times 10^{-4}}}.$$

(b) Work out (2)

$$\frac{3 \times 10^5}{4 \times 10^3}.$$

Give your answer as an ordinary number.

Solution

$$\begin{aligned} \frac{3 \times 10^5}{4 \times 10^3} &= \frac{3}{4} \times \frac{10^5}{10^3} \\ &= 0.75 \times 10^2 \\ &= 0.75 \times 100 \\ &= \underline{\underline{75}}. \end{aligned}$$

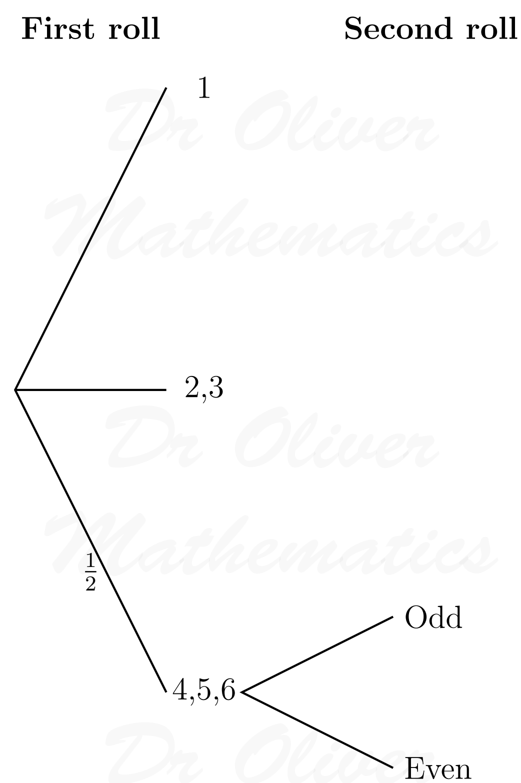
6. Anna plays a game with an ordinary, fair dice.

- If she rolls 1, she wins.
- If she rolls 2 or 3, she loses.
- If she rolls 4, 5 or 6, she rolls again.

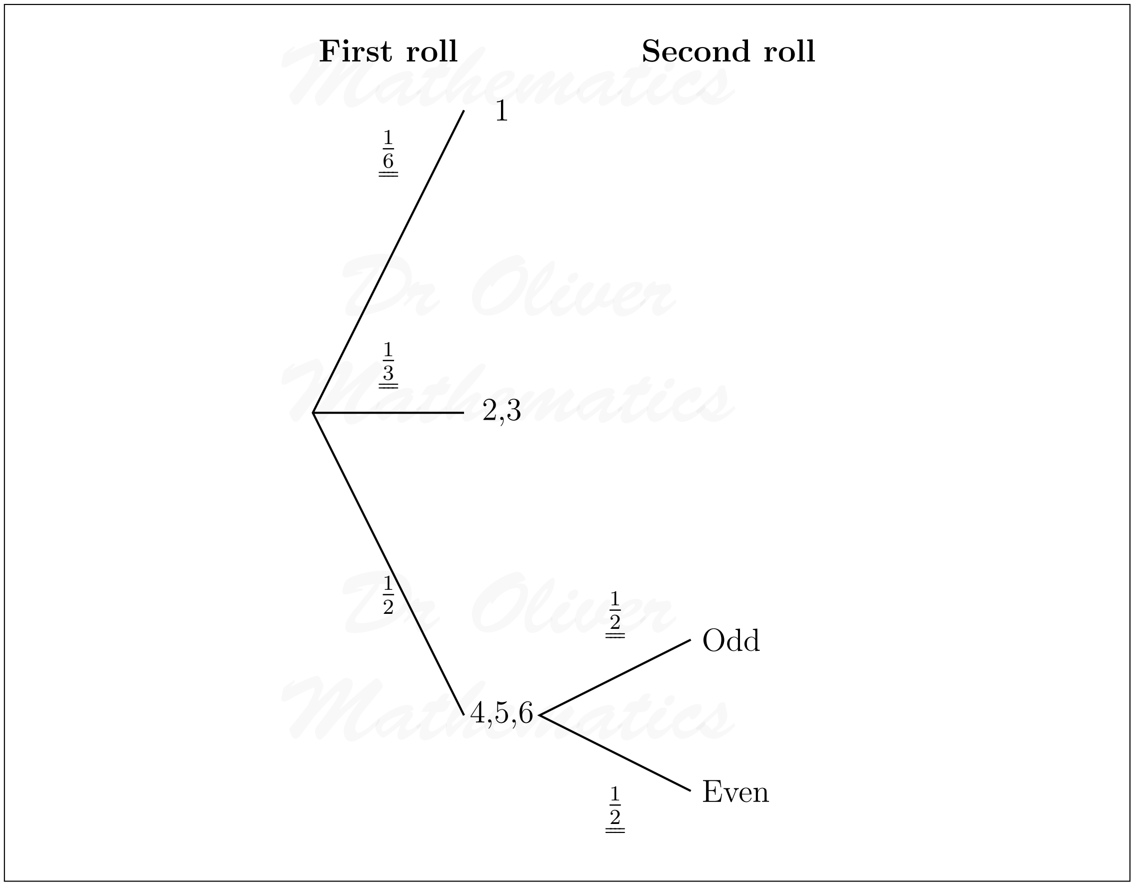
When she has to roll again,

- if she rolls an odd number, she wins or
- if she rolls an even number she loses.

(a) Complete the tree diagram with the four missing probabilities. (2)



Solution



(b) Is Anna more likely to win or to lose? (4)

You **must** work out the probability that she wins.

Solution

$$\begin{aligned} \text{P(wins)} &= \text{P(wins on the first go)} + \text{P(wins on the second go)} \\ &= \tfrac{1}{6} + (\tfrac{1}{2} \times \tfrac{1}{2}) \\ &= \tfrac{1}{6} + \tfrac{1}{4} \\ &= \tfrac{2}{12} + \tfrac{3}{12} \\ &= \tfrac{5}{12}; \end{aligned}$$

hence, she is more likely to lose.

7. Three friends arrive at a party. (2)

Their arrival increases the number of people at the party by 20%.

In total, how many people are now at the party?

Solution

Let there x people at the party before the friends arrive. Now,

$$\begin{aligned}\left(\frac{x+3}{x}\right)\times 100\% = 120\% &\Rightarrow \left(\frac{x+3}{x}\right)\times 5 = 6\\ &\Rightarrow \left(\frac{x+3}{x}\right) = \tfrac{6}{5}\\ &\Rightarrow 5(x+3) = 6x\\ &\Rightarrow 5x+15 = 6x\\ &\Rightarrow x = 15.\end{aligned}$$

Hence, there are

$$15 + 3 = \underline{\underline{18 \text{ people}}}$$

at the party.

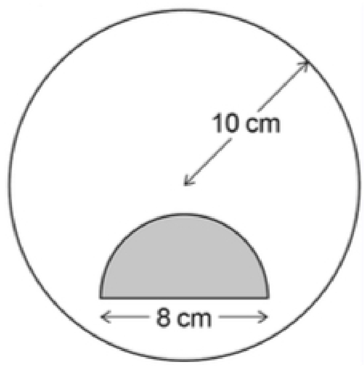
8. Work out the value of (3)

$$(3^{12} \div 3^5) \div (3^2 \times 3).$$

Solution

$$\begin{aligned}(3^{12} \div 3^5) \div (3^2 \times 3) &= \frac{3^{12} \div 3^5}{3^2 \times 3}\\ &= \frac{3^7}{3^3}\\ &= 3^4\\ &= \underline{\underline{81}}.\end{aligned}$$

9. A shaded semi-circle is inside a circle as shown. (4)



- The **radius** of the circle is 10 cm.
- The **diameter** of the semicircle is 8 cm.

How many times bigger is the unshaded area than the shaded area?

Solution

Well,

$$\begin{aligned}\text{shaded area} &= \tfrac{1}{2} \times \pi \times 4^2 \\ &= \tfrac{1}{2} \times \pi \times 16 \\ &= 8\pi\end{aligned}$$

and

$$\begin{aligned}\text{unshaded area} &= (\pi \times 10^2) - 8\pi \\ &= 100\pi - 8\pi \\ &= 92\pi.\end{aligned}$$

Finally,

$$\begin{aligned}\frac{92\pi}{8\pi} &= \frac{23 \times 4}{2 \times 4} \\ &= \underline{\underline{\tfrac{23}{2}}}.\end{aligned}$$

10. The number of items, n, made in 1 hour by a machine is given by

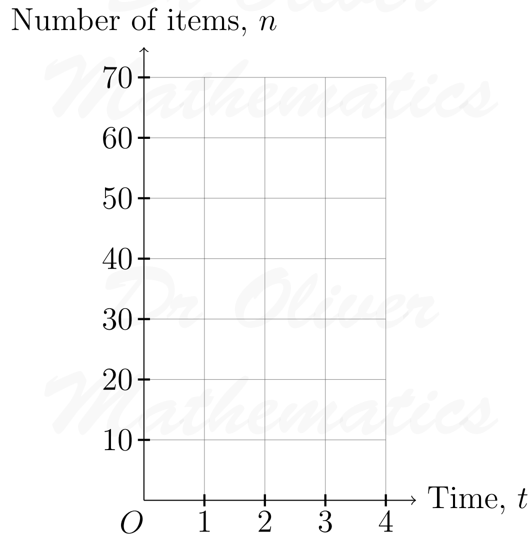
$$n = \frac{60}{t}.$$

- t is the time in minutes the machine takes to make one item.
- The value of t changes for different types of item.

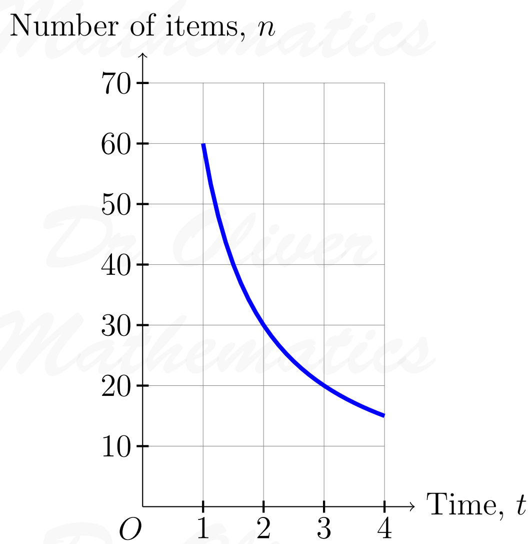
(a) On the grid below, draw the graph of (2)

$$n = \frac{60}{t},$$

for values of t from 1 to 4.



Solution

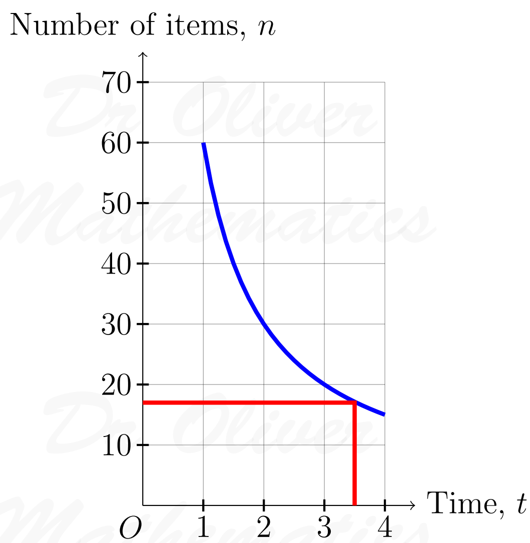


The machine takes 3 minutes 30 seconds to make one item.

(b) **Use your graph** to estimate the value of n. (2)

Solution

Correct read-off:



$\underline{\underline{n = 17}}$.

11. Ed and Fay shared £330 in the ratio 7 : 4. (3)

- Ed gives Fay some of his money.
- Fay now has the same amount as Ed.

How much does Ed give Fay?

Solution

Ed gets

$$\begin{aligned}\left(\frac{7}{7+4}\right) \times 330 &= \tfrac{7}{11} \times 330 \\ &= 7 \times 30 \\ &= £210\end{aligned}$$

and Fay gets

$$330 - 210 = £120.$$

Now,

$$\frac{330}{2} = 165$$

and he gives her

$$210 - 165 = \underline{\underline{£45}}.$$

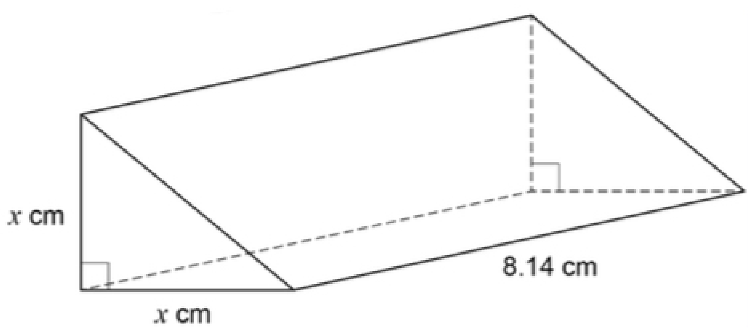
12. The next term of a sequence is made by adding the previous two terms. (1)
Which of these sequences follows this rule?
Circle your answer.

$-9, 2, -7, -5, -12 \quad -3, 5, -2, 3, 1 \quad 0, -3, -3, 0, -3 \quad -1, -1, -2, -3, 1$

Solution

$\underline{\underline{-9, 2, -7, -5, -12}} \quad -3, 5, -2, 3, 1 \quad 0, -3, -3, 0, -3 \quad -1, -1, -2, -3, 1$

13. The triangular cross section of a prism is an isosceles right-angled triangle. (3)



The volume of the prism is 102 cm^3.

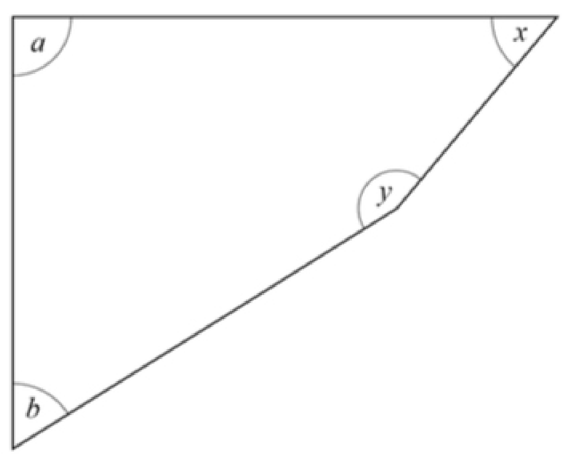
Use approximations to estimate the value of x.

You **must** show your working.

Solution

$$\begin{aligned}\text{Volume} = 102 &\Rightarrow \tfrac{1}{2} \times x^2 \times 8.14 = 102 \\ &\Rightarrow \tfrac{1}{2}x^2 \times 8 \approx 100 \\ &\Rightarrow 4x^2 = 100 \\ &\Rightarrow x^2 = 25 \\ &\Rightarrow \underline{\underline{x = 5}}.\end{aligned}$$

14. Here is a quadrilateral. (3)



Not drawn accurately

- $a = 90°$.
- $a : b = 5 : 3$.
- $x : y = 1 : 3$.

Show that

$$b = x.$$

Solution

Well,

$$a : b = 5 : 3 \Rightarrow \frac{b}{90} = \frac{3}{5}$$
$$\Rightarrow b = \tfrac{3}{5} \times 90$$
$$\Rightarrow b = 3 \times 18$$
$$\Rightarrow b = 54.$$

Now,

$$a + b = 90 + 54 = 144 \Rightarrow x + y = 216.$$

Next,

$$x = \tfrac{1}{4} \times 216 = 54.$$

Hence,

$$\underline{\underline{b = x}}.$$

15. Here is some information about the test marks of 120 students.

Mark, m	$0 < m \leqslant 10$	$10 < m \leqslant 20$	$20 < m \leqslant 30$	$30 < m \leqslant 40$	$40 < m \leqslant 50$
Frequency	20	28	40	20	12

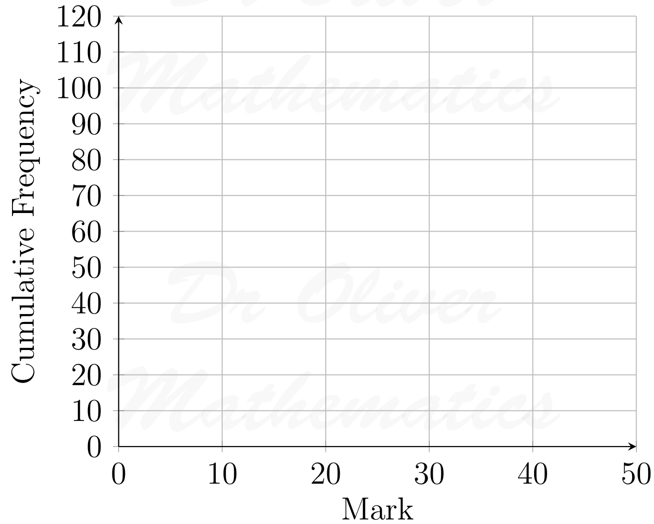
(a) Complete the cumulative frequency table. (1)

Mark, m	$m \leqslant 10$	$m \leqslant 20$	$m \leqslant 30$	$m \leqslant 40$	$m \leqslant 50$
Frequency	20	48			

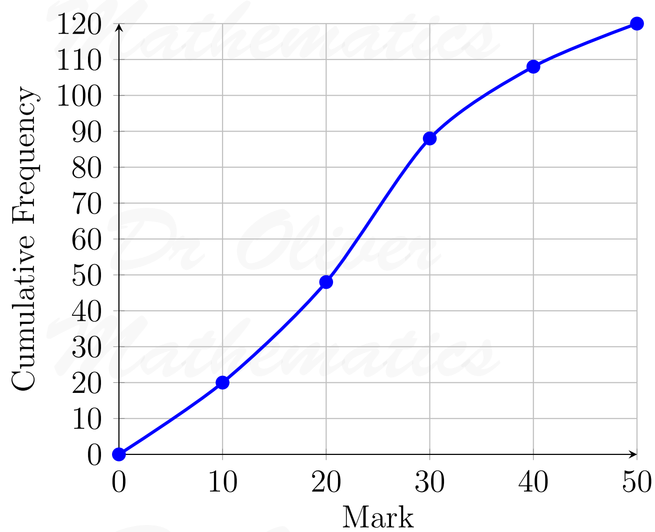
Solution

Mark, m	$m \leqslant 10$	$m \leqslant 20$	$m \leqslant 30$	$m \leqslant 40$	$m \leqslant 50$
Frequency	20	48	$\underline{\underline{88}}$	$\underline{\underline{108}}$	$\underline{\underline{120}}$

(b) Draw a cumulative frequency graph. (2)



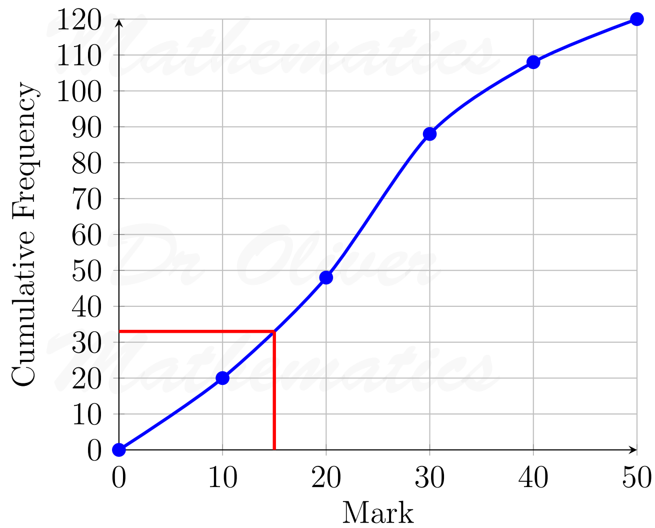
Solution



Students who scored 15 marks or fewer take another test.

(c) Use your graph to estimate how many students take another test. (2)

Solution



Correct read-off: $\underline{\underline{33}}$.

16. Simplify fully (3)

$$\frac{4x - 8x^2}{12x - 6}.$$

Solution

$$\begin{aligned}\frac{4x - 8x^2}{12x - 6} &= \frac{-4x(2x - 1)}{6(2x - 1)} \\ &= \underline{\underline{-\tfrac{2}{3}x}}.\end{aligned}$$

17. Toby is forming and solving equations.

(a) Toby uses y to represent the number. (2)

The product of half of a number and three more than the number
is the same as
the square of the number

Write an equation that Toby could form.

Solution

E.g., $\underline{\underline{\frac{1}{2}y(y+3) = y^2}}$.

Toby forms another equation.

$$x = \frac{9}{8x}.$$

He wants to work out the values of x.

Here is his working.

$$x = \frac{9}{8x}$$

$$8x^2 = 9$$

$$8x = 3 \text{ or } 8x = -3$$

$$x = \frac{3}{8} \text{ or } x = -\frac{3}{8}$$

(b) What error has he made in his working? (1)

Solution

On the second line, he has got $8x^2 = 9$ (correct) which leads to

$$8x^2 = 9 \Rightarrow \sqrt{8x^2} = \pm 3$$
$$\Rightarrow \underline{\underline{2\sqrt{2}\,x = \pm 3}}.$$

18. Here is an identity:

$$x^2 - y^2 \equiv (x+y)(x-y).$$

(a) Use the identity to work out the value of (2)

$$193^2 - 7^2.$$

You **must** show your working.

Solution

$$\begin{aligned}193^2 - 7^2 &= (193 + 7)(193 - 7)\\ &= (200)(186)\\ &= \underline{\underline{37\,200}}.\end{aligned}$$

(b) Factorise (1)

$$100a^2 - 81b^2.$$

Solution

Difference of two squares:

$$\begin{aligned}100a^2 - 81b^2 &= (10a)^2 - (9b)^2\\ &= \underline{\underline{(10a + 9b)(10a - 9b)}}.\end{aligned}$$

19. Circle the fraction that is equivalent to $0.\dot{1}$. (1)

$$\tfrac{1}{9} \quad \tfrac{1}{99} \quad \tfrac{1}{10} \quad \tfrac{11}{100}$$

Solution

Well,

$$\begin{aligned}x &= 0.\dot{1} \quad (1)\\ 10x &= 1.\dot{1} \quad (2).\end{aligned}$$

Do (2) − (1):

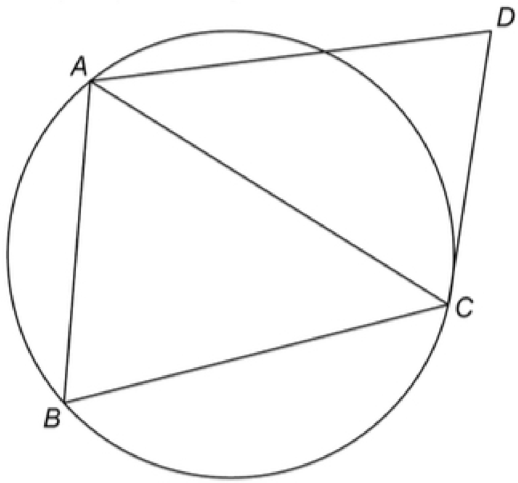
$$9x = 1 \Rightarrow x = \tfrac{1}{9}$$

so

$$\underline{\underline{\tfrac{1}{9}}} \quad \tfrac{1}{99} \quad \tfrac{1}{10} \quad \tfrac{11}{100}$$

20. A, B, and C are points on a circle.
CD is a tangent.

Not drawn accurately



Assume that triangle ABC is isosceles with $AC = BC$.

(a) Prove that AB is parallel to DC. (4)

Solution
Let $\angle BAC = \angle ABC = x$.
Now, $\angle ACD = x$ (alternate segment theorem)
Finally, $\angle BAC = \angle ACD$ (alternate angles) so AB is parallel to DC

In fact, triangle ABC is equilateral.

(b) Tick the **two** boxes for the statements that **must** be correct. (1)

☐ *AB* is parallel to *DC*

☐ *AC* bisects angle *BCD*

☐ *AC* bisects angle *BAD*

Solution
AB is parallel to DC and AC bisects angle BCD.

21. Solve the simultaneous equations (4)

$$2x + 3y = 5p$$
$$y = 2x + p,$$

where p is a constant.

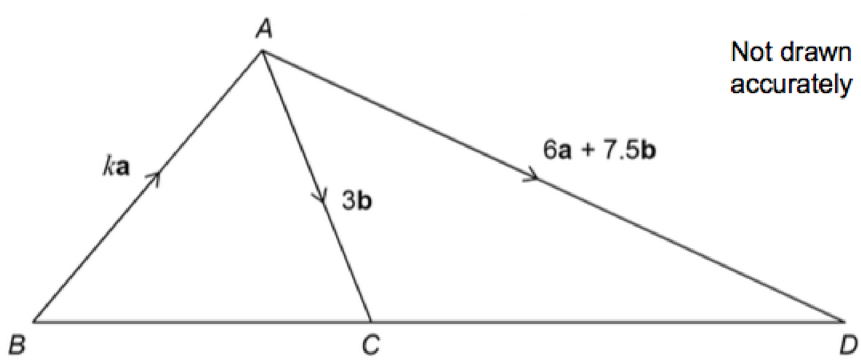
Give your answers in terms of p in their simplest form.

Solution

Well,

$$\begin{aligned} 2x + 3y = 5p &\Rightarrow 2x + 3(2x + p) = 5p \\ &\Rightarrow 2x + 6x + 3p = 5p \\ &\Rightarrow 8x = 2p \\ &\Rightarrow \underline{\underline{x = \tfrac{1}{4}p}} \\ &\Rightarrow y = 2(\tfrac{1}{4}p) + p \\ &\Rightarrow \underline{\underline{y = \tfrac{3}{2}p}}. \end{aligned}$$

22. ABC and ACD are triangles.
k is a constant.



(a) Show that (1)

$$\overrightarrow{CD} = 6\mathbf{a} + 4.5\mathbf{b}.$$

Solution

$$\begin{aligned}\overrightarrow{CD} &= \overrightarrow{CA} + \overrightarrow{AD} \\ &= -\overrightarrow{AC} + \overrightarrow{AD} \\ &= -3\mathbf{b} + (6\mathbf{a} + 7.5\mathbf{b}) \\ &= \underline{\underline{6\mathbf{a} + 4.5\mathbf{b}}},\end{aligned}$$

as required.

BCD is a straight line.

(b) Work out the value of k. (3)
You **must** show your working.

Solution

Now,

$$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{BA} + \overrightarrow{AC} \\ &= k\mathbf{a} + 3\mathbf{b}\end{aligned}$$

and

$$\frac{k}{3} = \frac{6}{4.5} \Rightarrow k = \tfrac{4}{3} \times 3$$
$$\Rightarrow \underline{\underline{k = 4}}.$$

23. Simplify (3)

$$8^4 \div 32^{\frac{2}{5}}.$$

Give your answer in the form

$$2^m,$$

where m is an integer.

Solution

Well,

$$
\begin{aligned}
8^4 \div 32^{\frac{2}{5}} &= \frac{(2^3)^4}{(2^5)^{\frac{2}{5}}} \\
&= \frac{2^{12}}{2^2} \\
&= \underline{\underline{2^{10}}}.
\end{aligned}
$$

24. (1)

$$\mathrm{f}(x) = \sin(x - 90)^\circ.$$

Circle the value of $\mathrm{f}(0^\circ)$.

$$1 \quad 0 \quad -\tfrac{1}{2} \quad -1$$

Solution

Now,

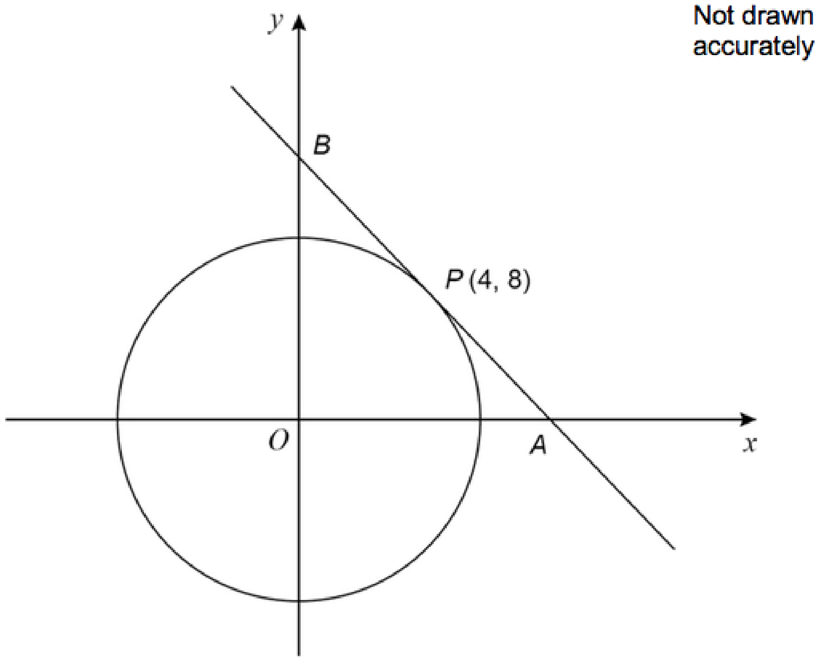
$$\mathrm{f}(0^\circ) = \sin(-90)^\circ = -1$$

so

$$1 \quad 0 \quad -\tfrac{1}{2} \quad \underline{\underline{-1}}$$

25. $P(4, 8)$ is a point on a circle, centre O.

The tangent at P intersects the axes at points A and B.



(a) Show that the gradient of the tangent is $-\frac{1}{2}$. (2)

Solution

Well,

$$m_{OP} = \frac{8-0}{4-0} = 2$$

which means the gradient of the tangent is

$$m_{AB} = \underline{\underline{-\tfrac{1}{2}}}.$$

(b) Work out the length AB. (4)

Give your answer in the form $a\sqrt{5}$, where a is an integer.

You **must** show your working.

Solution

Now, the equation of AB is

$$y - 8 = -\tfrac{1}{2}(x - 4) \Rightarrow y - 8 = -\tfrac{1}{2}x + 2$$
$$\Rightarrow y = -\tfrac{1}{2}x + 10.$$

Next,

$$x = 0 \Rightarrow y = 10$$

and

$$\begin{aligned} y = 0 \Rightarrow 0 &= -\tfrac{1}{2}x + 10 \\ \Rightarrow \tfrac{1}{2}x &= 10 \\ \Rightarrow x &= 20; \end{aligned}$$

so $A(20, 0)$ and $B(0, 10)$.

Finally,

$$\begin{aligned} AB &= \sqrt{(20 - 0)^2 + (0 - 10)^2} \\ &= \sqrt{400 + 100} \\ &= \sqrt{500} \\ &= \sqrt{100 \times 5} \\ &= \sqrt{100} \times \sqrt{5} \\ &= \underline{\underline{10\sqrt{5}}}; \end{aligned}$$

hence, $\underline{\underline{a = 10}}$.

26. The turning point of the graph (3)

$$y = (x + a)^2 + b$$

has x-coordinate -2.

$(3, 1)$ is another point on the graph.

Work out the y-coordinate of the turning point.

Solution

Now, the equation is

$$y = (x + 2)^2 + b.$$

Next,

$$\begin{aligned} x = 3, y = 1 \Rightarrow 1 &= 5^2 + b \\ \Rightarrow 1 &= 25 + b \\ \Rightarrow \underline{\underline{b = -24}}. \end{aligned}$$

27. Angle x is acute. (3)

$$\cos x = \sin 60^\circ \times \tan 30^\circ.$$

Work out the size of angle x.
You **must** show your working.

Solution

$$\begin{aligned} \cos x = \sin 60^\circ \times \tan 30^\circ &\Rightarrow \cos x = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{3} \\ &\Rightarrow \cos x = \frac{3}{6} \\ &\Rightarrow \cos x = \tfrac{1}{2} \\ &\Rightarrow \underline{\underline{x = 60^\circ}}. \end{aligned}$$