

Dr Oliver Mathematics
GCSE Mathematics
2017 Paper 3H: Calculator
1 hour 30 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

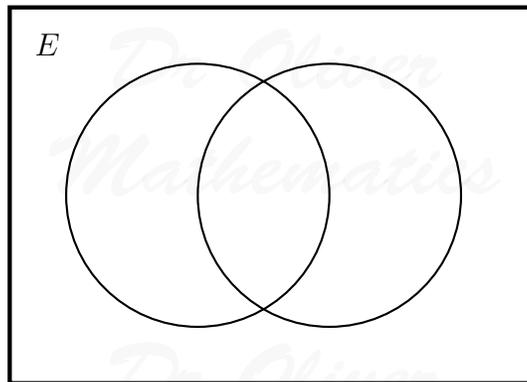
1. $\mathcal{E} = \{\text{odd numbers less than } 30\}$.

$A = \{3, 9, 15, 21, 27\}$.

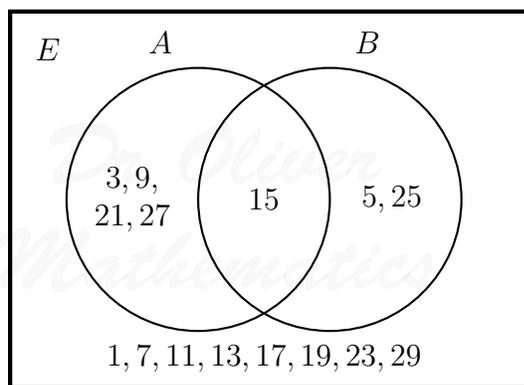
$B = \{5, 15, 25\}$.

(a) Complete the Venn diagram to represent this information.

(4)



Solution



A number is chosen at random from the universal set, E .

(b) What is the probability that the number is in the set $A \cup B$?

(2)

Solution

$$P(A \cup B) = \frac{4+1+2}{15} = \frac{7}{15}.$$

2. Solve the simultaneous equations

(3)

$$3x + y = -4$$

$$3x - 4y = 6.$$

Solution

Subtract:

$$5y = -10 \Rightarrow \underline{\underline{y = -2}}$$

$$\Rightarrow 3x = -2$$

$$\Rightarrow \underline{\underline{x = -\frac{2}{3}}}.$$

3. The table shows some information about the dress sizes of 25 women.

Dress size	Number of women
8	2
10	9
12	8
14	6

(a) Find the median dress size.

(1)

Solution

Dress size	Number of women	Cumulative Frequency
8	2	2
10	9	11
12	8	19
14	6	25

There are 25 women and the median is $\frac{25+1}{2} = 13$ th piece of information. Hence, the median dress size is 12.

3 of the 25 women have a shoe size of 7.

Zoe says that if you choose at random one of the 25 women, the probability that she has either a shoe size of 7 or a dress size of 14 is $\frac{9}{25}$ because

$$\frac{3}{25} + \frac{6}{25} = \frac{9}{25}.$$

(b) Is Zoe correct?

You must give a reason for your answer.

(1)

Solution

No: you cannot tell if they are mutually exclusive or not.

4. Daniel bakes 420 cakes.

He bakes only vanilla cakes, banana cakes, lemon cakes, and chocolate cakes.

$\frac{2}{7}$ of the cakes are vanilla cakes.

35% of the cakes are banana cakes.

The ratio of the number of lemon cakes to the number of chocolate cakes is 4 : 5.

Work out the number of lemon cakes Daniel bakes.

(5)

Solution

Vanilla: $\frac{2}{7} \times 420 = 120$ cakes.

Banana: $\frac{35}{100} \times 420 = 147$ cakes.

Vanilla and banana: $120 + 147 = 267$ cakes.

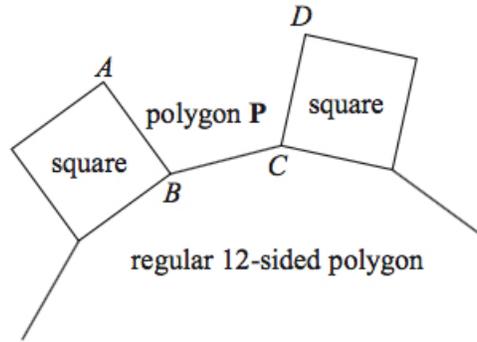
Lemon and chocolate: $420 - 267 = 153$ cakes.

Finally,

$$\text{lemon cakes} = \frac{4}{9} \times 153 = \underline{\underline{68 \text{ cakes}}}.$$

5. In the diagram, AB , BC , and CD are three sides of a regular polygon P .

(4)



Show that polygon **P** is a hexagon.
You must show your working.

Solution

Let the polygon be n sided. First,

$$\text{exterior angle} = \frac{360}{12} = 30^\circ$$

and

$$\text{all interior angles} = 180 - 30 = 150^\circ.$$

Now,

$$360 - 90 - 150 = 120^\circ$$

and, finally,

$$\begin{aligned} 120 &= \frac{180(n-2)}{n} \Rightarrow 120 = 180(n-2) \\ &\Rightarrow 120n = 180n - 360 \\ &\Rightarrow 60n = 360 \\ &\Rightarrow n = 6, \end{aligned}$$

and we have a hexagon.

6. The density of apple juice is 1.05 grams per cm^3 .
The density of fruit syrup is 1.4 grams per cm^3 .
The density of carbonated water is 0.99 grams per cm^3 .
25 cm^3 of apple juice are mixed with 15 cm^3 of fruit syrup and 280 cm^3 of carbonated water to make a drink with a volume of 320 cm^3 .
Work out the density of the drink.
Give your answer correct to 2 decimal places.

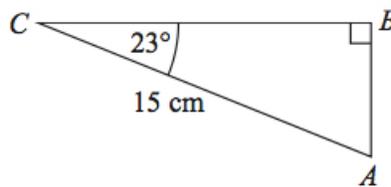
(4)

Solution

$$\begin{aligned}\text{Density} &= \frac{(1.05 \times 25) + (1.4 \times 15) + (0.99 \times 280)}{320} \\ &= 1 \frac{89}{6400} \\ &= \underline{\underline{1.01 \text{ grams per cm}^3 \text{ (2 dp)}}}.\end{aligned}$$

7. ABC is a right-angled triangle.

(2)



Calculate the length of AB .

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}AB &= 15 \sin 23^\circ \\ &= 5.860966927 \text{ (FCD)} \\ &= \underline{\underline{5.86 \text{ cm (3 sf)}}}.\end{aligned}$$

8. A square, with sides of length x cm, is inside a circle. Each vertex of the square is on the circumference of the circle. The area of the circle is 49 cm^2 . Work out the value of x . Give your answer correct to 3 significant figures.

(4)

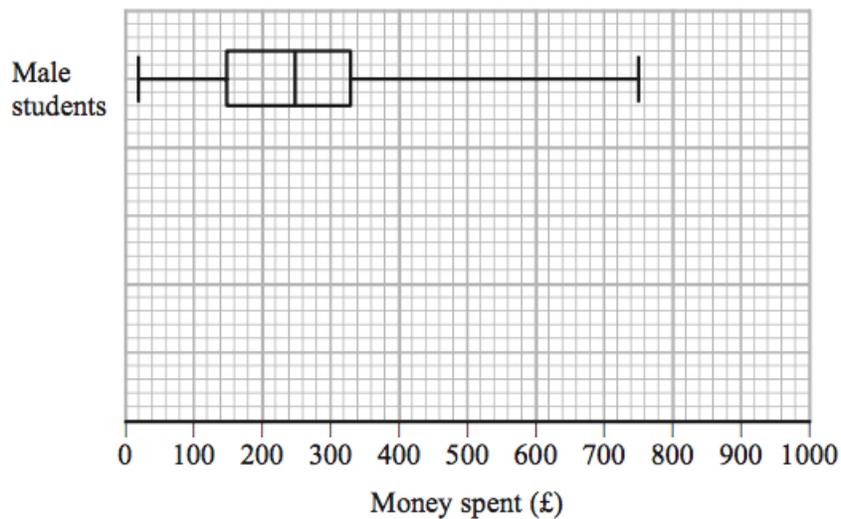
Solution

$$\pi r^2 = 49 \Rightarrow r^2 = \frac{49}{\pi}$$

and

$$\begin{aligned} \left(\frac{1}{2}x\right)^2 + \left(\frac{1}{2}x\right)^2 &= \frac{49}{\pi} \Rightarrow \frac{1}{2}x^2 = \frac{49}{\pi} \\ \Rightarrow x^2 &= \frac{98}{\pi} \\ \Rightarrow x &= \sqrt{\frac{98}{\pi}} \\ \Rightarrow x &= 5.585\ 191\ 926 \text{ (FCD)} \\ \Rightarrow x &= \underline{\underline{5.59 \text{ cm (3 sf)}}}. \end{aligned}$$

9. The box plot shows information about the distribution of the amounts of money spent by some male students on their holidays.



- (a) Work out the interquartile range for the amounts of money spent by these male students. (2)

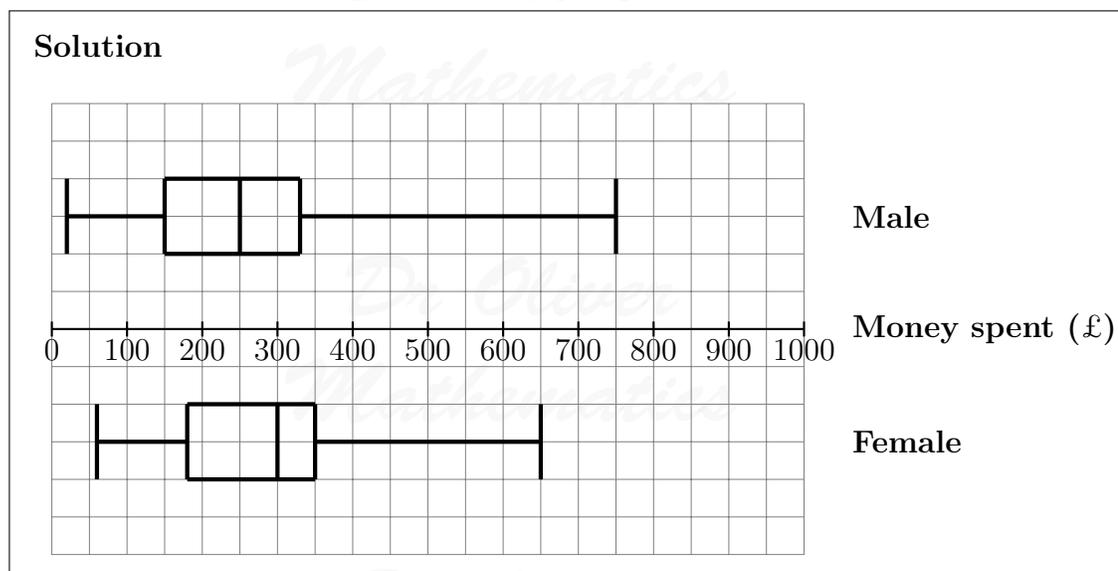
Solution

$$330 - 150 = \underline{\underline{\pounds 180}}.$$

The table below shows information about the distribution of the amounts of money spent by some female students on their holidays.

	Smallest	Lower Quartile	Median	Upper Quartile	Largest
Money spent (£)	60	180	300	350	650

- (b) On the grid above, draw a box plot for the information in the table. (2)



Chris says, “The box plots show that the female students spent more money than the male students.”

- (c) Is Chris correct? (1)
 Give a reason for your answer.

Solution

The women spent, on average, £300 and the men spent, on average, £250, so Chris is correct.

10. Naoby invests £6000 for 5 years. (3)
 The investment gets compound interest of $x\%$ per annum.
 At the end of 5 years the investment is worth £8029.35.
 Work out the value of x .

Solution

$$\begin{aligned}
8029.35 &= 6000\left(1 + \frac{x}{100}\right)^5 \Rightarrow \left(1 + \frac{x}{100}\right)^5 = 1.338\,255 \\
&\Rightarrow 1 + \frac{x}{100} = \sqrt[5]{1.338\,255} \\
&\Rightarrow 1 + \frac{x}{100} = 1.059\,999\,908 \text{ (FCD)} \\
&\Rightarrow \frac{x}{100} = 0.059\,999\,908 \text{ (FCD)} \\
&\Rightarrow x = 5.999\,990\,8 \text{ (FCD);}
\end{aligned}$$

hence, it is 6%.

11. Jeff is choosing a shrub and a rose tree for his garden. (2)
- At the garden centre there are 17 different types of shrubs and some rose trees. Jeff says, "There are 215 different ways to choose one shrub and one rose tree." Could Jeff be correct?
You must show how you get your answer.

Solution

$$\frac{215}{17} = 12\frac{11}{17};$$

this is *not* a natural number and so, no, Jeff is not correct.

12. The points A , B , C , and D lie in order on a straight line. (3)
- $AB : BD = 1 : 5$.
 $AC : CD = 7 : 11$.
Work out $AB : BC : CD$.

Solution

$$\begin{aligned}
AB : BD = 1 : 5 &\Rightarrow 5AB = BD \\
&\Rightarrow 5AB = BC + CD \\
&\Rightarrow 55AB = 11BC + 11CD \quad (1)
\end{aligned}$$

and

$$\begin{aligned}
AC : CD = 7 : 11 &\Rightarrow 11AC = 7CD \\
&\Rightarrow 11AB + 11BC = 7CD \\
&\Rightarrow 11AB = 7CD - 11BC \\
&\Rightarrow 55AB = 35CD - 55BC. \quad (2)
\end{aligned}$$

Subtract (2) – (1):

$$\begin{aligned}0 &= 24CD - 66BC \Rightarrow 24CD = 66BC \\ &\Rightarrow CD = \frac{11}{4}BC \\ &\Rightarrow BC : CD = 4 : 11.\end{aligned}$$

Next,

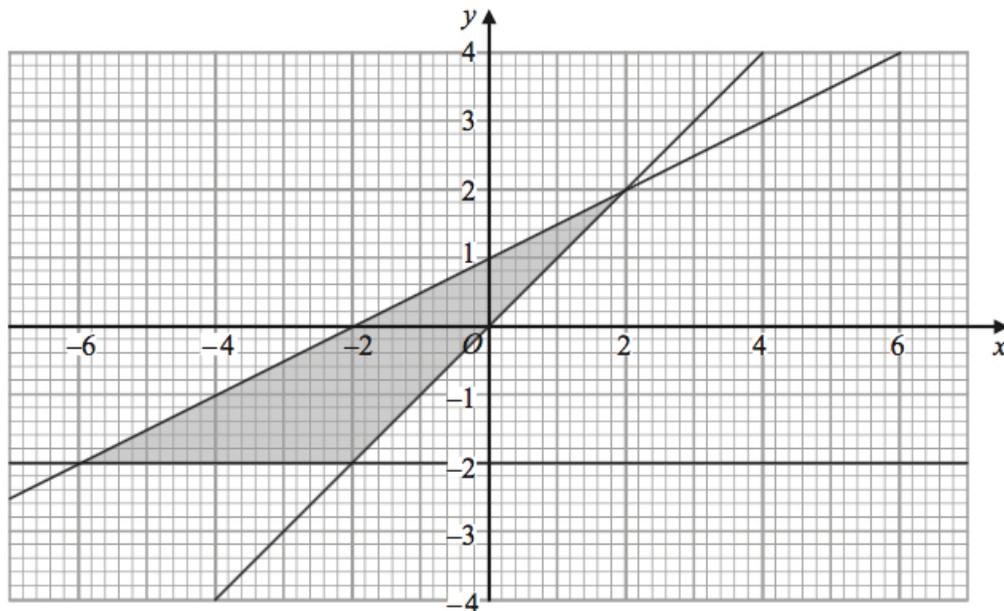
$$\begin{aligned}11AB &= 7CD - 11BC \Rightarrow 11AB = 7CD - 4CD \\ &\Rightarrow 11AB = 3CD \\ &\Rightarrow AB : CD = 3 : 11.\end{aligned}$$

Finally,

$$AB : BC : CD = \underline{\underline{3 : 4 : 11}}.$$

13. Write down the three inequalities that define the shaded region.

(4)



Solution

$$\underline{\underline{y \geq x.}}$$

$$\underline{\underline{y \leq -2.}}$$

$$\underline{\underline{y \leq \frac{1}{2}x + 1.}}$$

14. (a) Simplify

$$\frac{x^2 - 6}{2x^2 - 5x - 12}.$$

(3)

Solution

$$\begin{aligned} \frac{x^2 - 6}{2x^2 - 5x - 12} &\equiv \frac{(x - 4)(x + 4)}{(2x + 3)(x - 4)} \\ &\equiv \frac{x + 4}{\underline{\underline{2x + 3}}}. \end{aligned}$$

(b) Make v the subject of the formula

$$w = \frac{15(t - 2v)}{v}.$$

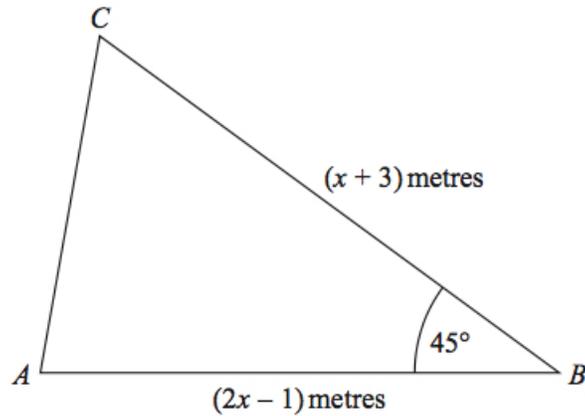
(3)

Solution

$$\begin{aligned} w = \frac{15(t - 2v)}{v} &\Rightarrow vw = 15(t - 2v) \\ &\Rightarrow vw = 15t - 30v \\ &\Rightarrow vw + 30v = 15t \\ &\Rightarrow v(w + 30) = 15t \\ &\Rightarrow v = \underline{\underline{\frac{15t}{w + 30}}}. \end{aligned}$$

15. The area of triangle ABC is $6\sqrt{2}$ m².

(5)



Calculate the value of x .
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} \frac{1}{2} \times (2x - 1) \times (x + 3) \times \sin 45^\circ &= 6\sqrt{2} \\ \Rightarrow \frac{1}{2\sqrt{2}}(2x - 1)(x + 3) &= 6\sqrt{2} \\ \Rightarrow 2x^2 + 5x - 3 &= 24 \\ \Rightarrow 2x^2 + 5x - 27 &= 0 \\ \Rightarrow x &= \frac{-5 \pm \sqrt{(-5)^2 - 4 \times 2 \times (-27)}}{4} \\ \Rightarrow x &= \frac{-5 \pm \sqrt{241}}{4} \\ \Rightarrow x &= -5.131\dots \text{ and } x = 2.631\dots; \end{aligned}$$

as $x = -5.131\dots$ is negative, $x = 2.63$ (3 sf).

16. Using

$$x_{n+1} = -2 - \frac{4}{x_n^2} \text{ with } x_0 = -2.5,$$

(a) find the values of x_1 , x_2 , and x_3 .

(3)

Solution

$$x_1 = -2 - \frac{4}{(-2.5)^2}$$

$$= \underline{\underline{-2.64}};$$

$$x_2 = -2 - \frac{4}{(-2.64)^2}$$

$$= \underline{\underline{-2\frac{625}{1089}}};$$

$$x_3 = -2 - \frac{4}{(-2\frac{625}{1089})^2}$$

$$= \underline{\underline{-2.603\ 767\ 255}} \text{ (FCD)}.$$

- (b) Explain the relationship between the values of x_1 , x_2 , and x_3 and the equation $x^3 + 2x^2 + 4 = 0$. (2)

Solution

$$x = -2 - \frac{4}{x^2} \Rightarrow x^3 = -2x^2 - 4$$

$$\Rightarrow x^3 + 2x^2 + 4 = 0;$$

iteration is an estimation of a solution.

17. A train travelled along a track in 110 minutes, correct to the nearest 5 minutes. Jake finds out that the track is 270 km long. He assumes that the track has been measured correct to the nearest 10 km.

- (a) Could the average speed of the train have been greater than 160 km/h? (4)
You must show how you get your answer.

Solution

$$265 \leq \text{length (km)} < 275$$

and

$$107.5 \leq \text{minutes (min)} < 112.5.$$

But we want km/h and so

$$\text{the average speed} = \frac{275 \times 60}{107.5} = 153\frac{21}{43}$$

and the answer is no.

Jake's assumption was wrong.

The track was measured correct to the nearest 5 km.

(b) Explain how this could affect your decision in part (a).

(1)

Solution

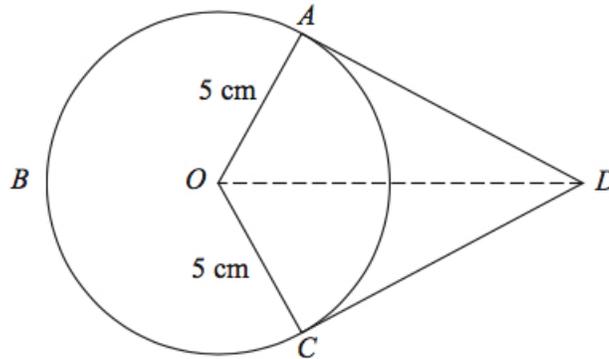
This time,

$$267.5 \leq \text{length (km)} < 272.5$$

and the average speed would drop.

18. A , B , and C are points on a circle of radius 5 cm, centre O .
 DA and DC are tangents to the circle.
 $DO = 9$ cm.

(5)



Work out the length of arc ABC .

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} \cos \angle AOD &= \frac{5}{9} \Rightarrow \angle AOD = 56.251\,011\,4 \text{ (FCD)} \\ &\Rightarrow \angle AOC = 112.502\,022\,8 \text{ (FCD)} \\ &\Rightarrow \angle ABC = 247.497\,977\,2 \text{ (FCD)} \\ &\Rightarrow \text{arc } ABC = \frac{247.497\dots}{360} \times 2 \times 5 \times \pi \\ &\Rightarrow \text{arc } ABC = 21.598\,272\,97 \text{ (FCD)} \\ &\Rightarrow \text{arc } ABC = \underline{\underline{21.6 \text{ cm (3 sf)}}}. \end{aligned}$$

19. Solve

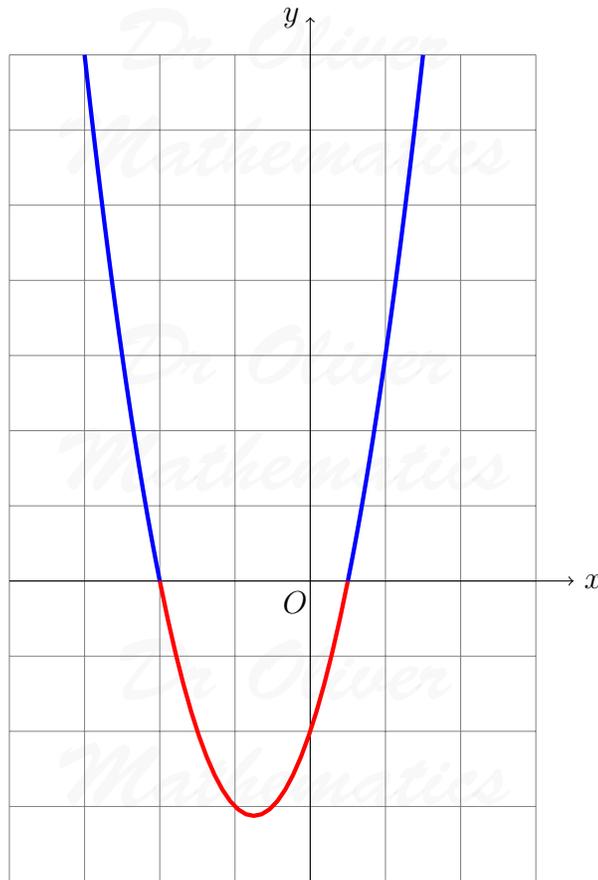
(3)

$$2x^2 + 3x - 2 > 0.$$

Solution

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to: } 2 \times (-2) = -4 \end{array} \right\} -1, +4$$

$$\begin{aligned} 2x^2 + 3x - 2 = 0 &\Rightarrow 2x^2 + 4x - x - 2 = 0 \\ &\Rightarrow 2x(x + 2) - (x + 2) = 0 \\ &\Rightarrow (2x - 1)(x + 2) = 0 \\ &\Rightarrow x = -2 \text{ or } x = \frac{1}{2}. \end{aligned}$$



Finally,

$$\underline{\underline{x < -2 \text{ or } x > \frac{1}{2}}}$$

20. The equation of a curve is $y = a^x$.
A is the point where the curve intersects the y -axis.

(a) State the coordinates of A.

(1)

Solution

(0, 1).

(What happens if $a = 0$?)

The equation of circle **C** is $x^2 + y^2 = 16$.

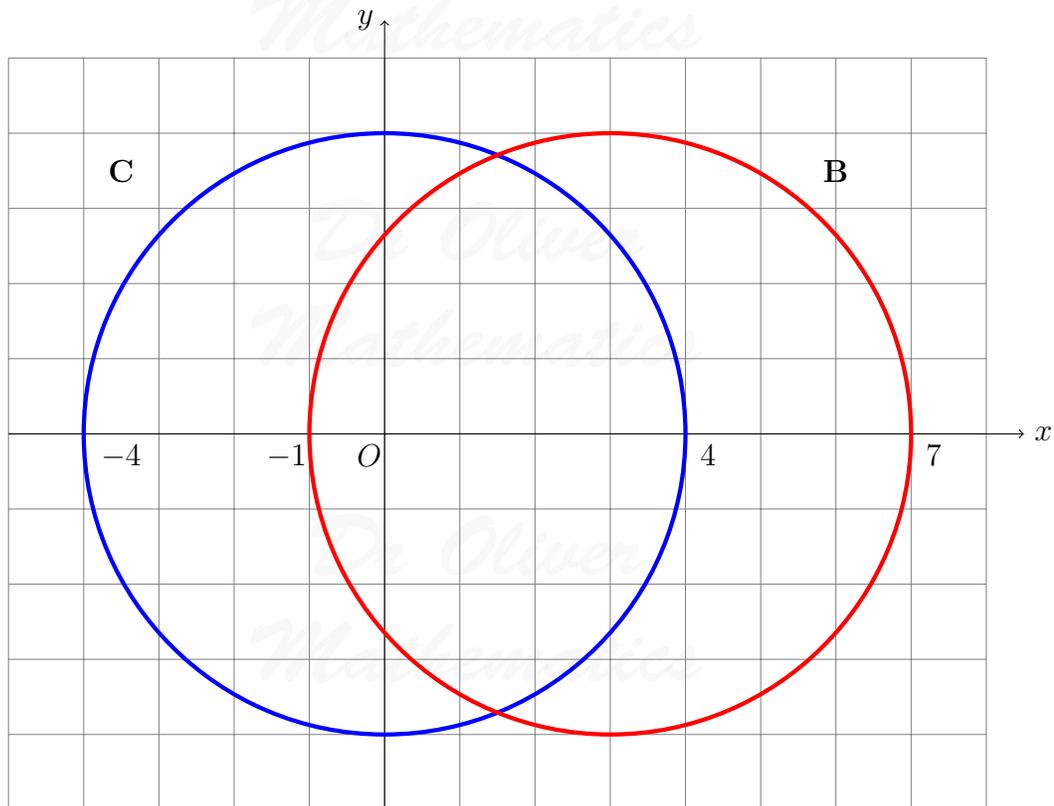
The circle **C** is translated by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ to give circle **B**.

(b) Draw a sketch of circle **B**.

(3)

Label with coordinates of the circle **B** and any points of intersection with the x -axis.

Solution



(-1, 0) and (7, 0)