

Dr Oliver Mathematics
AQA GCSE Mathematics
2017 June Paper 3: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. ?

(1)

$$\mathbf{a} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

Circle the vector $2\mathbf{a} + \mathbf{b}$.

$$\begin{pmatrix} -5 \\ -3 \end{pmatrix} \quad \begin{pmatrix} -11 \\ -3 \end{pmatrix} \quad \begin{pmatrix} -5 \\ -1 \end{pmatrix} \quad \begin{pmatrix} -11 \\ -1 \end{pmatrix}$$

Solution

Well,

$$\begin{aligned} 2\mathbf{a} + \mathbf{b} &= 2 \begin{pmatrix} -4 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} -8 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} -5 \\ -3 \end{pmatrix} \end{aligned}$$

and so

$$\underline{\underline{\begin{pmatrix} -5 \\ -3 \end{pmatrix}}} \quad \begin{pmatrix} -11 \\ -3 \end{pmatrix} \quad \begin{pmatrix} -5 \\ -1 \end{pmatrix} \quad \begin{pmatrix} -11 \\ -1 \end{pmatrix}$$

2. Which of these values of n makes

$$2.7 \times 10^n$$

(1)

a cube number?

Circle your answer.

0 1 2 3

Solution

n	Number	Cube
2.7×10^0	2.7	No
27×10^1	27	Yes: 3^3
2.7×10^2	270	No
2.7×10^3	2700	No

so

$$0 \quad \underline{1} \quad 2 \quad 3$$

3. Rearrange

$$2x = \frac{y}{w}$$

(1)

to make w the subject.

Circle your answer.

$$w = \frac{2y}{x} \quad w = \frac{2x}{y} \quad w = \frac{y}{2x} \quad w = \frac{x}{2y}$$

Solution

Well,

$$2x = \frac{y}{w} \Rightarrow 2xw = y$$

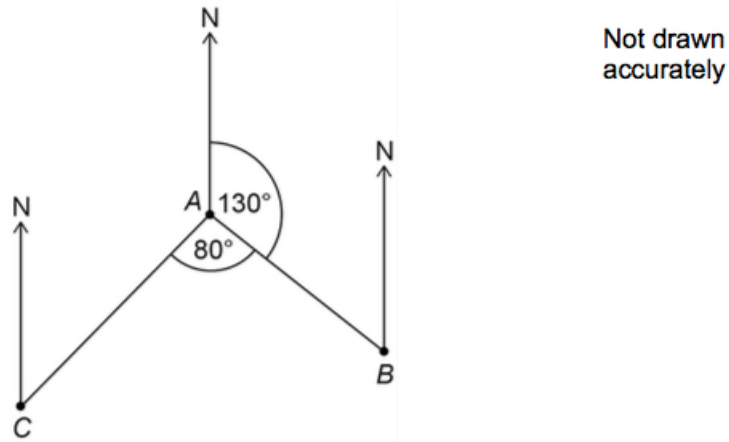
$$\Rightarrow w = \frac{y}{2x}$$

and so

$$w = \frac{2y}{x} \quad w = \frac{2x}{y} \quad w = \underline{\underline{\frac{y}{2x}}} \quad w = \frac{x}{2y}$$

4. Work out the bearing of C from A .

(1)



Circle your answer.

- 030° 130° 150° 210°

Solution

Well,

$$130 + 80 = 210$$

and so

- 030° 130° 150° 210°

5. A coin lands on Tails 200 times. (2)
The relative frequency of Tails is 0.4.

Work out the number of times the coin was thrown.

Solution

$$\frac{200}{0.4} = \underline{500 \text{ throws.}}$$

6. How are the whole number solutions to A and B different? (2)

A : Solve $3 \leq 3x < 18$

B : Solve $3 < 3x \leq 18$.

Solution

A:

$$3 \leq 3x < 18 \Rightarrow 1 \leq x < 6$$
$$\Rightarrow x = 1, 2, 3, 4, \text{ or } 5.$$

B:

$$3 < 3x \leq 18 \Rightarrow 1 < x \leq 6$$
$$\Rightarrow x = 2, 3, 4, 5, \text{ or } 6.$$

Hence, A includes the number 1 (B does not) and B includes the number 6 (A does not).

7. (a) The length of a pipe is 6 metres to the nearest metre. (2)
Complete the error interval for the length of the pipe.

Solution

$$\underline{\underline{5.5 \text{ m} \leq \text{length} < 6.5 \text{ m}}}$$

- (b) The length of a different pipe is 4 metres to the nearest metre. (2)

Olly says, "The total length of the two pipes is 11 metres to the nearest metre."

Give an example to show that he could be correct.

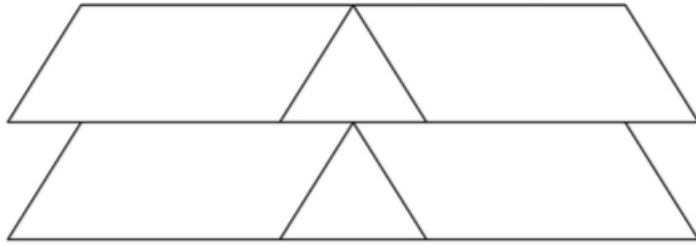
Solution

E.g., he could take one of length 6.4 m and one of length 4.4 m:

$$6.4 + 4.4 = 10.8 \text{ m.}$$

What is that to the nearest metre? It is 11 m!

8. This shape is made from two triangles and four congruent parallelograms.



Not drawn accurately

For each statement, tick the correct box.

(a) The triangles are equilateral.

(1)

Must be true

Could be true

Must be false

Solution

Could be true.

(b) The triangles are congruent.

(1)

Must be true

Could be true

Must be false

Solution

Must be true.

9. There are 720 boys and 700 girls in a school.

- The probability that a boy chosen at random studies French is $\frac{2}{3}$.
 - The probability that a girl chosen at random studies French is $\frac{3}{5}$.
- (a) Work out the number of students in the school who study French. (3)

Solution

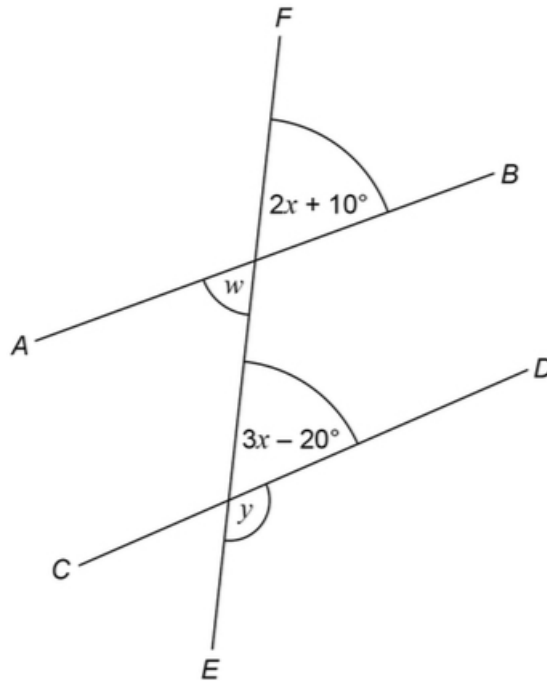
$$\begin{aligned}\text{Study French} &= \left(\frac{2}{3} \times 720\right) + \left(\frac{3}{5} \times 700\right) \\ &= 480 + 420 \\ &= \underline{\underline{900 \text{ pupils}}}.\end{aligned}$$

- (b) Work out the probability that a student chosen at random from the whole school does **not** study French. (2)

Solution

$$\begin{aligned}P(\text{not study French}) &= \frac{720 + 700 - 900}{720 + 700} \\ &= \frac{520}{1420} \\ &= \underline{\underline{\frac{26}{71} \text{ or } 0.366 \text{ (3 sf)}}}.\end{aligned}$$

10. AB , CD , and EF are straight lines.



Not drawn accurately

- (a) Ava assumes that AB and CD are parallel. (4)
 What answer should she get for the size of angle y ?

Solution

Corresponding angles are the same:

$$\begin{aligned}
 3x - 20 &= 2x + 10 \Rightarrow x = 30 \\
 &\Rightarrow y = 180 - [3(30) - 20] \\
 &\Rightarrow y = 180 - [90 - 20] \\
 &\Rightarrow y = 180 - 70 \\
 &\Rightarrow \underline{\underline{y = 110^\circ}}
 \end{aligned}$$

- (b) In fact, (3)
- AB and CD are **not** parallel.
 - w is 60° .

What effect does this have on the size of angle y ?
 Tick a box.

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y is bigger

y is the same

y is smaller

Show working to support your answer.

Solution

Now,

$$w = 60 \Rightarrow 2x + 10 = 60$$

$$\Rightarrow 2x = 50$$

$$\Rightarrow x = 25$$

$$\Rightarrow 3x = 75$$

$$\Rightarrow 3x - 20 = 55$$

$$\Rightarrow y = 125$$

so y is bigger.

11. Purple paint is made by mixing red paint and blue paint in the ratio 5 : 2. (3)

Yan has 30 litres of red paint and 9 litres of blue paint.

What is the maximum amount of purple paint he can make?

Solution

Well,

$$5 : 2 = \frac{5}{2} : 1 = 22\frac{1}{2} : 9.$$

So, he can make

$$22\frac{1}{2} + 9 = \underline{\underline{31\frac{1}{2}}} \text{ litres}$$

of purple paint.

12.

(2)

$$(ar^b)^4 = 16r^{20},$$

where a and b are positive integers.

Work out a and b .

Solution

Well,

$$(ar^b)^4 = a^4r^{4b}.$$

Now,

$$a^4 = 16 \Rightarrow \underline{a = 2}$$

and

$$4b = 20 \Rightarrow \underline{b = 5}.$$

13. In a class of 28 students

(4)

- the mean height of the 12 boys is 1.58 metres and
- the mean height of all 28 students is 1.52 metres.

Work out the mean height of the girls.

Solution

Let x cm be mean height of all of the girls. Now, there are

$$28 - 12 = 16$$

girls in the class. Then

$$\begin{aligned} \frac{(12 \times 1.58) + (16 \times x)}{28} &= 1.52 \Rightarrow 18.96 + 16x = 42.56 \\ &\Rightarrow 16x = 23.6 \\ &\Rightarrow \underline{x = 1.475}. \end{aligned}$$

14.

(1)

$$xy = c,$$

where c is a constant.

Circle the correct statement.

y is directly proportional to x

y is directly proportional to $\frac{1}{x}$

y is inversely proportional to $\frac{1}{x}$

x is directly proportional to y

Solution

y is directly proportional to x

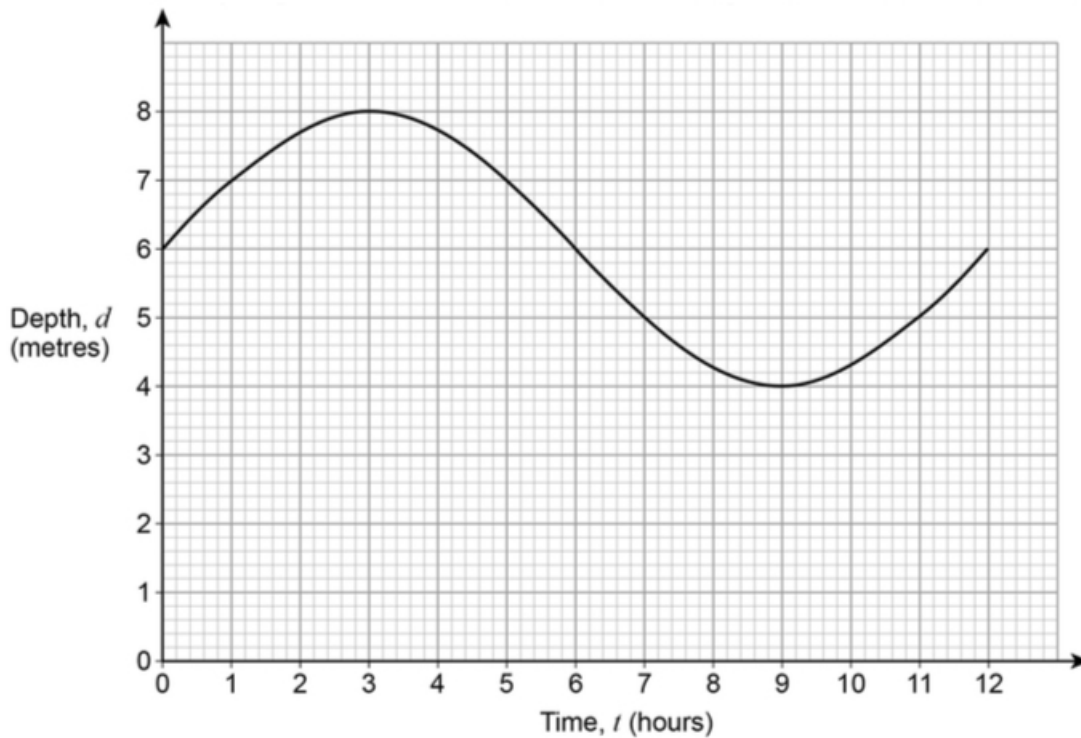
y is directly proportional to $\frac{1}{x}$

y is inversely proportional to $\frac{1}{x}$

x is directly proportional to y

15. The graph shows the depth of water in a harbour for 12 hours.

- d is the depth of water in a harbour in metres.
- t is the number of hours after 9 am.



- (a) For how many of the 12 hours is the depth more than 5 metres? (1)

Solution

$$(7 - 0) + (12 - 11) = 7 + 1 = \underline{8 \text{ hours.}}$$

- (b) By how much does the depth change between 12 noon and 4 pm? (1)

Solution

12 noon is $t = 3$ and 4 pm is $t = 7$:

$$8 - 5 = \underline{3 \text{ feet.}}$$

16. The value of a new car is £18 000. (3)

The value of the car decreases by

- 25% in the first year and
- 12% in each of the next 4 years.

Work out the value of the car after 5 years.

Solution

After 5 years, the value of the car is

$$\begin{aligned}18\,000 \times (1 - 0.25) \times (1 - 0.12)^4 &= 8\,095.887\,36 \text{ (FCD)} \\ &= \underline{\underline{\pounds 8\,100 \text{ (3 sf)}}}.\end{aligned}$$

17. Liam drives his car.

(5)

- He drives the first 9 miles in 9 minutes.
- He then drives at an average speed of 70 miles per hour for 1 hour 36 minutes.

He finds this information about his car.

Average speed	Miles travelled per gallon
65 miles per hour or less	50
More than 65 miles per hour	40

Use the information to show that his car uses less than 3 gallons of petrol for the drive.

Solution

In the first part of the journey, he goes at a speed of

$$\frac{9}{\frac{9}{60}} = 60 \text{ mph}$$

and he uses

$$\frac{9}{50} = 0.18 \text{ gallons.}$$

In the second part of the journey, he goes at a speed of

$$70 \times \frac{96}{60} = 112 \text{ mph}$$

and he uses

$$\frac{112}{40} = 2.8 \text{ gallons.}$$

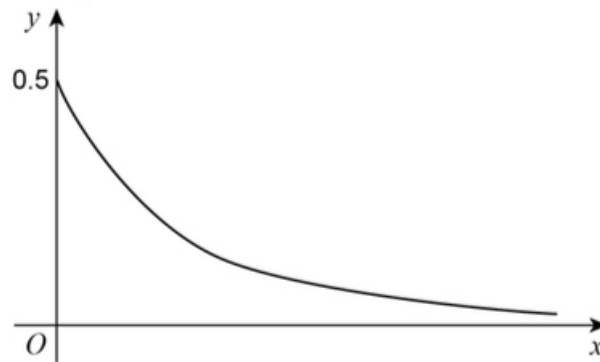
Hence, he uses

$$0.18 + 2.8 = \underline{\underline{2.98 \text{ gallons}}} < 3.$$

18. Nick sketches the graph of

$$y = 0.5^x \text{ for } x \geq 0.$$

(1)



Make **one** criticism of his sketch.

Solution

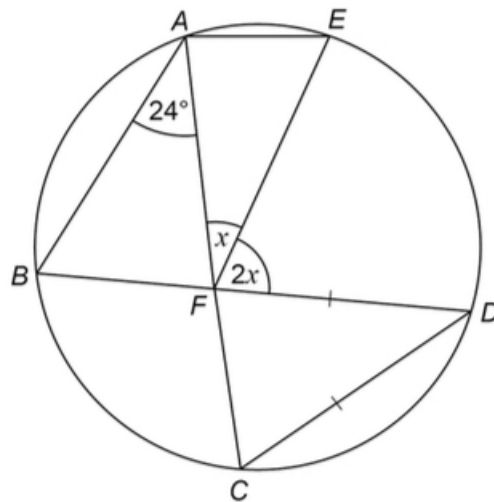
E.g., the y -intercept should be 1, not 0.5.

19. $A, B, C, D,$ and E are points on a circle.

BFD and AFC are straight lines.

$DC = DF.$

(4)



Not drawn accurately

Work out the size of angle x .

You **must** show your working which may be on the diagram.

Solution

$$\angle BDC = 24^\circ \text{ (angles in the same arc)}$$

$$\angle DFC = \frac{1}{2}(180 - 24) = 78^\circ \text{ (base angles)}$$

$$\angle AFD = 180 - 78 = 102^\circ \text{ (supplementary angles)}$$

$$\angle AFE = \frac{1}{3} \times 102 = \underline{\underline{34^\circ}} \text{ (splits } AFE \text{ in the ratio } 1 : 2)$$

20. This sign shows when a lift is safe to use.

(4)

Total mass of people must be 450 kg or less

Ben and some other people are in the lift.
Their total mass is 525 kg to the nearest 5 kg.

Ben gets out.
He has a mass of 78 kg to the nearest kg.

Is the lift now safe to use?
You **must** show your working.

Solution

Well,

$$522.5 \leq \text{total mass} < 527.5$$

and Ben has a mass of

$$77.5 \leq \text{Ben's mass} < 78.5.$$

So, doing a subtraction,

$$\text{min. total} - \text{max. Ben} < \text{total mass} - \text{Ben} < \text{max. total} - \text{min. Ben}$$

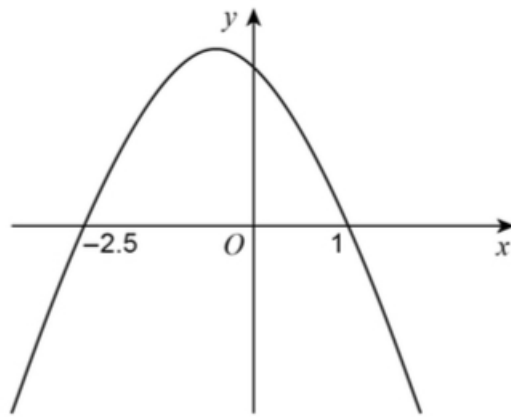
which leads to

$$444 < \text{total mass} - \text{Ben} < 450.$$

Yes: the lift is safe.

21. Here is a sketch of $y = f(x)$ where $f(x)$ is a quadratic function.

(1)



Not drawn accurately

The graph intersects the x -axis where $x = -2.5$ and $x = 1$.

Circle the solution of $f(x) > 0$.

- $x < -2.5$ or $x > 1$
 $-2.5 < x < 1$
 $x > -2.5$ or $x > 1$
 $x > -2.5$ or $x < 1$

Solution

- $x < -2.5$ or $x > 1$
 $-2.5 < x < 1$
 $x > -2.5$ or $x > 1$
 $x > -2.5$ or $x < 1$

22. Work out an expression for the n th term of the quadratic sequence

(3)

2 17 40 71 ...

Give your answer in the form

$$an^2 + bn + c,$$

where a , b , and c are constants.

Solution

2	17	40	71
15	23	31	
8	8		

$$a + b + c$$

$$3a + b$$

$$4a + 2b + c$$

$$2a$$

$$5a + b$$

$$9a + 4b + c$$

$$2a$$

$$7a + b$$

$$16a + 4b + c$$

Now,

$$2a = 8 \Rightarrow a = 4$$

$$3a + b = 15 \Rightarrow 12 + b = 15$$

$$\Rightarrow b = 3$$

$$a + b + c = 2 \Rightarrow 4 + 3 + c = 2$$

$$\Rightarrow c = -5;$$

hence, n th term is

$$\underline{4n^2 + 3n - 5.}$$

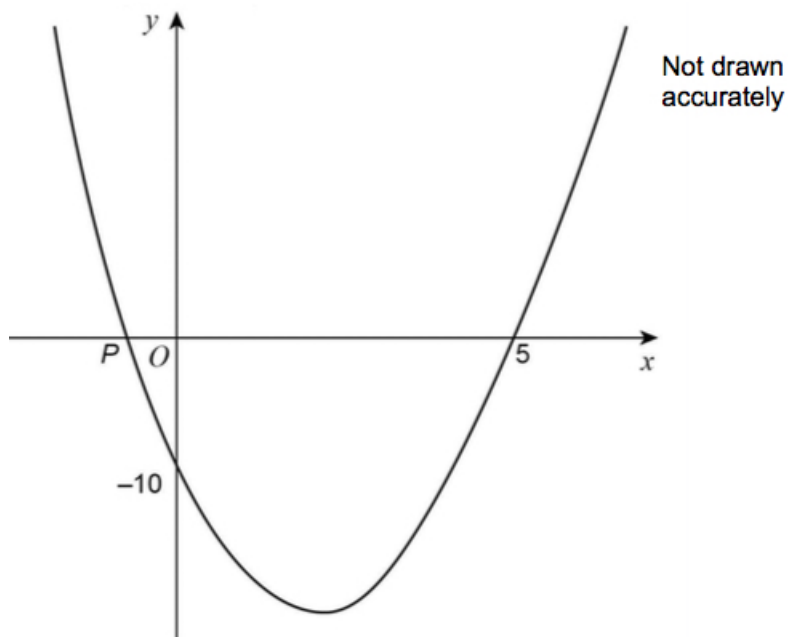
23. Here is a sketch of

$$y = x^2 + bx + c.$$

(4)

The curve intersects

- the x -axis at $(5, 0)$ and point P and
- the y -axis at $(0, -10)$.



Work out the x -coordinate of the turning point of the graph.

Solution

Well, clearly $y = (x - 5)(x + a)$ for some value of a (why?). Now,

$$\begin{aligned}x = 0, y = -10 &\Rightarrow -10 = 0 + 0 + c \\ &\Rightarrow c = -10.\end{aligned}$$

Next,

\times	x	-5
x	x^2	$-5x$
$+a$	$+ax$	$-5a$

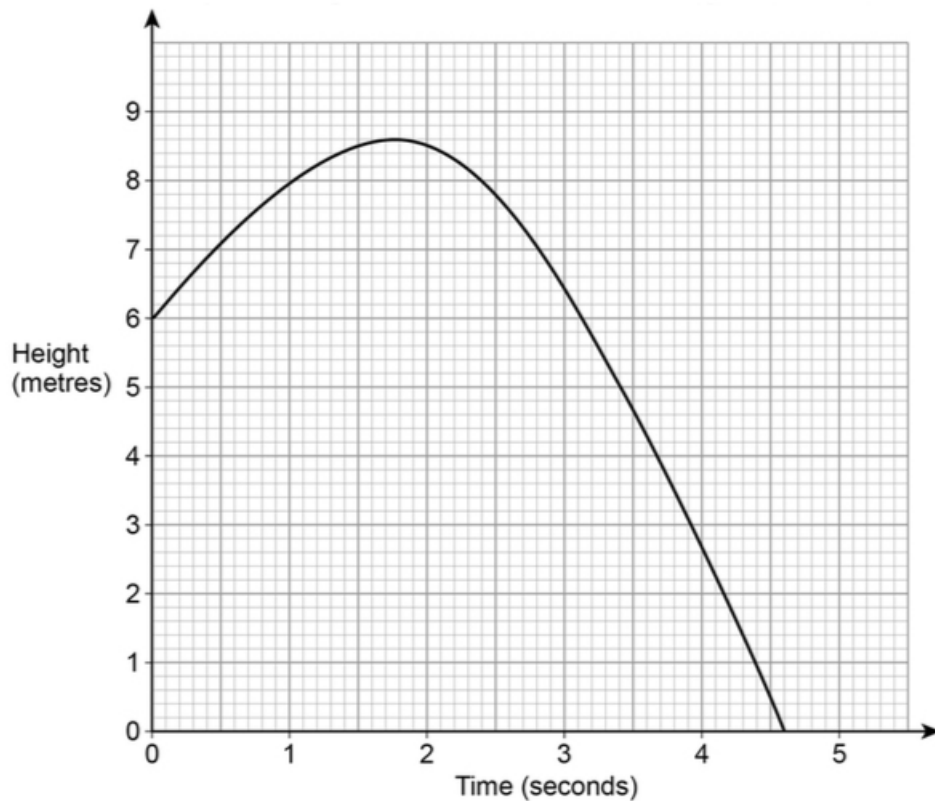
so it is clear that

$$-5a = -10 \Rightarrow a = 2.$$

Hence, $y = (x - 5)(x + 2)$ and the turning point of the graph is

$$\frac{5 + (-2)}{2} = \underline{\underline{1\frac{1}{2}}}.$$

24. A ball is thrown from a point 6 metres above the ground. (2)
The graph shows the height of the ball above the ground, in metres.



Estimate the speed of the ball, in m/s, after 1 second.
 You **must** show your working.

Solution

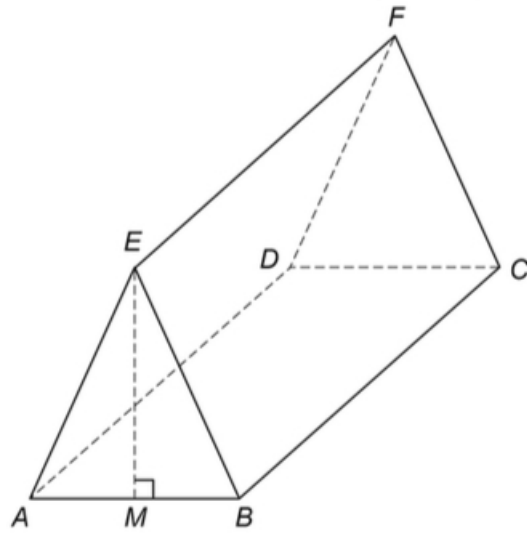
The tangent to the curve goes through (0, 6.2) and (1.5, 9) so the

$$\begin{aligned} \text{gradient} &= \frac{9 - 6.2}{1.5 - 0} \\ &= \underline{\underline{2\frac{2}{5} \text{ m/s.}}} \end{aligned}$$

25. Rectangle $ABCD$ is the horizontal base of a triangular prism $ABCDEF$.

- $AE = BE$.
- E is vertically above M , the midpoint of AB .
- $AB = 16$ cm.
- $AE = 17$ cm.

- $BC = 30$ cm.



- (a) Show that $EM = 15$ cm. (2)

Solution

Well,

$$AM = \frac{1}{2}AB = 8 \text{ cm}$$

and we need Pythagoras' theorem:

$$\begin{aligned} AE^2 &= AM^2 + EM^2 \Rightarrow 17^2 = 8^2 + EM^2 \\ &\Rightarrow 289 = 64 + EM^2 \\ &\Rightarrow EM^2 = 225 \\ &\Rightarrow \underline{\underline{EM = 15 \text{ cm},}} \end{aligned}$$

as required.

- (b) Work out the size of angle ECM . (4)

Solution

Now,

$$\begin{aligned} EC^2 &= EM^2 + EB^2 + BC^2 \Rightarrow EC^2 = 15^2 + 8^2 + 30^2 \\ &\Rightarrow EC^2 = 1189 \\ &\Rightarrow EC = \sqrt{1189} \end{aligned}$$

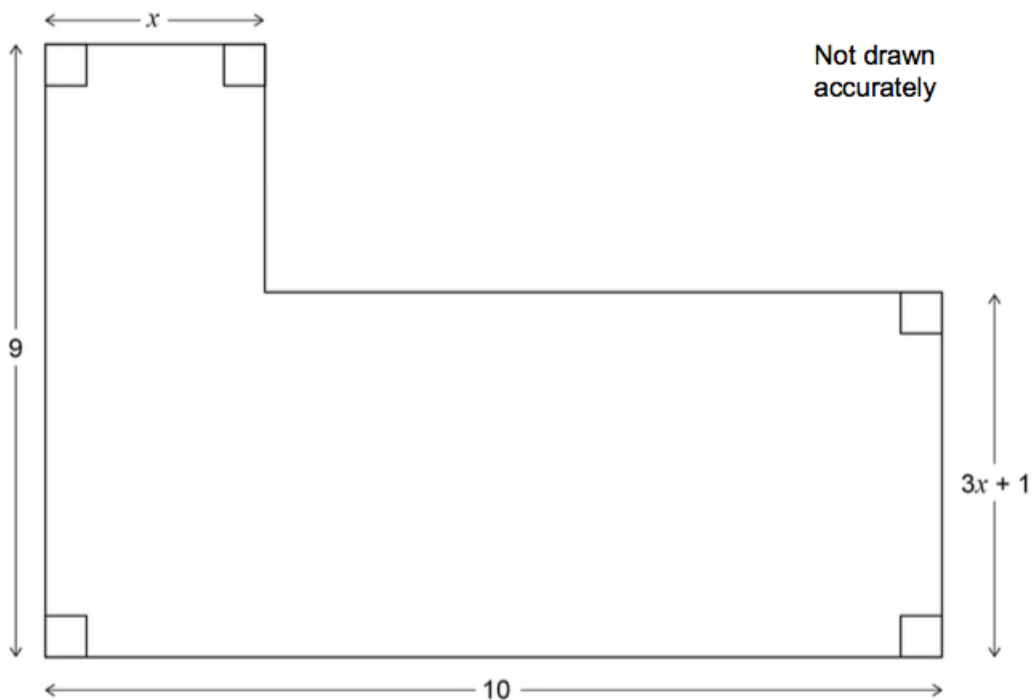
and

$$\sin = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin ECM = \frac{15}{\sqrt{1189}}$$
$$\Rightarrow \angle ECM = 25.78599974 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle ECM = 25.8 \text{ (3 sf)}}}$$

26. Here is an L-shape.

All dimensions are in centimetres.

(6)



The area of the L-shape is 65 cm².

Work out the value of x.

Solution

Well,

$$\begin{aligned} & \text{big rectangle} - \text{small rectangle} = 65 \\ \Rightarrow & (10 \times 9) - [(10 - x) \times (8 - 3x)] = 65 \\ \Rightarrow & 90 - (10 - x)(8 - 3x) = 65 \end{aligned}$$

\times	10	$-x$
8	80	$-8x$
$-3x$	$-30x$	$+3x^2$

$$\begin{aligned} \Rightarrow & 90 - (80 - 38x + 3x^2) = 65 \\ \Rightarrow & 10 + 38x - 3x^2 = 65 \\ \Rightarrow & 3x^2 - 38x + 55 = 0 \end{aligned}$$

$$\begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array} \left. \begin{array}{l} -38 \\ (+3) \times (+55) = +165 \end{array} \right\} -33, -5$$

$$\begin{aligned} \Rightarrow & 3x^2 - 33x - 5x + 55 = 0 \\ \Rightarrow & 3x(x - 11) - 5(x - 11) = 0 \\ \Rightarrow & (3x - 5)(x - 11) = 0 \\ \Rightarrow & 3x - 5 = 0 \text{ or } x - 11 = 0 \\ \Rightarrow & x = 1\frac{2}{3} \text{ or } x = 11. \end{aligned}$$

Clearly, $x \neq 11$ (why?) and so

$$\underline{\underline{x = 1\frac{2}{3}}}$$

27. Prove that

$$x^2 + x + 1$$

(3)

is always positive.

Solution

$$\begin{aligned}x^2 + x + 1 &= [x^2 + x] + 1 \\&= [(x^2 + x + \frac{1}{4}) - \frac{1}{4}] + 1 \\&= [(x + \frac{1}{2})^2 - \frac{1}{4}] + 1 \\&= (x + \frac{1}{2})^2 - \frac{1}{4} + 1 \\&= (x + \frac{1}{2})^2 + \frac{3}{4} \\&\underline{\underline{\geq 0}}\end{aligned}$$

on account that

$$(x + \frac{1}{2})^2 \geq 0.$$

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