Dr Oliver Mathematics
Mathematics
Trigonometry Part 1
Past Examination Questions

This booklet consists of 24 questions across a variety of examination topics.
The total number of marks available is 203.

1. (a) Show that the equation (2)

$$5\cos^2 x = 3(1 + \sin x)$$

can be written as

$$5\sin^2 x + 3\sin x - 2.$$

Solution

$$\begin{aligned}3(1 + \sin x) = 5\cos^2 x &\Rightarrow 3 + 3\sin x = 5(1 - \sin^2 x)\\ &\Rightarrow 3 + 3\sin x = 5 - 5\sin^2 x\\ &\Rightarrow \underline{\underline{5\sin^2 x + 3\sin x - 2 = 0}}.\end{aligned}$$

(b) Hence solve, for $0° \leqslant x < 360°$, the equation (5)

$$5\cos^2 x = 3(1 + \sin x),$$

giving your answer to 1 decimal place where appropriate.

Solution

$$\begin{aligned}5\cos^2 x = 3(1 + \sin x) &\Rightarrow 5\sin^2 x + 3\sin x - 2 = 0\\ &\Rightarrow (5\sin x - 2)(\sin x + 1) = 0\\ &\Rightarrow \sin x = \tfrac{2}{5} \text{ or } \sin x = -1.\end{aligned}$$

$\underline{\sin x = \frac{2}{5}}$:

$$\sin x = \tfrac{2}{5} \Rightarrow x = 23.571\ldots \text{ or } x = 156.421\ldots$$

$\underline{\sin x = -1}$:

$$\sin x = -1 \Rightarrow x = 270.$$

Hence, the solutions are

$$\underline{\underline{23.6 \text{ (1 dp)}, 156.4 \text{ (1 dp)}, \text{ or } 270}}.$$

2. Solve, for $0^\circ \leqslant x \leqslant 180^\circ$, the equation

   (a) $\sin(x+10)^\circ = \frac{\sqrt{3}}{2}$, (4)

   Solution

   $$\begin{aligned}\sin(x+10)^\circ = \tfrac{\sqrt{3}}{2} &\Rightarrow x+10 = 60 \text{ or } x+10 = 120 \\ &\Rightarrow \underline{\underline{x = 50 \text{ or } x = 110}}.\end{aligned}$$

   (b) $\cos 2x = -0.9$, giving your answer to 1 decimal place. (4)

   Solution

   $$\begin{aligned}\cos 2x = -0.9 &\Rightarrow 2x = 154.158\,067\,2 \text{ or } 2x = 205.841\,932\,2 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 77.1 \text{ or } x = 102.9 \text{ (1 dp)}}}.\end{aligned}$$

3. (a) Find all the values of θ, to 1 decimal place, in the interval $0^\circ \leqslant \theta < 360^\circ$ for which (4)

   $$5\sin(\theta+30)^\circ = 3.$$

   Solution

   $$\begin{aligned}5\sin(\theta+30)^\circ = 3 &\Rightarrow \sin(\theta+30)^\circ = \tfrac{3}{5} \\ &\Rightarrow \theta+30 = 36.869\,897\,65 \text{ or } \theta+30 = 143.130\,102\,4 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 6.9 \text{ or } x = 113.1 \text{ (1 dp)}}}.\end{aligned}$$

   (b) Find all the values of θ, to 1 decimal place, in the interval $0^\circ \leqslant x < 360^\circ$ for which (5)

   $$\tan^2\theta = 4.$$

   Solution

   $$\begin{aligned}\tan^2\theta = 4 &\Rightarrow \tan\theta = 2 \text{ or } \tan\theta = -2 \\ &\Rightarrow \theta = 63.434\ldots, 243.434\ldots \text{ or } \theta = 116.565\ldots, 296.565\ldots \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\theta = 63.4, 116.6, 243.4, \text{ or } 296.6 \text{ (1 dp)}}}.\end{aligned}$$

4. (a) Given that $\sin\theta = 5\cos\theta$, find the value of $\tan\theta$. (1)

Solution

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \underline{\underline{5}}.$$

(b) Hence, or otherwise, find the values of θ in the interval $0° \leqslant x < 360°$ for which (3)

$$\sin\theta = 5\cos\theta,$$

giving your answer to 1 decimal place where appropriate.

Solution

$$\begin{aligned}\sin\theta = 5\cos\theta &\Rightarrow \tan\theta = 5\\ &\Rightarrow \theta = 78.690\,067\,53 \text{ or } \theta = 258.690\,067\,53 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{\theta = 78.7 \text{ or } 258.7 \text{ (1 dp)}}}.\end{aligned}$$

5. Find all the solutions, in the interval $0 \leqslant x < 2\pi$, of the equation (6)

$$2\cos^2 x + 1 = 5\sin x,$$

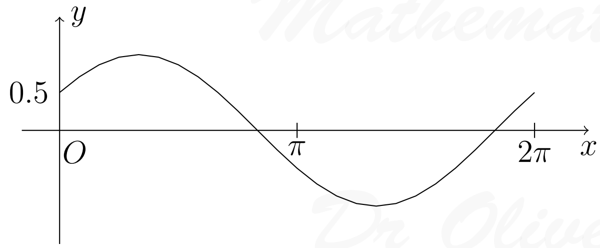
giving each solution in terms of π.

Solution

$$\begin{aligned}2\cos^2 x + 1 = 5\sin x &\Rightarrow 2(1 - \sin^2 x) + 1 = 5\sin x\\ &\Rightarrow (2 - 2\sin^2 x) + 1 = 5\sin x\\ &\Rightarrow 3 - 2\sin^2 x = 5\sin x\\ &\Rightarrow 2\sin^2 x + 5\sin x - 3 = 0\\ &\Rightarrow (2\sin x - 1)(\sin x + 3) = 0\\ &\Rightarrow \sin x = \tfrac{1}{2} \text{ (as } \sin x = -3 \text{ does not have a solution)}\\ &\Rightarrow \underline{\underline{\theta = \tfrac{\pi}{6} \text{ or } \tfrac{5\pi}{6}}}.\end{aligned}$$

6. (a) Sketch, for $0 \leqslant x \leqslant 2\pi$, the graph of $y = \sin(x + \frac{\pi}{6})$. (2)

Solution



(b) Write down the exact coordinates of the points where the graph meets the coordinates axes. (3)

Solution

$(0, \frac{1}{2})$, $(\frac{5\pi}{6}, 0)$, and $(\frac{11\pi}{6}, 0)$.

(c) Solve, for $0 \leqslant x \leqslant 2\pi$, the equation (5)

$$\sin(x + \tfrac{\pi}{6}) = 0.65,$$

giving your answer in radians to 2 decimal places.

Solution

$$\begin{aligned}
\sin(x + \tfrac{\pi}{6}) = 0.65 &\Rightarrow x + \tfrac{\pi}{6} = 0.707\,584\ \ldots \text{ or } 2.434\,008\ldots \\
&\Rightarrow x = 0.183\,985\ldots \text{ or } 1.910\,040\ldots \\
&\Rightarrow \underline{\underline{x = 0.18 \text{ or } 1.91 \text{ (2 dp)}}}.
\end{aligned}$$

7. (a) Show that the equation (2)

$$3\sin^2\theta - 2\cos^2\theta = 1$$

can be written as

$$5\sin^2\theta = 1.$$

Solution

$$\begin{aligned}
3\sin^2\theta - 2\cos^2\theta = 1 &\Rightarrow 3\sin^2\theta - 2(1 - \sin^2\theta) = 1 \\
&\Rightarrow 3\sin^2\theta - 2 + 2\sin^2\theta = 1 \\
&\Rightarrow \underline{\underline{5\sin^2\theta = 3}}.
\end{aligned}$$

(b) Hence solve, for $0^\circ \leqslant x < 360^\circ$, the equation (7)

$$3\sin^2\theta - 2\cos^2\theta = 1,$$

giving your answer to 1 decimal place.

Solution

$$\begin{aligned} 3\sin^2\theta - 2\cos^2\theta = 1 &\Rightarrow 5\sin^2\theta = 3 \\ &\Rightarrow \sin^2\theta = \tfrac{3}{5} \\ &\Rightarrow \sin\theta = \tfrac{\sqrt{15}}{5} \text{ or } \sin\theta = -\tfrac{\sqrt{15}}{5}. \end{aligned}$$

$\underline{\sin x = \tfrac{\sqrt{15}}{5}}$:

$$\sin x = \tfrac{\sqrt{15}}{5} \Rightarrow x = 50.768\ldots \text{ or } x = 129.231\ldots$$

$\underline{\sin x = -\tfrac{\sqrt{15}}{5}}$:

$$\sin x = -\tfrac{\sqrt{15}}{5} \Rightarrow x = 230.768\ldots \text{ or } x = 309.231\ldots$$

Hence, the solutions are

$$\underline{\underline{50.8,\ 129.2,\ 230.7,\ \text{or } 309.2 \text{ (1 dp)}}}.$$

8. Solve, for $0^\circ \leqslant x < 360^\circ$,

(a) $\sin(x - 20)^\circ = \dfrac{1}{\sqrt{2}}$, (4)

Solution

$$\begin{aligned} \sin(x-20)^\circ = \frac{1}{\sqrt{2}} &\Rightarrow x - 20 = 45 \text{ or } x - 20 = 135 \\ &\Rightarrow \underline{\underline{x = 65 \text{ or } x = 155}}. \end{aligned}$$

(b) $\cos 3x^\circ = -\dfrac{1}{2}$. (6)

Solution

$$\cos 3x^\circ = -\frac{1}{2} \Rightarrow 3x = 120,\ 240,\ 480,\ 600,\ 840,\ 960$$
$$\Rightarrow \underline{\underline{x = 40,\ 80,\ 160,\ 200,\ 280,\ 320}}.$$

9. (a) Show that the equation (2)

$$4\sin^2 x + 9\cos x - 6 = 0$$

can be written as

$$4\cos^2 x - 9\cos x + 2 = 0.$$

Solution

$$\begin{aligned} 4\sin^2 x + 9\cos x - 6 = 0 &\Rightarrow 4(1-\cos^2 x) + 9\cos x - 6 = 0 \\ &\Rightarrow 4 - 4\cos^2 x + 9\cos x - 6 = 0 \\ &\Rightarrow \underline{\underline{4\cos^2 x - 9\cos x + 2 = 0}}. \end{aligned}$$

(b) Hence solve, for $0^\circ \leqslant x < 720^\circ$, (6)

$$4\sin^2 x - 9\cos x - 6 = 0$$

giving your answer to 1 decimal place.

Solution

$$\begin{aligned} 4\sin^2 x - 9\cos x - 6 = 0 &\Rightarrow 4\cos^2 x - 9\cos x + 2 = 0 \\ &\Rightarrow (4\cos x - 1)(\cos x - 2) = 0 \\ &\Rightarrow \cos x = \tfrac{1}{4} \text{ (as } \cos x = 2 \text{ does not have a solution)} \\ &\Rightarrow x = 75.522\ldots,\ 284.477\ldots,\ 435.522\ldots,\ 644.477\ldots \\ &\Rightarrow \underline{\underline{x = 75.5,\ 284.5,\ 435.5,\ 644.5 \text{ (1 dp)}}}. \end{aligned}$$

10. (a) Solve, for $-180^\circ \leqslant \theta < 180^\circ$, (4)

$$(1 + \tan\theta)(5\sin\theta - 2) = 0.$$

Solution

$$(1 + \tan\theta)(5\sin\theta - 2) = 0 \Rightarrow \tan\theta = -1 \text{ or } \sin\theta = \tfrac{2}{5}.$$

$\tan x = -1$:

$$\tan x = -1 \Rightarrow x = -45 \text{ or } 135.$$

$\sin x = \frac{2}{5}$:

$$\begin{aligned}\sin x = \tfrac{2}{5} &\Rightarrow 23.578\,178\,48,\ 156.421\,821\,5 \text{ (FCD)}\\ &= 23.6,\ 156.4 \text{ (1 dp)}.\end{aligned}$$

(b) Solve, for $0° \leqslant x < 360°$, (6)

$$4\sin x = 3\tan x.$$

Solution

$$\begin{aligned}4\sin x = 3\tan x &\Rightarrow 4\sin x - \frac{3\sin x}{\cos x} = 0\\ &\Rightarrow \sin x\left(4 - \frac{3}{\cos x}\right) = 0\\ &\Rightarrow \sin x = 0 \text{ or } \cos x = \tfrac{1}{4}.\end{aligned}$$

$\sin x = 0$:

$$\sin x = 0 \Rightarrow x = 0,\ 180.$$

$\cos x = \frac{3}{4}$:

$$\begin{aligned}\cos = \tfrac{3}{4} &\Rightarrow 41.409\,622\,11,\ 318.590\,377\,9 \text{ (FCD)}\\ &= 41.9,\ 318.6 \text{ (1 dp)}.\end{aligned}$$

11. (a) Show that the equation (2)

$$5\sin x = 1 + 2\cos^2 x$$

can be written as

$$2\sin^2 x + 5\sin x - 3 = 0.$$

Solution

$$\begin{aligned} 5\sin x = 1 + 2\cos^2 x &\Rightarrow 5\sin x = 1 + 2(1 - \sin^2 x) \\ &\Rightarrow 5\sin x = 1 + 2 - 2\sin^2 x \\ &\Rightarrow \underline{\underline{2\sin^2 x + 5\sin x - 3 = 0}}. \end{aligned}$$

(b) Solve, for $0° \leqslant x < 360°$, (4)

$$2\sin^2 x + 5\sin x - 3 = 0.$$

Solution

$$\begin{aligned} 2\sin^2 x + 5\sin x - 3 = 0 &\Rightarrow (2\sin x - 1)(\sin x + 3) = 0 \\ &\Rightarrow \sin x = \tfrac{1}{2} \text{ (as } \sin x = -3 \text{ does not have a solution)} \\ &\Rightarrow \underline{\underline{x = 30,\ 150}}. \end{aligned}$$

12. (a) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$. (1)

Solution

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \underline{\underline{\frac{2}{5}}}.$$

(b) Solve, for $0° \leqslant x < 360°$, (5)

$$5\sin 2x = 2\cos 2x,$$

giving your answer to 1 decimal place.

Solution

$$\begin{aligned} & 5\sin 2x = 2\cos 2x \\ \Rightarrow\ & \tan 2x = \tfrac{2}{5} \\ \Rightarrow\ & 2x = 21.801\ldots\ 201.801\ldots,\ 381.801\ldots,\ 561.801\ldots\ \text{(FCD)} \\ \Rightarrow\ & \underline{\underline{x = 10.9,\ 100.9,\ 190.9,\ 280.9\ \text{(1 dp)}}}. \end{aligned}$$

13. (a) Show that the equation (2)

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

can be written in the form

$$4\sin^2 x + 7\sin x + 3 = 0.$$

Solution

$$\begin{aligned} 3\sin^2 x + 7\sin x = \cos^2 x - 4 &\Rightarrow 3\sin^2 x + 7\sin x = (1 - \sin^2 x) - 4 \\ &\Rightarrow \underline{\underline{4\sin^2 x + 7\sin x + 3 = 0}}. \end{aligned}$$

(b) Hence solve, for $0^\circ \leqslant x < 360^\circ$, (7)

$$3\sin^2 x + 7\sin x = \cos^2 x - 4,$$

giving your answer to 1 decimal place where appropriate.

Solution

$$\begin{aligned} 3\sin^2 x + 7\sin x = \cos^2 x - 4 &\Rightarrow 4\sin^2 x + 7\sin x + 3 = 0 \\ &\Rightarrow (4\sin x + 3)(\sin x + 1) = 0 \\ &\Rightarrow 4\sin x + 3 = 0 \text{ or } \sin x + 1 = 0 \\ &\Rightarrow \sin x = -\tfrac{3}{4} \text{ or } \sin x = -1. \end{aligned}$$

$\underline{\sin x = -\frac{3}{4}}$:

$$\begin{aligned} \sin x = -\tfrac{3}{4} &\Rightarrow x = 288.590\,377\,9,\ 311.409\,622\,1 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 288.6,\ 311.4 \text{ (1 dp)}}}. \end{aligned}$$

$\underline{\sin x = -1}$:

$$\sin x = -1 \Rightarrow \underline{\underline{x = 270 \text{ (exact)}}}.$$

14. (a) Solve for $0^\circ \leqslant x < 360^\circ$, giving your answer to 1 decimal place, (4)

$$3\sin(x + 45)^\circ = 2.$$

Solution

$$\begin{aligned}3\sin(x+45)^\circ = 2 &\Rightarrow \sin(x+45)^\circ = \tfrac{2}{3}\\ &\Rightarrow x + 45 = 41.810\,314\,9 \text{ (but this does not count!)},\\ &\qquad\qquad 138\,189\,685\,1,\ 401.810\,314,\ 9 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{x = 93.2,\ 356.8 \text{ (1 dp)}}}.\end{aligned}$$

(b) Find, for $0 \leqslant x < 2\pi$, all the solutions of (6)

$$2\sin^2 x + 2 = 7\cos x,$$

giving your answer in radians.

Solution

$$\begin{aligned}2\sin^2 x + 2 = 7\cos x &\Rightarrow 2(1-\cos^2)x + 2 = 7\cos x\\ &\Rightarrow 2 - 2\cos^2 x + 2 = 7\cos x\\ &\Rightarrow 2\cos^2 x + 7\cos x - 4 = 0\\ &\Rightarrow (2\cos x - 1)(\cos x + 4) = 0\\ &\Rightarrow \cos x = \tfrac{1}{2} \text{ (as } \cos x = -4 \text{ does not have a solution)}\\ &\Rightarrow \underline{\underline{x = \tfrac{\pi}{3},\ \tfrac{5\pi}{3}}}.\end{aligned}$$

15. (a) Find all the solutions of the equation $\sin(3x-15)^\circ = \frac{1}{2}$, for which $0^\circ \leqslant x \leqslant 180^\circ$. (6)

Solution

$$\begin{aligned}\sin(3x-15)^\circ = \tfrac{1}{2} &\Rightarrow 3x - 15 = 30,\ 150,\ 390\,510\\ &\Rightarrow 3x = 45,\ 165,\ 405,\ 525\\ &\Rightarrow \underline{\underline{x = 15,\ 55,\ 135,\ 175}}.\end{aligned}$$

Figure 1 shows part of the curve with equation

$$y = \sin(ax - b),$$

where $a > 0$, $0 < b < \pi$.

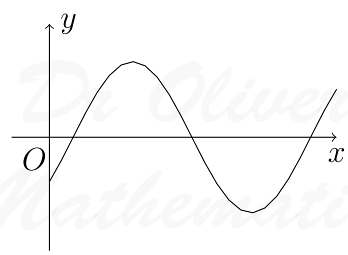


Figure 1: $y = \sin(ax - b)$

The curve cuts the x-axis at the points P, Q, and R.

(b) Given that the coordinates of $P(\frac{\pi}{10}, 0)$, $Q(\frac{3\pi}{5}, 0)$, and $R(\frac{11\pi}{10}, 0)$ respectively, find the values of a and b. (4)

Solution

We want

$$\tfrac{a\pi}{10} - b = 0,\ \tfrac{3a\pi}{5} - b = \pi,\ \text{and}\ \tfrac{11a\pi}{10} - b = 2\pi.$$

We'll take the first two:

$$\begin{aligned} b = \tfrac{a\pi}{10} &\Rightarrow \tfrac{3a\pi}{5} - \tfrac{a\pi}{10} = \pi \\ &\Rightarrow \tfrac{3a}{5} - \tfrac{a}{10} = 1 \\ &\Rightarrow \tfrac{a}{2} = 1 \\ &\Rightarrow \underline{\underline{a = 2}} \end{aligned}$$

and

$$b = \tfrac{2\pi}{10} = \underline{\underline{\tfrac{\pi}{5}}}.$$

16. (a) Show that the equation (2)

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5\cos 2x)\sin 2x = 0.$$

Solution

$$\tan 2x = 5\sin 2x \Rightarrow \frac{\sin 2x}{\cos 2x} - 5\sin 2x = 0$$
$$\Rightarrow \sin 2x - 5\sin 2x\cos 2x = 0$$
$$\Rightarrow \underline{\underline{(1 - 5\cos 2x)\sin 2x = 0}}.$$

(b) Hence solve, for $0^\circ \leqslant x \leqslant 180^\circ$, (5)

$$\tan 2x = 5\sin 2x,$$

giving your answer to 1 decimal place where appropriate.

Solution

$$\tan 2x = 5\sin 2x \Rightarrow (1 - 5\cos 2x)\sin 2x = 0$$
$$\Rightarrow \sin 2x = 0 \text{ or } \cos 2x = \tfrac{1}{5}.$$

$\underline{\sin 2x = 0}$:

$$\sin 2x = 0 \Rightarrow 2x = 0, 180, 360 \Rightarrow \underline{\underline{x = 0, 90, 180 \text{ (exact)}}}.$$

$\underline{\cos 2x = \tfrac{1}{5}}$:

$$\cos 2x = \tfrac{1}{5} \Rightarrow 2x = 78.463\,040\,097, 281.536\,959 \text{ (FCD)}$$
$$\Rightarrow x = 39.231\,520\,48, 140.768\,479\,5 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 39.2, 140.8 \text{ (1 dp)}}}.$$

17. Solve, for $0^\circ \leqslant x < 180^\circ$, (7)

$$\cos(3x - 10)^\circ = -0.4,$$

giving your answer to 1 decimal place.

Solution

$$\begin{aligned}\cos(3x-10)^\circ = -0.4 &\Rightarrow 3x - 10 = 113.578\ldots, 264.421\ldots, 473.578\ldots \\ &\Rightarrow 3x = 123.578\ldots, 274.421\ldots, 483.578\ldots \\ &\Rightarrow x = 41.192\ldots, 85.473\ldots, 161.192\ldots \\ &\Rightarrow \underline{\underline{x = 41.2, 85.5, 161.2 \text{ (1 dp)}}}.\end{aligned}$$

18. (a) Solve, for $-180^\circ \leqslant x < 180^\circ$, (3)

$$\tan(x-40)^\circ = 1.5,$$

giving your answer to 1 decimal place.

Solution

$$\begin{aligned}\tan(x-40)^\circ = 1.5 &\Rightarrow x - 40 = -123.690\,067\,5,\ 56.309\,932\,47 \text{ (FCD)} \\ &\Rightarrow x = -83.690\,067\,5,\ 96.309\,932\,47 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = -83.7,\ 96.3 \text{ (1 dp)}}}.\end{aligned}$$

(b) Show that the equation (3)

$$\sin\theta\tan\theta = 3\cos\theta + 2$$

can be written in the form

$$4\cos^2\theta + 2\cos\theta - 1.$$

Solution

$$\begin{aligned}\sin\theta\tan\theta = 3\cos\theta + 2 &\Rightarrow \frac{\sin^2\theta}{\cos\theta} = 3\cos\theta + 2 \\ &\Rightarrow \sin^2\theta = 3\cos^2\theta + 2\cos\theta \\ &\Rightarrow 1 - \cos^2\theta = 3\cos^2\theta + 2\cos\theta \\ &\Rightarrow \underline{\underline{4\cos^2\theta + 2\cos\theta - 1 = 0}}.\end{aligned}$$

(c) Hence solve, for $0^\circ \leqslant \theta < 360^\circ$, (5)

$$\sin\theta\tan\theta = 3\cos\theta + 2,$$

showing stage of your working.

Solution

$$\sin\theta\tan\theta = 3\cos\theta + 2 \Rightarrow 4\cos^2\theta + 2\cos\theta - 1 = 0$$
$$\Rightarrow \cos\theta = \frac{-1 \pm \sqrt{5}}{4}.$$

$\underline{\cos\theta = \frac{-1+\sqrt{5}}{4}}$:

$$\cos\theta = \frac{-1+\sqrt{5}}{4} \Rightarrow \underline{\underline{\theta = 72,\ 288}}.$$

$\underline{\cos\theta = \frac{-1-\sqrt{5}}{4}}$:

$$\cos\theta = \frac{-1-\sqrt{5}}{4} \Rightarrow \underline{\underline{\theta = 144,\ 216}}.$$

19. (a) Solve, for $0^\circ \leqslant \theta < 180^\circ$, (5)

$$\sin(2\theta - 30)^\circ + 1 = 0.4,$$

giving your answer to 1 decimal place.

Solution

$$\begin{aligned}\sin(2\theta - 30)^\circ + 1 = 0.4 &\Rightarrow \sin(2\theta - 30)^\circ = -0.6\\ &\Rightarrow 2\theta - 30 = 216.869\,897\,6,\ 323.130\,102\,4 \text{ (FCD)}\\ &\Rightarrow 2\theta = 246.869\,897\,6,\ 353.130\,102\,4 \text{ (FCD)}\\ &\Rightarrow \theta = 123.434\,948\,8,\ 176.565\,051\,2 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{\theta = 123.4,\ 176.6 \text{ (1 dp)}}}.\end{aligned}$$

(b) Find all the values of x, in the interval for $0^\circ \leqslant x < 360^\circ$, for which (7)

$$9\cos^2 x - 11\cos x + 3\sin^2 x = 0,$$

giving your answer to 1 decimal place.

Solution

$$
\begin{aligned}
& 9\cos^2 x - 11\cos x + 3\sin^2 x = 0 \\
\Rightarrow\quad & 9\cos^2 x - 11\cos x + 3(1-\cos^2 x) = 0 \\
\Rightarrow\quad & 6\cos^2 x - 11\cos x + 3 = 0 \\
\Rightarrow\quad & (3\cos x - 1)(2\cos x - 3) = 0 \\
\Rightarrow\quad & \cos x = \tfrac{1}{3} \text{ (as } \cos x = \tfrac{3}{2} \text{ does not have a solution)} \\
\Rightarrow\quad & x = 70.528\,779\,37,\ 289.471\,220\,6 \text{ (FCD)} \\
\Rightarrow\quad & \underline{\underline{x = 70.5,\ 289.5 \text{ (1 dp)}}}.
\end{aligned}
$$

20. (a) Solve, for $0^\circ \leqslant \theta < 360^\circ$, (4)

$$9\sin(\theta + 60)^\circ = 4,$$

giving your answer to 1 decimal place.

Solution

$$
\begin{aligned}
9\sin(\theta+60)^\circ = 4 &\Rightarrow \sin(\theta+60)^\circ = \tfrac{4}{9} \\
&\Rightarrow \theta + 60 = 153.612\,2,\ 386.387\,8 \text{ (FCD)} \\
&\Rightarrow \theta = 93.612\,2,\ 326.387\,8 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{\theta = 93.6,\ 326.4 \text{ (1 dp)}}}.
\end{aligned}
$$

(b) Solve, for $-\pi \leqslant x < \pi$, the equation (6)

$$2\tan x - 3\sin x = 0,$$

giving your answer to 2 decimal place where appropriate.

Solution

$$
\begin{aligned}
2\tan x - 3\sin x = 0 &\Rightarrow \frac{2\sin x}{\cos x} - 3\sin x = 0 \\
&\Rightarrow \sin x\left(\frac{2}{\cos x} - 3\right) \\
&\Rightarrow \sin x = 0 \text{ or } \cos x = \tfrac{2}{3}.
\end{aligned}
$$

$\underline{\sin x = 0}$:

$$\sin x = 0 \Rightarrow \underline{\underline{x = -\pi,\ 0}}.$$

$\underline{\cos x = \frac{2}{3}}$:

$$\cos x = \tfrac{2}{3} \Rightarrow x = -0.841\,068\,670\,6,\ 0.841\,068\,670\,6 \text{ (FCD)}$$
$$\Rightarrow x = -0.84,\ 0.84 \text{ (2 dp)}.$$

21. (a) Solve, for $0^\circ \leqslant \theta \leqslant 180^\circ$, the equation (3)

$$\frac{\sin 2\theta}{4\sin 2\theta - 1} = 1,$$

giving your answer to 1 decimal place.

Solution

$$\begin{aligned}\frac{\sin 2\theta}{4\sin 2\theta - 1} = 1 &\Rightarrow \sin 2\theta = 4\sin 2\theta - 1\\ &\Rightarrow 3\sin 2\theta = 1\\ &\Rightarrow \sin 2\theta = \tfrac{1}{3}\\ &\Rightarrow 2\theta = 19.471\,220\,63,\ 160.528\,779\,4 \text{ (FCD)}\\ &\Rightarrow \theta = 9.735\,610\,317,\ 80.264\,389\,68 \text{ (FCD)}\\ &\Rightarrow \theta = 9.7,\ 80.3 \text{ (1 dp)}.\end{aligned}$$

(b) Solve, for $0 \leqslant x < 2\pi$, the equation (5)

$$5\sin^2 x - 2\cos x - 5 = 0,$$

giving your answer to 2 decimal places.

Solution

$$\begin{aligned}5\sin^2 x - 2\cos x - 5 = 0 &\Rightarrow 5(1 - \cos^2 x) - 2\cos x - 5 = 0\\ &\Rightarrow 5 - 5\cos^2 x - 2\cos x - 5 = 0\\ &\Rightarrow 5\cos^2 x + 2\cos x = 0\\ &\Rightarrow \cos x(5\cos x + 2) = 0\\ &\Rightarrow \cos x = 0 \text{ or } \cos x = -\tfrac{2}{5}.\end{aligned}$$

$\underline{\cos x = 0}$:

$$\cos x = 0 \Rightarrow x = \tfrac{\pi}{2},\ \tfrac{3\pi}{2} \text{ or } x = 1.57,\ 4.71 \text{ (2 dp)}.$$

$\underline{\cos x = -\frac{2}{5}}$:

$$\begin{aligned} \cos x = -\tfrac{2}{5} &\Rightarrow x = 1.982\,313\,173,\ 4.300\,872\,134 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 1.98,\ 4.30 \text{ (2 dp)}}}. \end{aligned}$$

22. (a) Solve, for $0 \leqslant \theta < \pi$, the equation (3)

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0,$$

giving your answer in terms of π.

Solution

$$\begin{aligned} \sin 3\theta - \sqrt{3}\cos 3\theta = 0 &\Rightarrow \sin 3\theta = \sqrt{3}\cos 3\theta \\ &\Rightarrow \tan 3\theta = \sqrt{3} \\ &\Rightarrow 3\theta = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \\ &\Rightarrow \underline{\underline{\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}}}. \end{aligned}$$

Given that

$$4\sin^2 x + \cos x = 4 - k,\ 0 \leqslant k \leqslant 3,$$

(b) find $\cos x$ in terms of k. (3)

Solution

$$\begin{aligned} 4\sin^2 x + \cos x = 4 - k &\Rightarrow 4(1 - \cos^2 x) + \cos x = 4 - k \\ &\Rightarrow 4 - 4\cos^2 x + \cos x = 4 - k \\ &\Rightarrow 4\cos^2 x - \cos x - k = 0 \\ &\Rightarrow \underline{\underline{\cos x = \frac{1 \pm \sqrt{1 + 16k}}{8}}}. \end{aligned}$$

(c) When $k = 3$, find the values of x in the range $0^\circ \leqslant x < 360^\circ$. (3)

Solution

$$\cos x = \frac{1 \pm 7}{8} = -\frac{3}{4} \text{ or } 1.$$

$\underline{\cos x = 1}$:

$$\cos x = 1 \Rightarrow \underline{\underline{x = 0}}.$$

$\underline{\cos x = -\frac{3}{4}}$:

$$\begin{aligned} \cos x = -\frac{3}{4} &\Rightarrow x = 138.590\,377\,9,\ 221.409\,622\,1 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 138.6,\ 221.4 \text{ (1 dp)}}}. \end{aligned}$$

23. (a) Solve, for $-\pi < \theta \leqslant \pi$, (3)

$$1 - 2\cos(\theta - \tfrac{\pi}{5}) = 0,$$

giving your answers in terms of π.

Solution

$$\begin{aligned} 1 - 2\cos(\theta - \tfrac{\pi}{5}) = 0 &\Rightarrow \cos(\theta - \tfrac{\pi}{5}) = \tfrac{1}{2} \\ &\Rightarrow \theta - \tfrac{\pi}{5} = -\tfrac{\pi}{3},\ \tfrac{\pi}{3} \\ &\Rightarrow \underline{\underline{\theta = -\tfrac{2\pi}{15},\ \tfrac{8\pi}{15}}}. \end{aligned}$$

(b) Solve, for $0^\circ \leqslant x < 360^\circ$, (6)

$$4\cos^2 x + 7\sin x - 2 = 0,$$

giving your answer to 1 decimal place.

Solution

$$\begin{aligned} 4\cos^2 x + 7\sin x - 2 = 0 &\Rightarrow 4(1 - \sin^2 x) + 7\sin x - 2 = 0 \\ &\Rightarrow 4 - 4\sin^2 x + 7\sin x - 2 = 0 \\ &\Rightarrow 4\sin^2 x - 7\sin x - 2 = 0 \\ &\Rightarrow (4\sin x + 1)(\sin x - 2) = 0 \\ &\Rightarrow \sin x = -\tfrac{1}{4} \text{ (as } \sin x = 2 \text{ does not have a solution)} \\ &\Rightarrow x = 194.477\,512\,19,\ 345.522\,487\,8 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 194.5,\ 345.5 \text{ (1 dp)}}}. \end{aligned}$$

24. (a) Show that the equation (3)

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2.$$

Solution

$$\begin{aligned}\cos^2 x = 8\sin^2 x - 6\sin x &\Rightarrow 1 - \sin^2 x = 8\sin^2 x - 6\sin x\\ &\Rightarrow 9\sin^2 x - 6\sin x = 1\\ &\Rightarrow 9\sin^2 x - 6\sin x + 1 = 2\\ &\Rightarrow \underline{\underline{(3\sin x - 1)^2 = 2}}.\end{aligned}$$

(b) Hence solve, for $0^\circ \leqslant x < 360^\circ$, (5)

$$\cos^2 x = 8\sin^2 x - 6\sin x,$$

giving your answers to 2 decimal place

Solution

$$\begin{aligned}\cos^2 x = 8\sin^2 x - 6\sin x &\Rightarrow (3\sin x - 1)^2 = 2\\ &\Rightarrow 3\sin x - 1 = \pm\sqrt{2}\\ &\Rightarrow 3\sin x = 1 \pm \sqrt{2}\\ &\Rightarrow \sin x = \tfrac{1\pm\sqrt{2}}{3}.\end{aligned}$$

$\underline{\sin x = \tfrac{1+\sqrt{2}}{3}}$:

$$\begin{aligned}\sin x = \tfrac{1+\sqrt{2}}{3} &\Rightarrow x = 53.584\,946\,05,\ 126.415\,053\,9 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{x = 53.58,\ 126.42 \text{ (2 dp)}}}.\end{aligned}$$

$\underline{\sin x = \tfrac{1-\sqrt{2}}{3}}$:

$$\begin{aligned}\sin x = \tfrac{1+\sqrt{2}}{3} &\Rightarrow x = 187.936\,249\,5,\ 352.063\,750\,5 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{x = 187.94,\ 352.06 \text{ (2 dp)}}}.\end{aligned}$$