

**Dr Oliver Mathematics**  
**Mathematics: Higher**  
**2017 Paper 1: Non-Calculator**  
**1 hour 10 minutes**

The total number of marks available is 60.

You must write down all the stages in your working.

1. Functions  $f$  and  $g$  are defined on suitable domains by

$$f(x) = 5x \text{ and } g(x) = 2 \cos x.$$

(a) Evaluate  $f(g(0))$ . (1)

(b) Find an expression for  $g(f(x))$ . (2)

2. The point  $P(-2, 1)$  lies on the circle (4)

$$x^2 + y^2 - 8x - 6y - 15 = 0.$$

Find the equation of the tangent to the circle at  $P$ .

3. Given (2)

$$y = (4x - 1)^{12},$$

find  $\frac{dy}{dx}$ .

4. Find the value of  $k$  for which the equation (3)

$$x^2 + 4x + (k - 5) = 0$$

has equal roots.

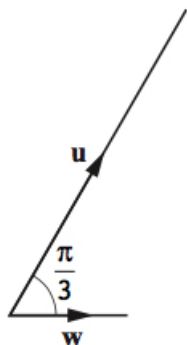
5. Vectors  $\mathbf{u}$  and  $\mathbf{v}$  are

$$\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix} \text{ and } \begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$$

respectively.

(a) Evaluate  $\mathbf{u} \cdot \mathbf{v}$ . (1)

Vector  $\mathbf{w}$  makes an angle of  $\frac{1}{3}\pi$  with  $\mathbf{u}$  and  $|\mathbf{w}| = \sqrt{3}$ .



(b) Calculate  $\mathbf{u} \cdot \mathbf{w}$ . (3)

6. A function,  $h$ , is defined by (3)

$$h(x) = x^3 + 7, \text{ where } x \in \mathbb{R}.$$

Determine an expression for  $h^{-1}(x)$ .

7.  $A(-3, 5)$ ,  $B(7, 9)$ , and  $C(2, 11)$  are the vertices of a triangle. (3)  
Find the equation of the median through  $C$ .

8. Calculate the rate of change of (3)

$$d(t) = \frac{1}{2t}, t \neq 0,$$

when  $t = 5$ .

9. A sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + 6,$$

where  $m$  is a constant.

(a) Given  $u_1 = 28$  and  $u_2 = 13$ , find the value of  $m$ . (2)

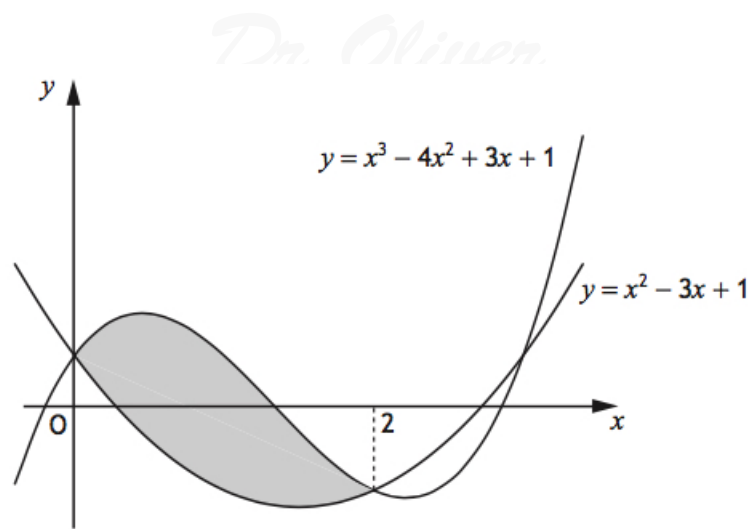
(b) (i) Explain why this sequence approaches a limit as  $n \rightarrow \infty$ . (1)

(ii) Calculate this limit. (2)

10. Two curves with equations

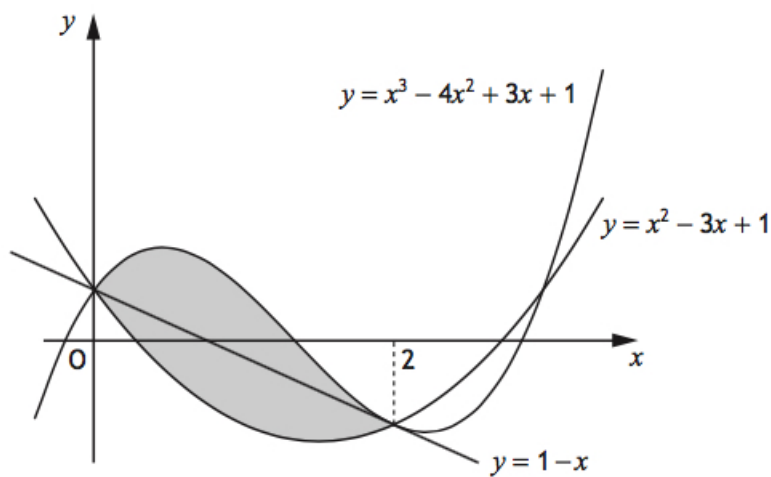
$$y = x^3 - 4x^2 + 3x + 1 \text{ and } y = x^2 - 3x + 1$$

intersect as shown in the diagram.



- (a) Calculate the shaded area. (5)

The line passing through the points of intersection of the curves has equation  $y = 1 - x$ .



- (b) Determine the fraction of the shaded area which lies below the line  $y = 1 - x$ . (4)

11.  $A$  and  $B$  are the points  $(-7, 2)$  and  $(5, a)$ . (3)

$AB$  is parallel to the line with equation

$$3y - 2x = 4.$$

Determine the value of  $a$ .

12. Given that (3)

$$\log_a 36 - \log_a 4 = \frac{1}{2},$$

find the value of  $a$ .

13. Find

$$\int \frac{1}{(5 - 4x)^{\frac{1}{2}}} dx, \quad x < \frac{5}{4}.$$

(4)

14. (a) Express

$$\sqrt{3} \sin x^\circ - \cos x^\circ$$

(4)

in the form

$$k \sin(x - a)^\circ,$$

where  $k > 0$  and  $0 < a < 360$ .

(b) Hence, or otherwise, sketch the graph with equation

(3)

$$y = \sqrt{3} \sin x^\circ - \cos x^\circ, \quad 0 \leq x \leq 360.$$

15. A quadratic function,  $f$ , is defined on  $\mathbb{R}$ , the set of real numbers.

Diagram 1 shows part of the graph with equation  $y = f(x)$ .  
The turning point is  $(2, 3)$ .

Diagram 2 shows part of the graph with equation  $y = h(x)$ .  
The turning point is  $(7, 6)$ .

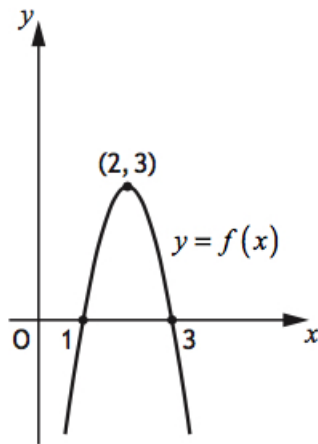


Diagram 1

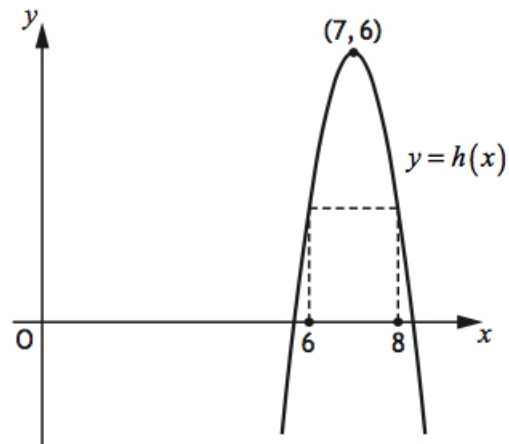


Diagram 2

Given that

$$h(x) = f(x + a) + b,$$

(a) write down the values of  $a$  and  $b$ . (2)

It is known that

$$\int_1^3 f(x) dx = 4.$$

(b) Determine the value of (1)

$$\int_6^8 h(x) dx.$$

(c) Given  $f'(1) = 6$ , state the value of  $h'(8)$ . (1)

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