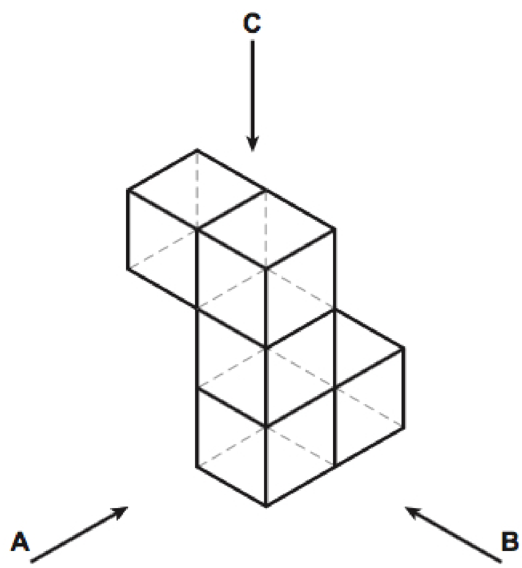
Dr Oliver Mathematics
AQA GCSE Mathematics
2013 June Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

1. This shape is made from **five** cubes. (3)



Draw what the shape looks like when seen from A, B, and C.

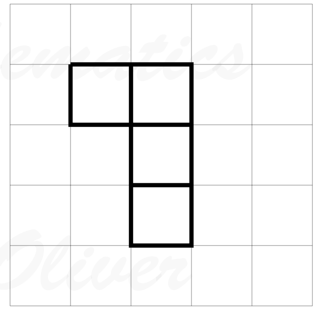
From A

From B

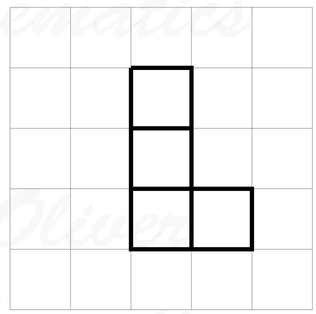
From C

Solution

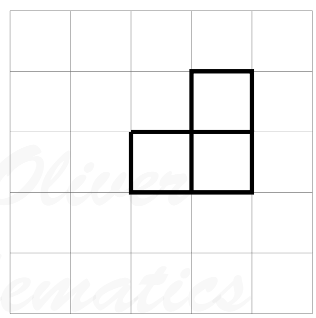
From A



From B



From C



2. Work out an approximate value of (2)

$$\frac{41 \times 198}{77}.$$

Solution

Approximate to 1 significant figure:

$$\begin{aligned}\frac{41 \times 198}{77} &\approx \frac{40 \times 200}{80} \\ &= \frac{8\,000}{80} \\ &= \underline{\underline{100}}.\end{aligned}$$

3. Which of the following expressions will give the median value when $n = 10$? (3)

$$\frac{1}{n} \qquad n-1 \qquad n+1 \qquad n^2 \qquad \sqrt{n}$$

You **must** show your working.

Solution

When $n = 10$, we get

$$\tfrac{1}{10} \qquad 10-1 \qquad 10+1 \qquad 10^2 \qquad \sqrt{10}$$

Write them in ascending order:

$$\tfrac{1}{10} \qquad \sqrt{10} \qquad 10-1 \qquad 10+1 \qquad 10^2$$

and pick the third one: $\underline{\underline{10-1}}$ or $\underline{\underline{9}}$.

4. p is an even number.
q is an odd number.

Tick the correct box for each part.

(a) Is pq an odd number, an even number, or could it be either? (1)

☐ odd ☐ even ☐ could be either

Solution

Even.

(b) Is $3(p + q)$ an odd number, an even number, or could it be either? (1)

odd | even | could be either

Solution

Odd.

(c) Is $p \div q$ an odd number, an even number, or could it be either? (1)

integer | not an integer | could be either

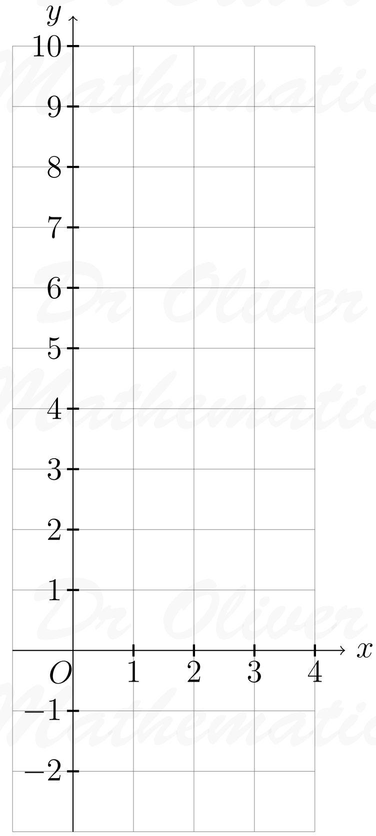
Solution

Could be either.

5. (a) Draw the graph of (3)

$$y = 2x - 1$$

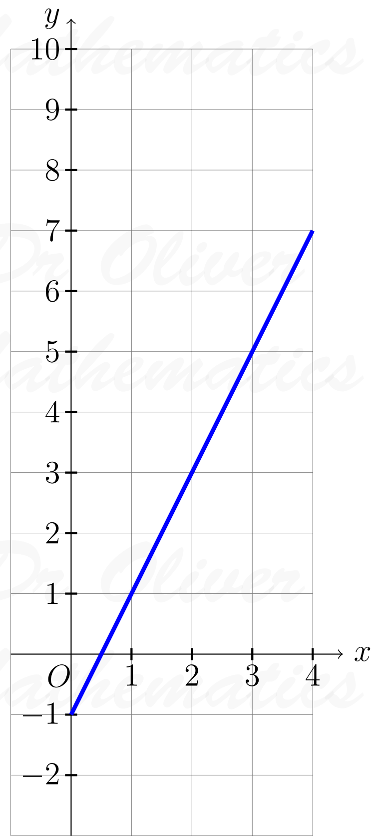
for values of x from 0 to 4.



Solution

We will make a table.

x	0	1	2	3	4
y	−1	1	3	5	7



(b) Solve (1)

$$2x - 1 = 2.$$

Solution

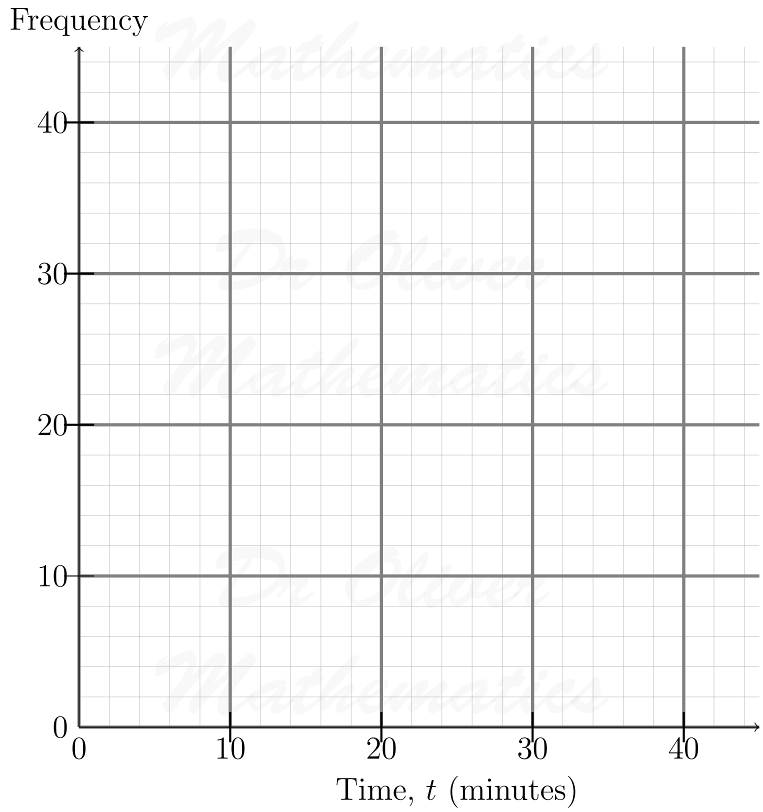
We will do this algebraically:

$$\begin{aligned} 2x - 1 = 2 &\Rightarrow 2x = 3 \\ &\Rightarrow \underline{\underline{x = 1\tfrac{1}{2}}}. \end{aligned}$$

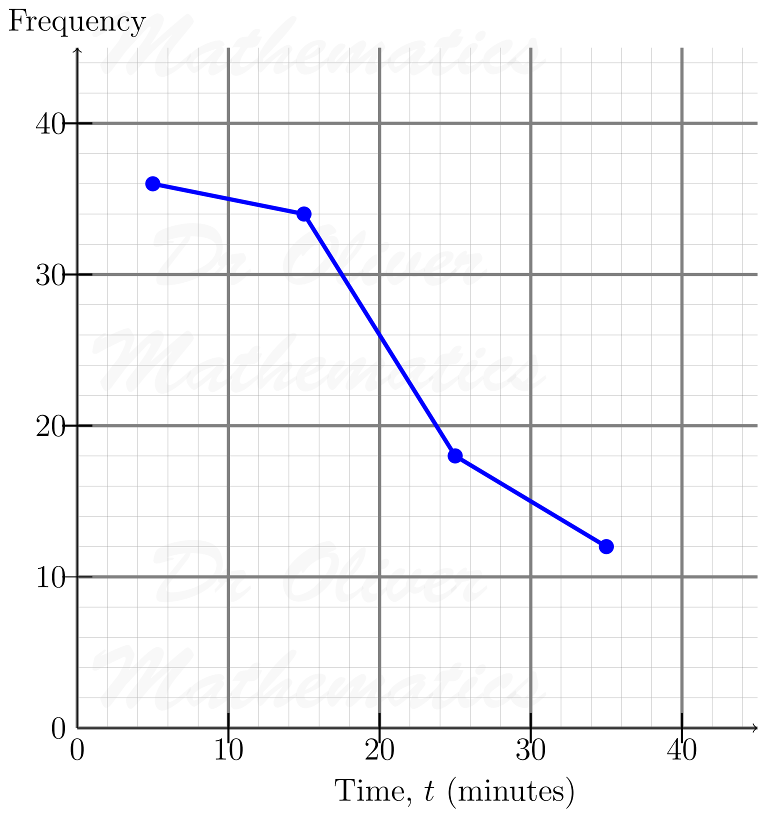
6. The times taken by 100 students to travel to school are shown.

Time, t (minutes)	Frequency
$0 < t \leqslant 10$	36
$10 < t \leqslant 20$	34
$20 < t \leqslant 30$	18
$30 < t \leqslant 40$	12

(a) Draw a frequency diagram for the data. (2)



Solution



The school has 600 students.

(b) Estimate how many students take more than 20 minutes to travel to school. (2)

Solution

$$\begin{aligned}\text{Estimate} &= \frac{18+12}{100} \times 600 \\ &= \frac{30}{100} \times 600 \\ &= 30 \times 6 \\ &= \underline{\underline{180 \text{ students}}}.\end{aligned}$$

7. The total number of people living in a street is 30. (3)
The table shows the number of people living in each house.

Number of people	Number of houses
2	4
3	3
4	a
5	1

Work out the value of a.
You **must** show your working.

Solution

$$\begin{aligned}(2\times 4)+(3\times 3)+(4\times a)+(5\times 1)=30 &\Rightarrow 8+9+4a+5=30\\ &\Rightarrow 4a=8\\ &\Rightarrow \underline{\underline{a=2}}.\end{aligned}$$

8. (a) Factorise (3)

$$3x-15.$$

Solution

$$3x-15=\underline{\underline{3(x-5)}}.$$

(b) Multiply out (2)

$$5(y+4t-2).$$

Solution

$$5(y+4t-2)=\underline{\underline{5y+20t-10}}.$$

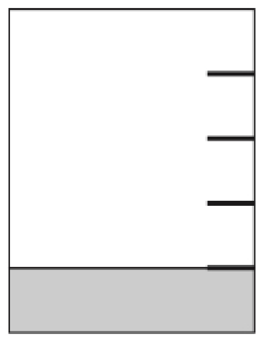
(c) Solve (1)

$$3(w+2)=2w-1.$$

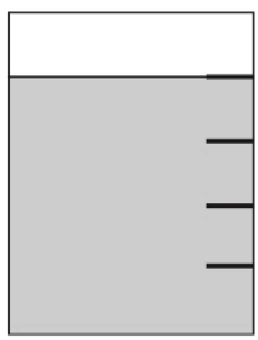
Solution

$$3(w+2) = 2w - 1 \Rightarrow 3w + 6 = 2w - 1$$
$$\Rightarrow \underline{\underline{w = -7}}.$$

9. When a jug is $\frac{1}{5}$ full of water it weighs 250 grams. (4)



When the same jug is $\frac{4}{5}$ full of water it weighs 550 grams.



How much does the jug weigh when it is empty?

Solution

Let j be the *mass* of the jug and l be the amount of water. Then

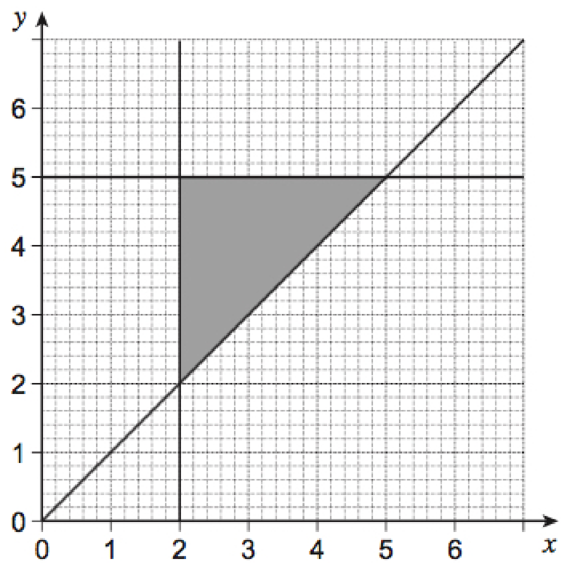
$$j + l = 250 \quad (1)$$
$$j + 4l = 550 \quad (2)$$

Do (2) − (1):

$$\begin{aligned} 3l = 300 &\Rightarrow l = 100 \\ &\Rightarrow j = 150; \end{aligned}$$

hence, jug 'weighs' (sigh) 150 grams.

10. Work out the three inequalities that describe the shaded region. (3)



Solution

$x \geqslant 2$, $y \leqslant 5$, and $y \geqslant x$.

11. A triangle, square, and pentagon have a total area of 48 cm^2. (3)
The areas of the shapes are in the ratio of their number of sides.

Work out the area of the pentagon.

Solution

Well,

$$\text{three} : \text{four} : \text{five} = 3 : 4 : 5.$$

Now,

$$3 + 4 + 5 = 12$$

and

$$\begin{aligned} \text{area of the pentagon} &= \frac{5}{12} \times 48 \\ &= 5 \times 4 \\ &= \underline{\underline{20 \text{ cm}^2}}. \end{aligned}$$

12. Rearrange (3)

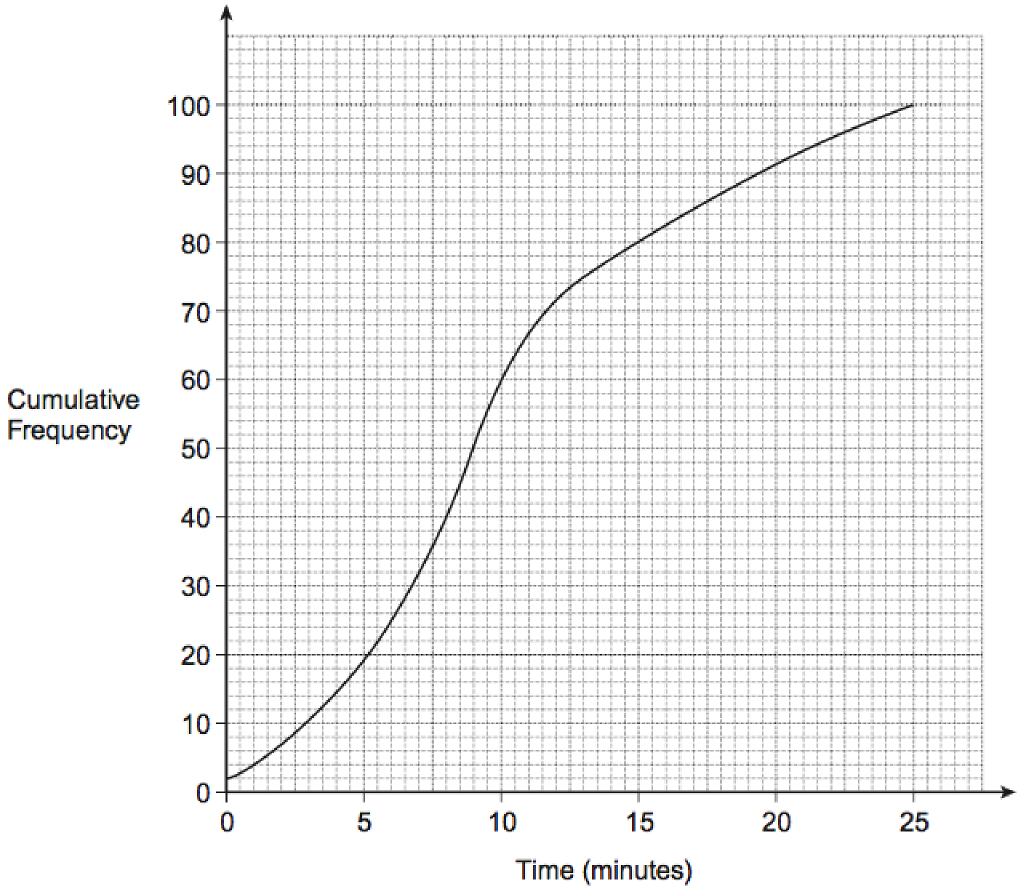
$$2(a + c) = 5(a - b)$$

to make c the subject.

Solution

$$\begin{aligned} 2(a + c) = 5(a - b) &\Rightarrow 2a + 2c = 5a - 5b \\ &\Rightarrow 2c = 3a - 5b \\ &\Rightarrow \underline{\underline{c = \tfrac{1}{2}(3a - 5b)}}. \end{aligned}$$

13. The times that 100 customers spent queuing in a post office were recorded. The cumulative frequency diagram shows the results.



(a) How many customers queued for more than 15 minutes? (1)

Solution

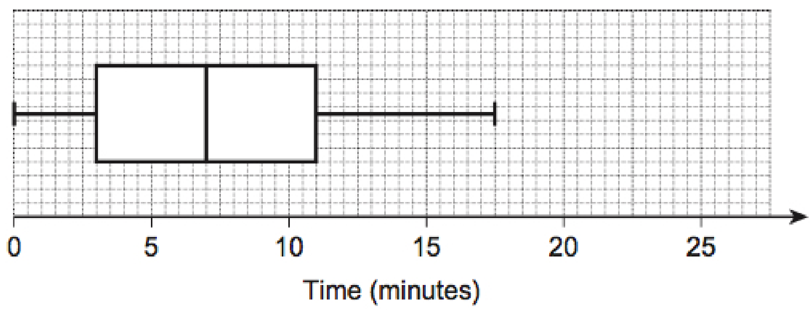
$100 - 80 =$ 20 customers.

(b) Work out the median queuing time. (1)

Solution

9 minutes.

A new serving window was opened in the post office.
The times that 100 customers spent queuing were then recorded.
The box plot shows the results.



(c) Work out the inter-quartile range of these times. (2)

Solution

$11 - 3 = $ 8 minutes.

(d) Compare the queuing times before and after the new serving window was opened. Give **two** comparisons. (2)

Solution

Average

Since the median for the new serving window (7) is lower than the median for the old serving window (9), the new serving window is shorter on average.

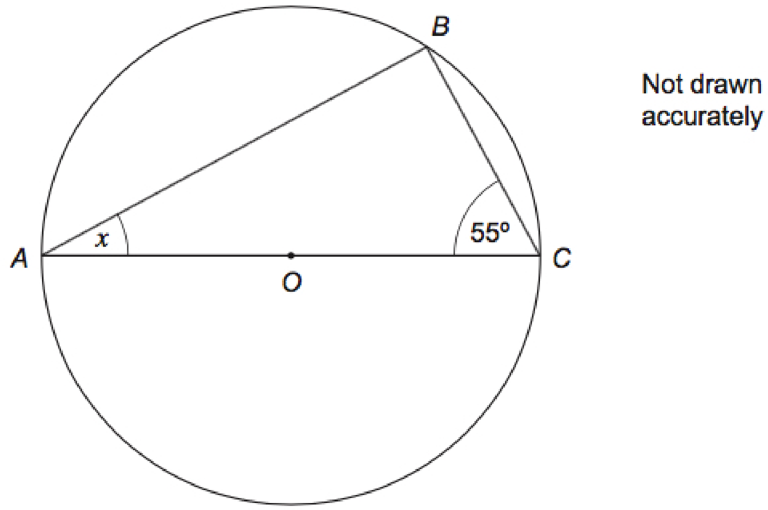
Spread

Since the range for the new serving window ($17.5 - 0 = 17.5$) is smaller than the range for the old serving window ($25 - 2 = 23$), the new serving window was more consistent.

OR

Since the IQR for the new serving window ($11 - 3 = 8$) is larger than the IQR for the old serving window ($13 - 6 = 7$), the old serving window was more consistent.

14. (a) A, B, and C are points on the circumference of a circle with centre O. (1)



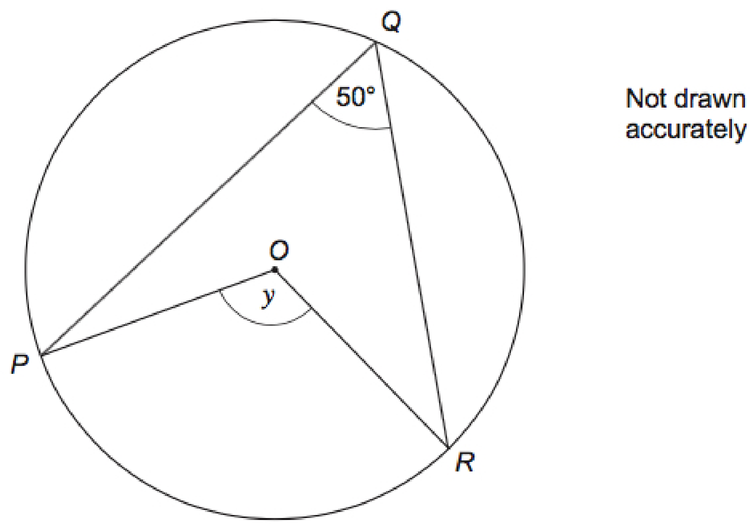
Work out the size of angle x.

Solution

Well, the angles in a triangle add up to 180°:

$$\begin{aligned} x &= 180 - (55 + 90) \\ &= 180 - 145 \\ &= \underline{\underline{35^\circ}}. \end{aligned}$$

(b) P, Q, and R are points on the circumference of a circle with centre O. (2)



Work out the size of angle y.
Give a reason for your answer.

Solution

The angle at the centre is twice the angle at the circumference:

$$y = 2 \times 50 = \underline{\underline{100^\circ}}.$$

15. (a) Expand and simplify (2)

$$(3x + 2)(2x + 5).$$

Solution

$\times$	$3x$	$+2$
$2x$	$6x^2$	$+4x$
$+5$	$+15x$	$+10$

Hence,

$$(3x + 2)(2x + 5) = \underline{\underline{6x^2 + 19x + 19}}.$$

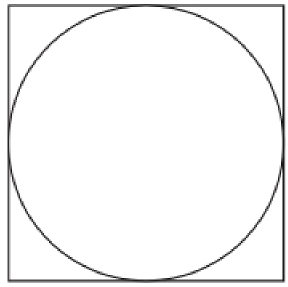
(b) Simplify fully (2)

$$(3x^2y^4)^2.$$

Solution

$$(3x^2y^4)^2 = \underline{\underline{9x^4y^8}}.$$

16. A circle is drawn inside a square as shown. (4)



Show that the area of the circle is more than 75% of the area of the square.

Solution

Let r be the radius of the circle. Then

$$\begin{aligned}\frac{\text{circle}}{\text{square}} &= \frac{\pi \times r^2}{(2r)^2} \times 100\% \\ &= \frac{\pi \times r^2}{4r^2} \times 100\% \\ &= \pi \times 25\% \\ &\approx 3.14 \times 25\% \\ &= \underline{\underline{78.5\%}}.\end{aligned}$$

17. n is an integer. (3)

Show that

$$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$$

is a square number.

Solution

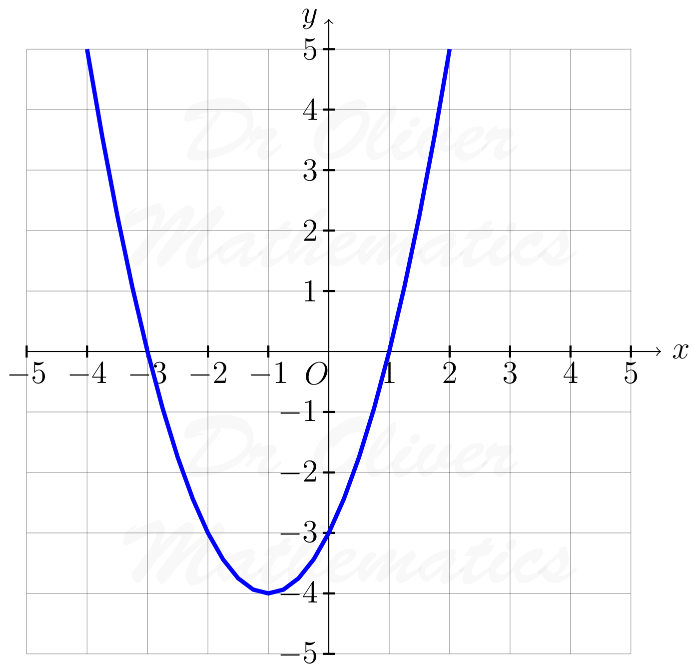
$$\begin{aligned}\frac{n(n-1)}{2} + \frac{n(n+1)}{2} &= \tfrac{1}{2}n\left[(n-1) + (n+1)\right] \\ &= \tfrac{1}{2}n(2n) \\ &= n^2;\end{aligned}$$

hence, if n is an integer, so is $\underline{\underline{n^2}}$.

18. The graph of (3)

$$y = x^2 + 2x - 3$$

is drawn.



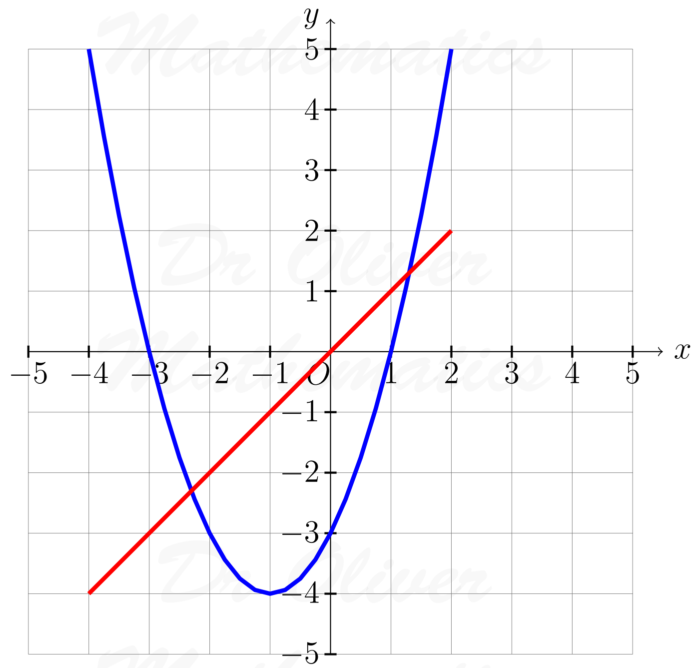
Draw an appropriate straight line on the graph to work out the approximate solutions of

$$x^2 + x - 3 = 0.$$

Solution

$$x^2 + x - 3 = 0 \Rightarrow x^2 + 2x - 3 = x$$

so we need to a line whose equation is $y = x$.



We now read-off the points:

$$\underline{\underline{x = -2.3 \text{ or } x = 1.3}}.$$

19. (a) Show clearly that (1)

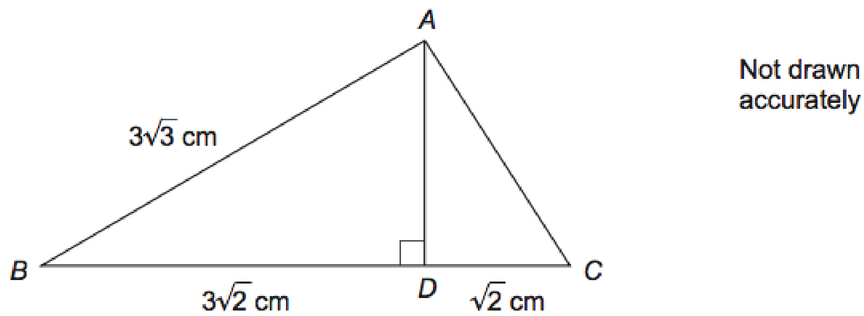
$$(3\sqrt{3})^2 = 27.$$

Solution

$$\begin{aligned}(3\sqrt{3})^2 &= (3\sqrt{3}) \times (3\sqrt{3}) \\ &= (3 \times 3) \times (\sqrt{3} \times \sqrt{3}) \\ &= 9 \times 3 \\ &= \underline{\underline{27}},\end{aligned}$$

as required.

(b) ABC is a triangle. (5)
AD is perpendicular to BC.
$AB = 3\sqrt{3}$ cm, $BD = 3\sqrt{2}$ cm, and $DC = \sqrt{2}$ cm.



Work out the area of triangle ABC.
Give your answer in the form $a\sqrt{2}$, where a is an integer.

Solution

Pythagoras' theorem:

$$\begin{aligned} BD^2 + AD^2 = AB^2 &\Rightarrow (3\sqrt{2})^2 + AD^2 = (3\sqrt{3})^2 \\ &\Rightarrow 18 + AD^2 = 27 \\ &\Rightarrow AD^2 = 9 \\ &\Rightarrow AD = 3. \end{aligned}$$

Finally,

$$\begin{aligned} \text{area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times (4\sqrt{2}) \times 3 \\ &= \underline{\underline{6\sqrt{2}}}; \end{aligned}$$

hence, $\underline{\underline{a = 6}}$.