Dr Oliver Mathematics
GCSE Mathematics
2004 November Paper 6H: Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. The table gives information about Year 10 and Year 11 at Mathstown School. (3)

Year	Number of girls	Number of boys
Year 10	108	132
Year 11	90	110

Mathstown School had an end of term party.
40% of the students in Year 10 and 70% of the students in Year 11 went to the party.
Work out the percentage of all students in Years 10 and 11 who went to the party.

Solution

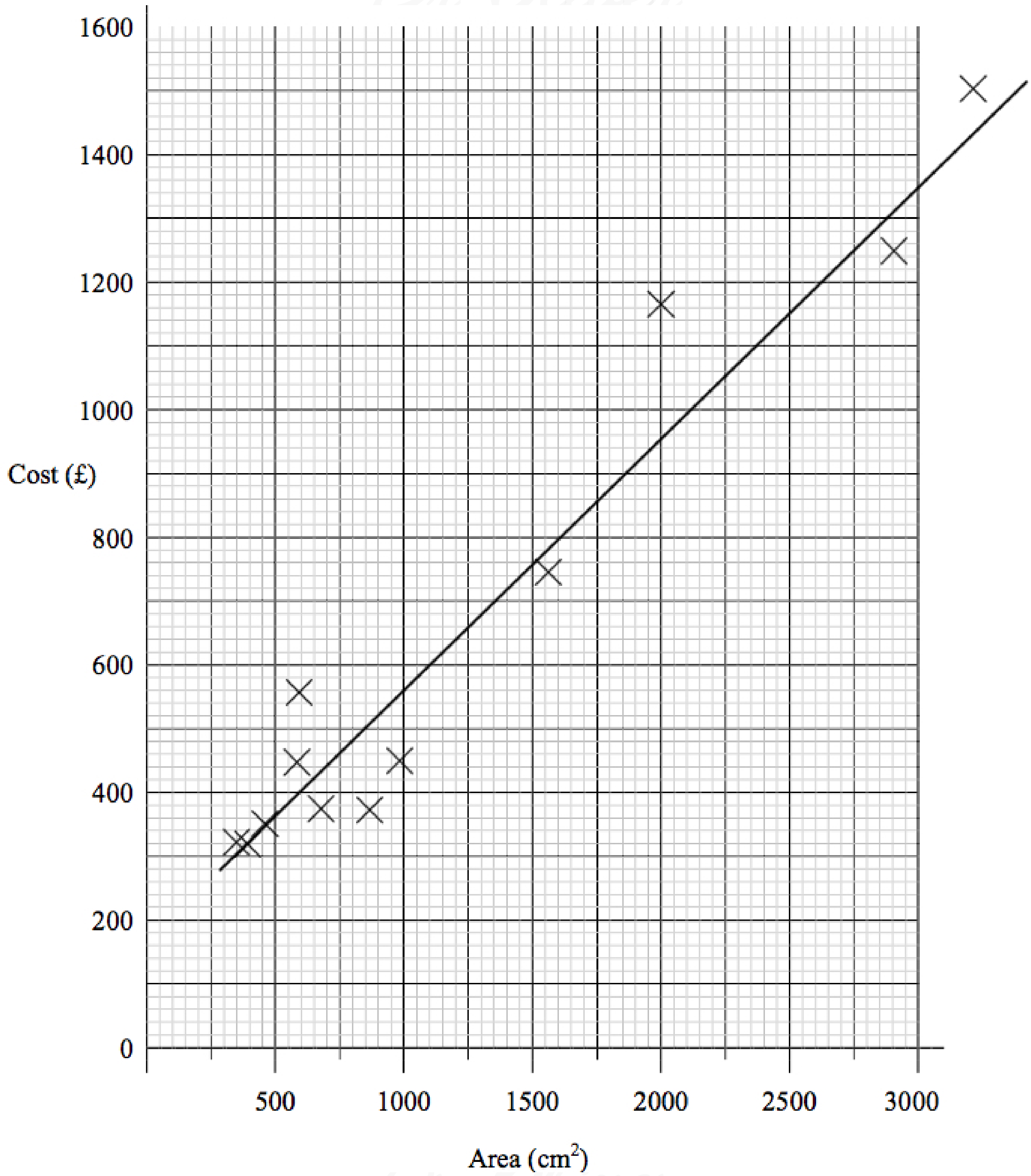
The number of all students in Years 10 and 11 who went to the party is

$$\begin{aligned} 0.4 \times (108 + 132) + 0.7 \times (90 + 110) &= 0.4 \times 240 + 0.7 \times 200 \\ &= 236 \end{aligned}$$

and the percentage is

$$\begin{aligned} \frac{236}{440} \times 100\% &= 53.\dot{6}\dot{3} \\ &= \underline{\underline{53.6\% \text{ (1 dp)}}}. \end{aligned}$$

2. Pablo is an artist. (2)
The scatter graph gives information about the area and the cost of some of his pictures.



The line of best fit has been drawn on the graph.
All Pablo's pictures are rectangles.
One of his pictures costs £1000.
Its length is 48 cm.
Use the line of best fit to estimate the width of the picture.

Solution

Area is approximately 2 100 and

$$\frac{2\,100}{48} = 43\tfrac{3}{4};$$

hence, it is $\underline{\underline{44 \text{ cm}}}$.

3. The equation (4)

$$x^3 + 4x = 100$$

has one solution which is a positive number.
Use the method of trial and improvement to find this solution.
Give your answer correct to 1 decimal place.
You must show **all** your working.

Solution

$$4^3 + 4 \times 4 = 80 \text{ and } 5^3 + 4 \times 5 = 145.$$

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
F(X)= and you type in $X^3 + 4X$; then you press [=].
Start? and you enter 4; then you press [=].
End? and you enter 5; then you press [=].
Step? and enter 0.05 – 1 decimal place divided by 2; then you press [=].

x	$f(x)$	Comment
4.35	99.712	too low
4.4	102.76	too high

Clearly,

$$4.35 < x < 4.4$$

and the answer is

$$\underline{\underline{x = 4.4 \text{ (1 dp)}}}.$$

4. (a) Solve (3)

$$4(2x + 1) = 2(3 - x).$$

Solution

$$\begin{aligned} 4(2x+1) = 2(3-x) &\Rightarrow 8x + 4 = 6 - 2x \\ &\Rightarrow 10x = 2 \\ &\Rightarrow \underline{\underline{x = \tfrac{1}{5}}}. \end{aligned}$$

(b) Factorise fully (2)

$$2p^2 - 4pq.$$

Solution

$$2p^2 - 4pq = \underline{\underline{2p(p-2q)}}.$$

(c) Factorise fully (2)

$$x^2 + 7x + 6.$$

Solution

$$x^2 + 7x + 6 = \underline{\underline{(x+1)(x+6)}}.$$

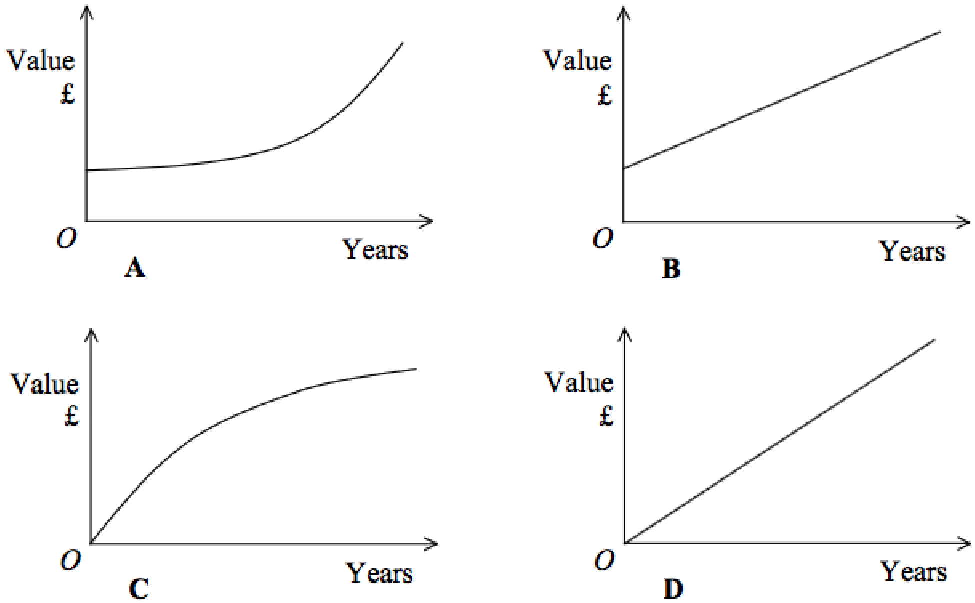
5. Nicola invests £8 000 for 3 years at 5% per annum **compound** interest.

(a) Calculate the value of her investment at the end of 3 years. (3)

Solution

$$8\,000 \times 1.05^3 = \underline{\underline{\pounds 9\,261}}.$$

Jim invests a sum of money for 30 years at 4% annum **compound** interest.



(b) Write down the letter of the graph which best shows how the value of Jim's investment changes over the 30 years. (1)

Solution

$\underline{\underline{A}}$.

Hannah invested an amount of money in an account paying 5% per annum **compound** interest.
After 1 year the value of her investment was £3 885.

(c) Work out the amount of money that Hannah invested. (3)

Solution

$$\frac{3\,885}{1.05} = \underline{\underline{£3\,700}}.$$

6. Fred runs 200 metres in 21.2 seconds.

(a) Work out Fred's average speed. (2)
Write down all the figures on your calculator display.

Solution

$$\frac{200}{21.2} = \underline{\underline{9.433\,962\,264 \text{ m/s (FCD)}}}.$$

(b) Round off your answer to part (a) to an appropriate degree of accuracy. (1)

Solution

$\underline{\underline{9.4 \text{ m/s (1 dp)}}}$.

7. Tony throws a biased dice 100 times.
The table shows his results.

Score	Frequency
1	12
2	13
3	17
4	10
5	18
6	30

He throws the dice once more.

(a) Find an estimate for the probability that he will get a 6. (1)

Solution

$\frac{30}{100} = \underline{\underline{0.3}}$.

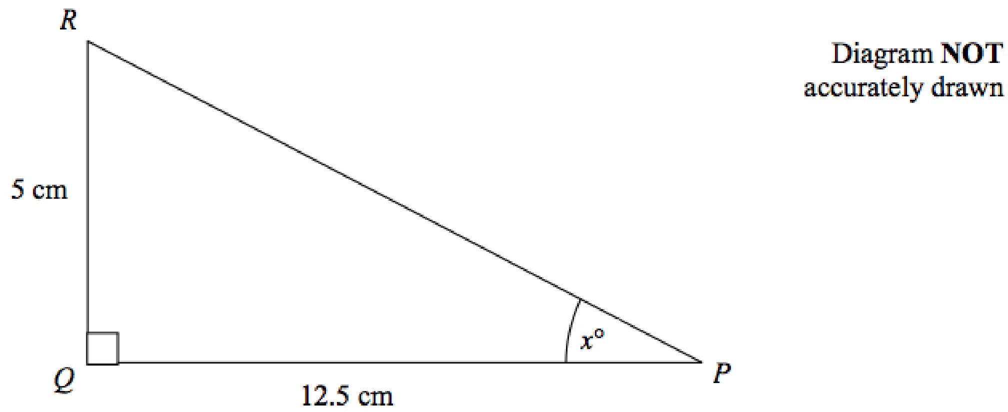
Emma has a biased coin.
The probability that the biased coin will land on a head is 0.7.
Emma is going to throw the coin 250 times.

(b) Work out an estimate for the number of times the coin will land on a head. (2)

Solution

$$250 \times 0.7 = \underline{\underline{175 \text{ times}}}.$$

8. PQR is a triangle. (3)

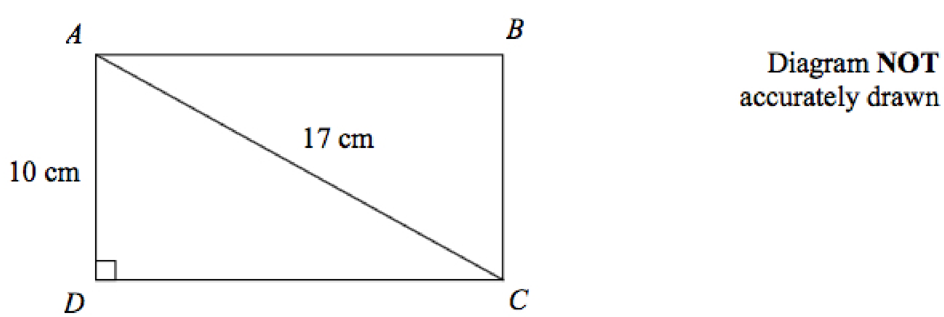


Angle $PQR = 90°$.
$PQ = 12.5$ cm.
$QR = 5$ cm.
Calculate the value of x.
Give your answer correct to 1 decimal place.

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan x = \frac{5}{12.5}$$
$$\Rightarrow x = 21.801\,409\,49 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 21.8 \text{ (1 dp)}}}.$$

9. $ABCD$ is a rectangle. (3)



$AC = 17$ cm.
$AD = 10$ cm.

Calculate the length of the side CD.
Give your answer correct to one decimal place.

Solution

$$\begin{aligned} CD &= \sqrt{AC^2 - AD^2} \\ &= \sqrt{17^2 - 10^2} \\ &= 13.747\,727\,08 \text{ (FCD)} \\ &= \underline{\underline{13.7 \text{ cm (1 dp)}}}. \end{aligned}$$

10.

$$y = \sqrt{\frac{r + t\sin x^\circ}{r - t\sin x^\circ}}.$$

$r = 8.8$.
$t = 7.2$.
$x = 40$.

(a) Calculate the value of y. (3)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} y &= \sqrt{\frac{8.8 + 7.2\sin 40^\circ}{8.8 - 7.2\sin 40^\circ}} \\ &= 1.794\,065\,71 \text{ (FCD)} \\ &= \underline{\underline{1.79 \text{ (3 sf)}}}. \end{aligned}$$

$y = 2$.
$t = 10$.
$x = 30$.

(b) Find the value of r. (3)

Solution

$$2=\sqrt{\frac{r+10\sin 30^\circ}{r-10\sin 30^\circ}} \Rightarrow 2=\sqrt{\frac{r+5}{r-5}}$$
$$\Rightarrow 4=\frac{r+5}{r-5}$$
$$\Rightarrow 4(r-5)=r+5$$
$$\Rightarrow 4r-20=r+5$$
$$\Rightarrow 3r=25$$
$$\Rightarrow \underline{\underline{r=8\tfrac{1}{3}}}.$$

11. The straight line L_1 has equation (3)

$$y=2x+3.$$

The straight line L_2 is parallel to the straight line L_1.
The straight line L_2 passes through the point $(3,2)$.
Find an equation of the straight line L_2.

Solution

The equation of L_2 is

$$y=2x+c$$

and we want to find c:

$$y-2=2(x-3) \Rightarrow y-2=2x-6$$
$$\Rightarrow \underline{\underline{y=2x-4}}.$$

12. A youth club has 60 members.
40 of the members are boys.
20 of the members are girls.
The mean number of videos watched last week by all 60 members was 2.8.
The mean number of videos watched last week by the 40 boys was 3.3.

(a) Calculate the mean number of videos watched last week by the 20 girls. (3)

Solution

Let the mean number of videos watched last week by the 20 girls be x. Then

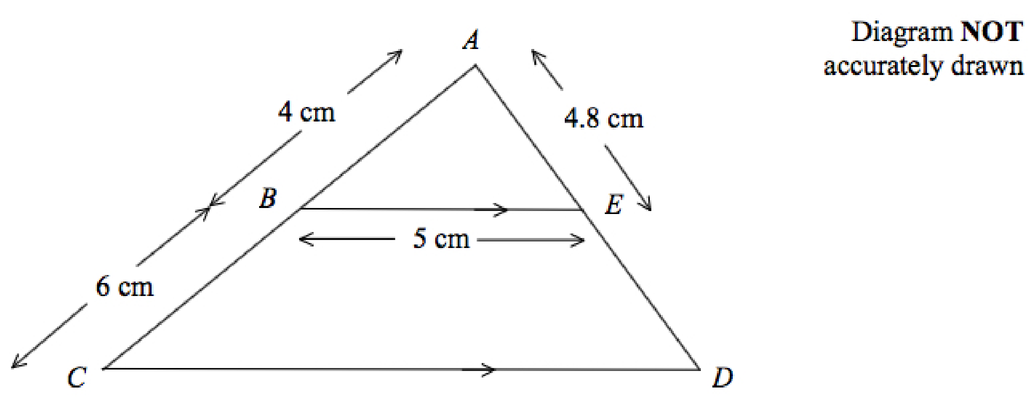
$$2.8 = \frac{(40 \times 3.3) + (20 \times x)}{60} \Rightarrow 168 = 132 + 20x$$
$$\Rightarrow 20x = 36$$
$$\Rightarrow \underline{\underline{x = 1.8}}.$$

Ibrahim has two lists of numbers.
The mean of the numbers in the first list is p.
The mean of the numbers in the second list is q.
Ibrahim combines the two lists into one new list of numbers.
Ibrahim says 'The mean of the new list of numbers is equal to $\frac{p+q}{2}$'.
One of two conditions must be satisfied for Ibrahim to be correct.

(b) Write down each of these conditions. (2)

Solution
E.g., lists have the same number of members and $\underline{\underline{p = q}}$.

13. BE is parallel to CD.



ABC and AED are straight lines.
$AB = 4$ cm.
$BC = 6$ cm.
$BE = 5$ cm.
$AE = 4.8$ cm.

(a) Calculate the length of CD. (2)

Solution

$$CD = \frac{10}{4} \times 5 = \underline{\underline{12\tfrac{1}{2}\text{ cm}}}.$$

(b) Calculate the length of ED. (2)

Solution

$$AD = \frac{10}{4} \times 4.8 = 12$$

and

$$ED = 12 - 4.8 = \underline{\underline{7.2\text{ cm}}}.$$

14. (a) Solve (3)

$$x^2 + x + 11 = 14.$$

Give your solutions correct to 3 significant figures.

Solution

$$x^2 + x + 11 = 14 \Rightarrow x^2 + x - 3 = 0.$$

$a = 1$, $b = 1$, $c = -3$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{(-1)^2 - 4 \times 1 \times (-3)}}{2} \\ &= \frac{-1 \pm \sqrt{13}}{2} \\ &= -2.302\,775\,638 \text{ or } 1.302\,775\,638 \text{ (FCD)} \\ &= \underline{\underline{-2.30 \text{ or } 1.30 \text{ (3 sf)}}}. \end{aligned}$$

$$y = x^2 + x + 11.$$

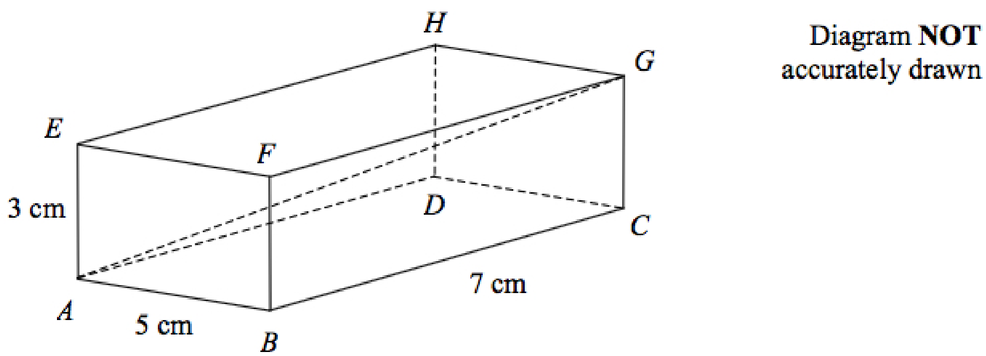
The value of y is a prime number when $x = 0$, 1, 2 and, 3.
The following statement is **not** true.
'$y = x^2 + x + 11$ is **always** a prime number when x is an integer.'

(b) Show that the statement is not true. (2)

Solution

Always take the $+c$ as the input: $\underline{\underline{x = 11}}$, $y = 143 = 11 \times 13$.

15. The diagram represents a cuboid $ABCDEFGH$.



$AB = 5$ cm.
$BC = 7$ cm.
$AE = 3$ cm.

(a) Calculate the length of AG. (2)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} AG &= \sqrt{AB^2 + BC^2 + CG^2} \\ &= \sqrt{5^2 + 7^2 + 3^2} \\ &= 9.110\,433\,579 \text{ (FCD)} \\ &= \underline{\underline{9.11 \text{ cm (3 sf)}}}. \end{aligned}$$

(b) Calculate the size of the angle between AG and the face $ABCD$. (2)
Give your answer correct to 1 decimal place.

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan GAC^\circ = \frac{3}{\sqrt{5^2+7^2}}$$
$$\Rightarrow GAC^\circ = 19.225\,855\,75 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{GAC^\circ = 19.2^\circ \text{ (1 dp)}}}.$$

16. In a factory, chemical reactions are carried out in spherical containers. (4)
The time, T minutes, the chemical reaction takes is directly proportional to the square of the radius, R cm, of the spherical container.
When $R = 120$, $T = 32$.
Find the value of T when $R = 150$.

Solution

$$T \propto R^2 \Rightarrow T = kR^2$$

for some k. Now,

$$32 = k \times 120^2 \Rightarrow k = \tfrac{1}{450}$$

and

$$T = \tfrac{1}{450} R^2.$$

Finally,

$$T = \tfrac{1}{450} \times 150^2 = \underline{\underline{50}}.$$

17. X and Y are two geometrically similar solid shapes. (3)
The total surface area of shape X is 450 cm^2.
The total surface area of shape Y is 800 cm^2.
The volume of shape X is 1 350 cm^3.
Calculate the volume of shape Y.

Solution

The area scale ratio (ASR) is

$$\frac{800}{450} = \frac{16}{9} = \left(\frac{4}{3}\right)^2$$

and the volume scale ratio (VSR) is

$$\left(\frac{4}{3}\right)^3 = \frac{64}{27}.$$

Finally, the volume of shape Y is

$$\frac{64}{27} \times 1\,350 = \underline{\underline{3\,200 \text{ cm}^3}}.$$

18. The time period, T seconds, of a pendulum is calculated using the formula (5)

$$T = 6.283 \times \sqrt{\frac{L}{g}},$$

where L metres is the length of the pendulum and g m/s^2 is the acceleration due to gravity.
$L = 1.36$ correct to 2 decimal places.
$g = 9.8$ correct to 1 decimal place.
Find the difference between the lower bound of T and the upper bound of T.

Solution

First,

$$1.355 \leqslant L < 1.365$$

and

$$9.75 \leqslant g < 9.85.$$

Next, the lower bound of T is

$$6.283 \times \sqrt{\frac{1.355}{9.85}}$$

and the upper bound of T is

$$6.283 \times \sqrt{\frac{1.365}{9.75}}.$$

Finally, the difference is

$$0.020\,547\,080\,23 \text{ (FCD)} = \underline{\underline{0.021 \text{ (3 dp)}}}.$$

19. Simplify (3)

$$\frac{4x^2 - 9}{2x^2 - 5x + 3}.$$

Solution

$$4x^2 - 9 = (2x)^2 - 3^3 = (2x-3)(2x+3).$$

$$\left.\begin{array}{ll} \text{add to:} & -5 \\ \text{multiply to:} & (+2)\times(+3) = +6 \end{array}\right\} -2,\ -3$$

$$\begin{aligned} 2x^2 - 5x + 3 &= 2x^2 - 2x - 3x + 3 \\ &= 2x(x-1) - 3(x-1) \\ &= (2x-3)(x-1). \end{aligned}$$

Finally,

$$\begin{aligned} \frac{4x^2-9}{2x^2-5x+3} &= \frac{(2x-3)(2x+3)}{(2x-3)(x-1)} \\ &= \underline{\underline{\frac{2x+3}{x-1}}}. \end{aligned}$$

20. In a game of chess, you can win, draw, or lose.
Gary plays two games of chess against Mijan.
The probability that Gary will win any game against Mijan is 0.55.
The probability that Gary will win draw game against Mijan is 0.3.

(a) Work out the probability that Gary will win **exactly** one of the two games against Mijan. (3)

Solution
The probability that Gary will lose draw game against Mijan is 0.15 and he probability that Gary will win **exactly** one of the two games against Mijan is

$$0.55 \times 0.45 + 0.45 \times 0.55 = \underline{\underline{0.495}}.$$

In a game of chess, you score 1 point for a win, $\frac{1}{2}$ point for a draw, and 0 points for a loss.

(b) Work out the probability that after two games, Gary's total score will be the same as Mijan's total score. (3)

Solution

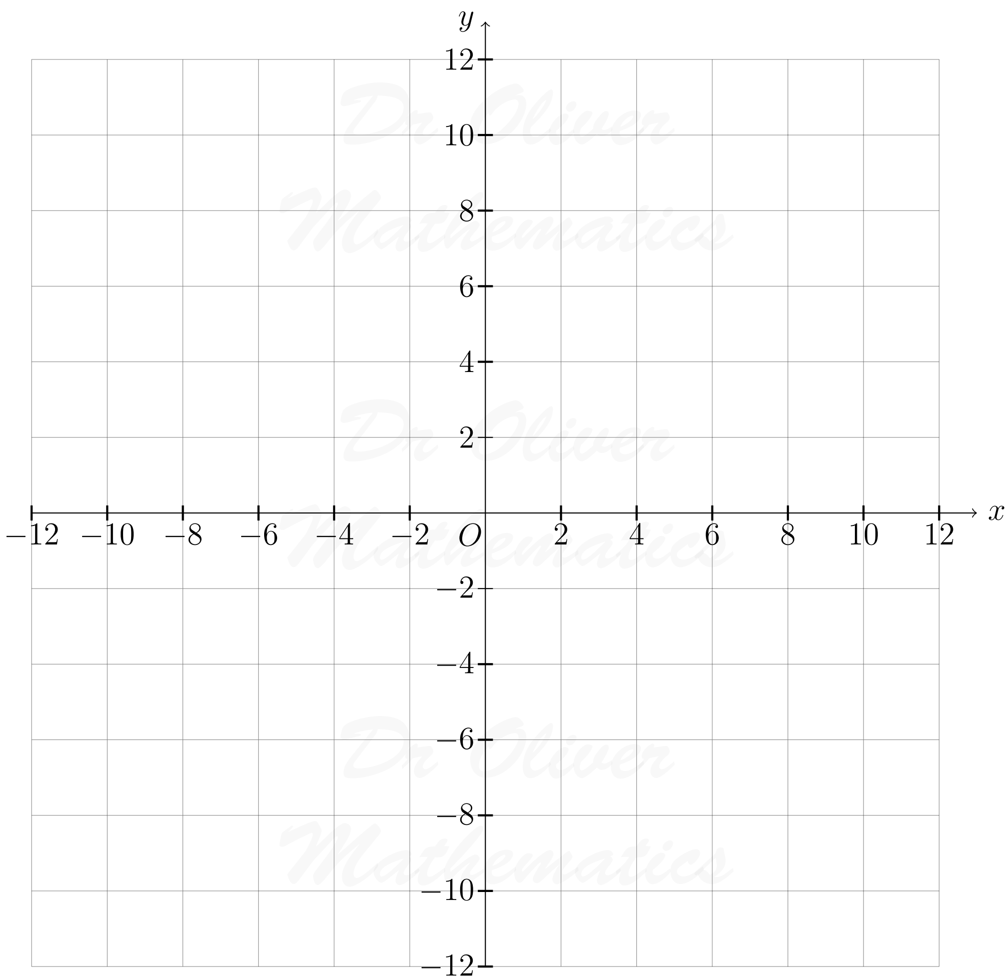
$$\begin{aligned}\text{P(exactly the same)} &= (0.55 \times 0.15) + (0.3 \times 0.3) + (0.15 \times 0.55) \\ &= 0.108\,5 + 0.09 + 0.108\,5 \\ &= \underline{\underline{0.255}}.\end{aligned}$$

21. (a) Draw the graphs of (3)

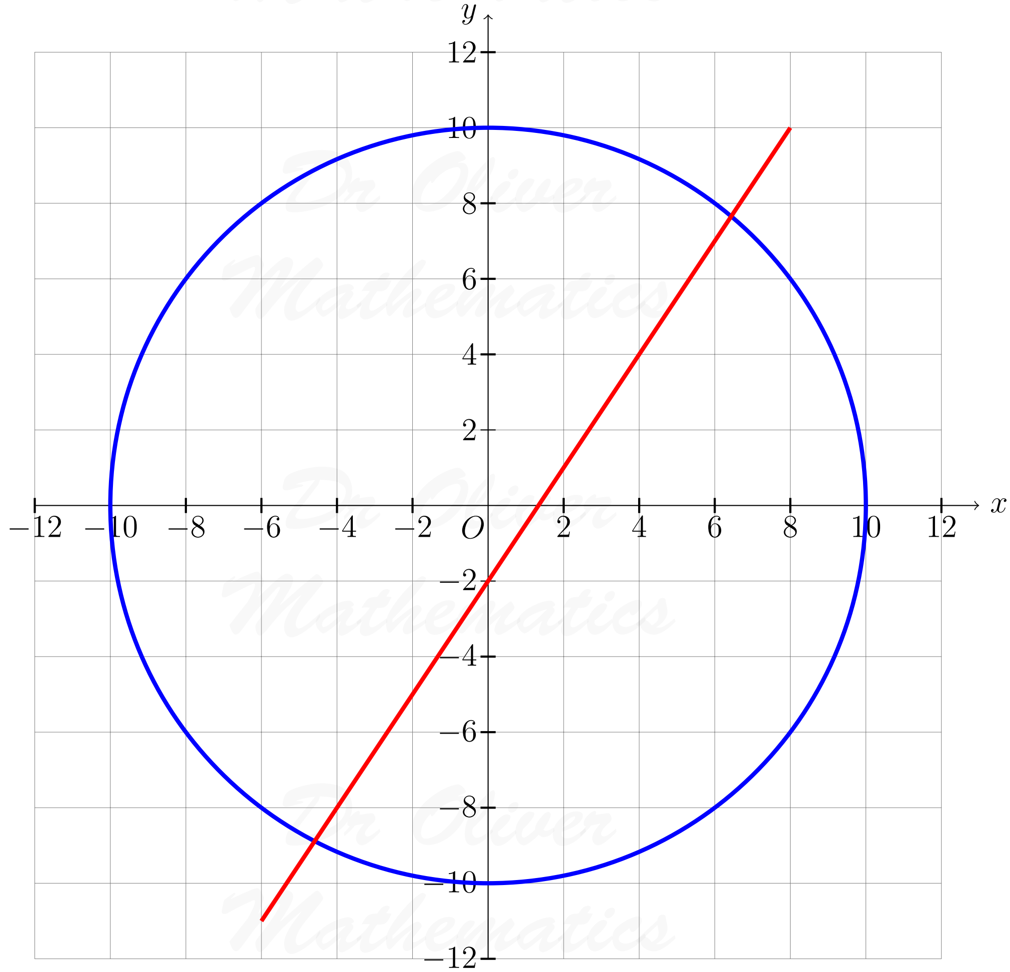
$$x^2 + y^2 = 100$$

and

$$2y = 3x - 4.$$



Solution



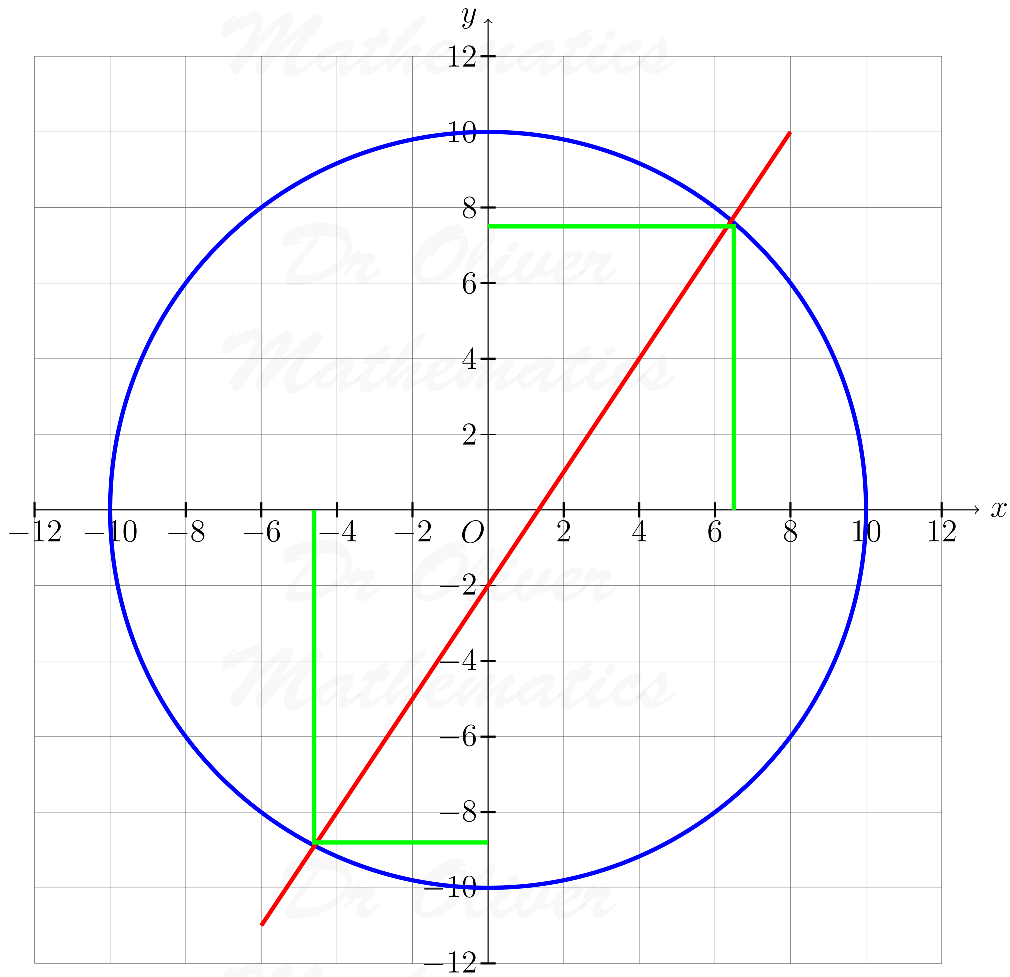
(b) Use the graphs to estimate the solutions of the simultaneous equations (2)

$$x^2 + y^2 = 100$$

and

$$2y = 3x - 4.$$

Solution



$\underline{\underline{(-4.6, -8.8)}}$ and $\underline{\underline{(6.5, 7.5)}}$.

For all the values of x,

$$x^2 + 6x = (x + 3)^2 - q.$$

(c) Find the value of q. (2)

Solution

$$\begin{aligned} x^2 + 6x &= (x^2 + 6x + 9) - 9 \\ &= (x + 3)^2 - 9; \end{aligned}$$

hence, $\underline{\underline{q = 9}}$.

One pair of integer values which satisfy the equation

$$x^2 + y^2 = 100$$

is $x = 6$ and $y = 8$.

(d) Find one pair of integer values which satisfy (3)

$$x^2 + 6x + y^2 - 4y - 87 = 0.$$

Solution

$$\begin{aligned} x^2 + 6x + y^2 - 4y - 87 = 0 &\Rightarrow x^2 + 6x + y^2 - 4y = 87 \\ &\Rightarrow x^2 + 6x + 9 + y^2 - 4y + 4 = 87 + 9 + 4 \\ &\Rightarrow (x+3)^2 + (y-2)^2 = 100; \end{aligned}$$

hence, $\underline{\underline{x = 3, y = 10}}$

22. In triangle PQR, $PQ = 10$ cm, $QR = 12$ cm, and angle $PQR = 45°$.

(a) Calculate the area of triangle PQR. (2)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} \text{Area} &= \tfrac{1}{2} \times 10 \times 12 \times \sin 45^\circ \\ &= 42.426\,406\,87 \text{ (FCD)} \\ &= \underline{\underline{42.4 \text{ cm}^2 \text{ (3 sf)}}}. \end{aligned}$$

The diagram shows triangle ABC and triangle ACD.

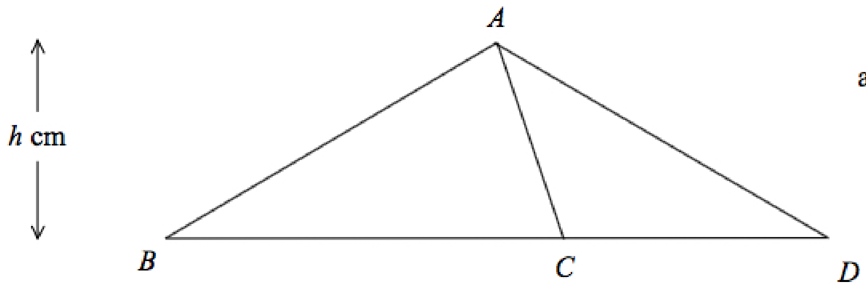


Diagram NOT accurately drawn

BCD is a straight line.
The perpendicular distance from A to the line BCD is h cm.

(b) Explain why (2)

$$\frac{\text{area of triangle } ABC}{\text{area of triangle } ACD} = \frac{BC}{CD}.$$

Solution

$$\frac{\text{area of triangle } ABC}{\text{area of triangle } ACD} = \frac{\frac{1}{2}h \times BC}{\frac{1}{2}h \times CD}$$
$$= \underline{\underline{\frac{BC}{CD}}},$$

as required.

The diagram shows triangle XYZ.

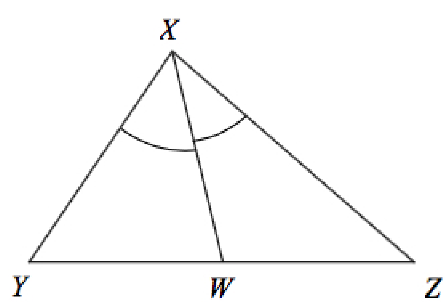


Diagram NOT accurately drawn

W is the point on YZ such that angle YXW = angle WXZ.

(c) Using expressions for the area of triangle YXW and the area of triangle WXZ, or (3) otherwise, show that

$$\frac{XY}{XZ} = \frac{YW}{WZ}.$$

Solution

$$\text{area of triangle } \; YXW = \tfrac{1}{2} \times XY \times XW \times \sin YXW$$

and

$$\text{area of triangle } \; WXZ = \tfrac{1}{2} \times XW \times XZ \times \sin WXZ.$$

Now,

$$\angle\, YXW = \angle\, WXZ$$

and so

$$\begin{aligned}\frac{XY}{XZ} &= \frac{\frac{2\times\text{area of triangle } \; YXW}{XW \sin YXW}}{\frac{2\times\text{area of triangle } \; WXZ}{XW \sin YXW}} \\ &= \frac{\text{area of triangle } \; YXW}{\text{area of triangle } \; WXZ} \\ &= \underline{\underline{\frac{YW}{WZ}}}.\end{aligned}$$

by part (b).