Dr Oliver Mathematics
Mathematics: Higher
2023 Paper 1: Non-Calculator
1 hour 15 minutes

The total number of marks available is 55.
You must write down all the stages in your working.

1. Given that (3)

$$y = x^{\frac{5}{3}} - \frac{10}{x^4}, \text{ where } x \neq 0,$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

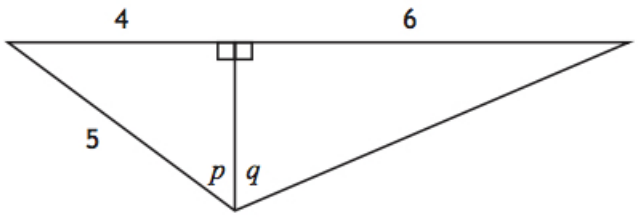
2. P and Q are the points $(-2, 6)$ and $(10, 0)$. (4)

Find the equation of the perpendicular bisector of PQ.

3. Solve (3)

$$\log_5 x - \log_5 3 = 2.$$

4. The diagram shows two right-angled triangles with angles p and q as marked.



(a) Determine the value of:
(i) $\cos p$, (1)
(ii) $\cos q$. (1)

(b) Hence determine the value of $\cos(p + q)$. (3)

5. The equation (3)

$$2x^2 + (3p - 2)x + p = 0$$

has equal roots.

Determine the possible values of p.

6. Find (4)

$$\int (2x^5 - 6\sqrt{x})\,\mathrm{d}x,\ x \geqslant 0.$$

7. (a) Evaluate (2)

$$\log_2 5 + \log_2 \tfrac{1}{40}.$$

(b) Given that $a \in \mathbb{R}$ and that $\log_8 a$ is negative, state the range of possible values of a. (1)

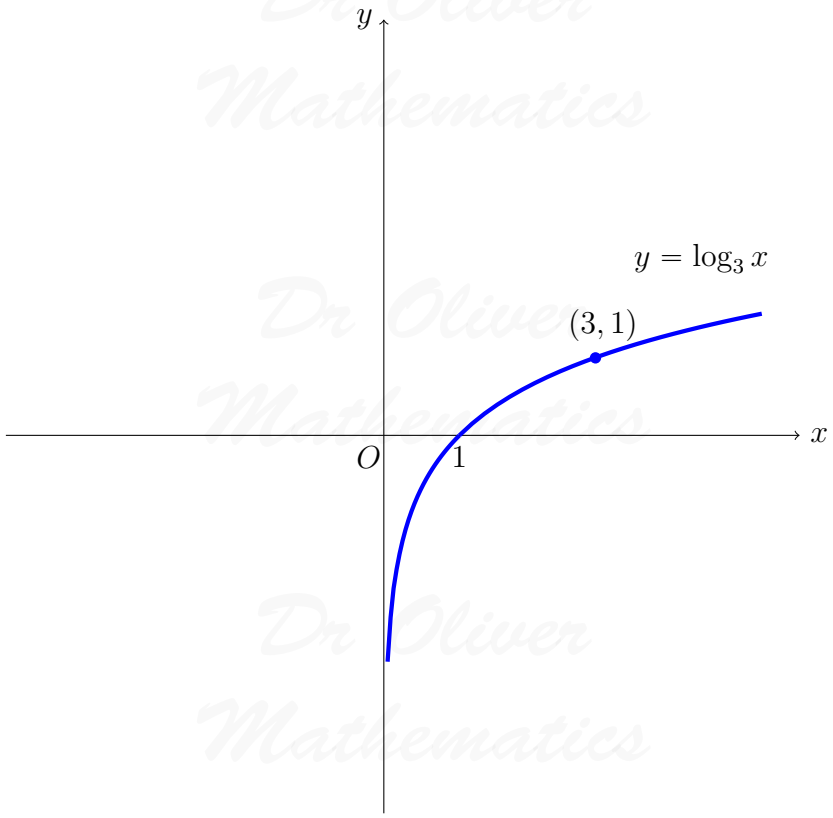
8. A function, f, is defined on $\mathbb{R}$, the set of real numbers, by (6)

$$\mathrm{f}(x) = x^3 + 3x^2 - 9x + 5.$$

Find the coordinates of the stationary points of f and determine their nature.

9. The diagram shows the graph of the function (3)

$$\mathrm{f}(x) = \log_3 x, \text{ where } x > 0.$$



The inverse function, f^{-1}, exists.

Sketch the graph of

$$y = \mathrm{f}^{-1}(x) - 1.$$

10. (a) Show that $(x + 5)$ is a factor of (2)

$$x^4 + 3x^3 - 7x^2 + 9x - 30.$$

(b) Hence, or otherwise, solve (5)

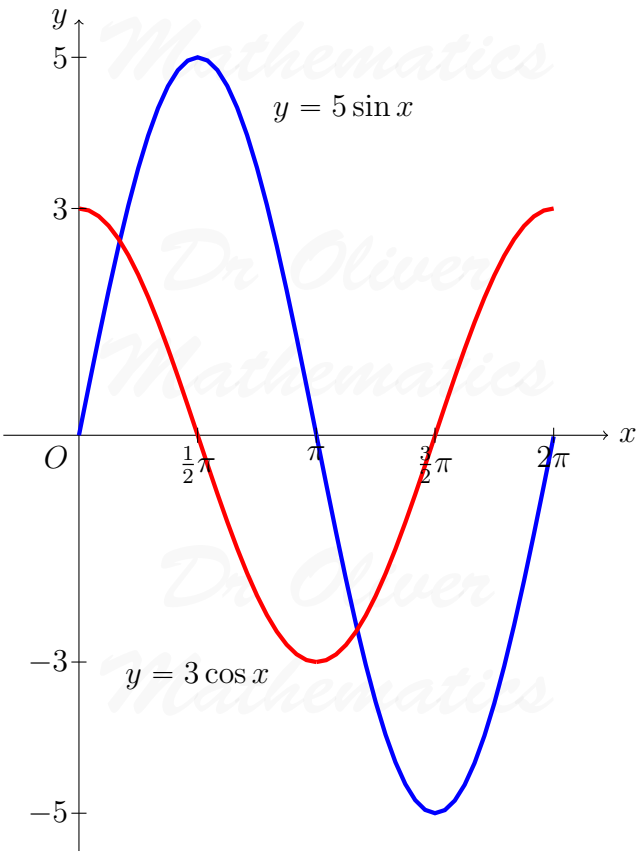
$$x^4 + 3x^3 - 7x^2 + 9x - 30 = 0,\ x \in \mathbb{R}.$$

11. (a) Evaluate (3)

$$\int_{\frac{1}{2}\pi}^{\pi} (5\sin x - 3\cos x)\,\mathrm{d}x.$$

The diagram below shows the graphs with equations

$$y = 5\sin x \text{ and } y = 3\cos x,\ 0 \leqslant x \leqslant 2\pi.$$



(b) Shade the area represented by the integral in (a). (1)

12. Express (3)

$$-2x^2 - 12x + 7$$

in the form

$$a(x + b)^2 + c.$$

13. Functions f and g are defined by:

- $\mathrm{f}(x) = 2\sin x$, where $0 < x < \frac{1}{2}\pi$ and
- $\mathrm{g}(x) = 2x$, where $0 < x < \frac{1}{4}\pi$.

(a) (i) Evaluate (1)

$$\mathrm{f}(\mathrm{g}(\tfrac{1}{6}\pi)).$$

(ii) Determine an expression for $\mathrm{f}(\mathrm{g}(x))$. (2)

(b) (i) Given that (1)

$$\mathrm{f}(p) = \tfrac{1}{3},$$

determine the exact value of $\sin p$.

(ii) Hence, determine the exact value of $\mathrm{f}(\mathrm{g}(p))$. (3)