Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2010 November Paper 2 Variant 3: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. The two variables x and y are such that

$$y = \frac{10}{(x+4)^3}.$$

   (a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. (2)

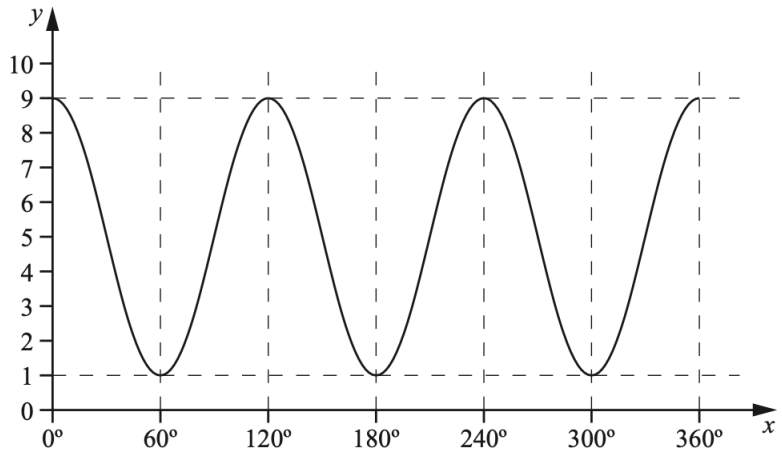
   (b) Hence find the approximate change in y as x increases from 6 to $6 + p$, where p is small. (2)

2. Find the equation of the curve which passes through the point $(4, 22)$ and for which (4)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x(x-2).$$

3. (a) The diagram shows the curve

$$y = A\cos Bx + C, \text{ for } 0^\circ \leqslant x \leqslant 360^\circ.$$



Find the value of

(b) (i) A, (1)

(ii) B, (1)

(iii) C. (1)

(c) Given that

$$\mathrm{f}(x) = 6\sin 2x + 7,$$

state

(i) the period of f, (1)

(ii) the amplitude of f. (1)

4. (a) Find, in ascending powers of x, the first 4 terms of the expansion of (2)

$$(1+x)^6.$$

(b) Hence find the coefficient of p^3 in the expansion of (3)

$$(1+p-p^2)^6.$$

5. (a) Given that (2)

$$\mathbf{A} = \begin{pmatrix} 2 & -4 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 3 & -1 \\ 0 & 5 \\ -2 & 7 \end{pmatrix},$$

find $\mathbf{AB}$.

(b) Given that

$$\mathbf{C} = \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 6 & -4 \\ 2 & 8 \end{pmatrix},$$

find

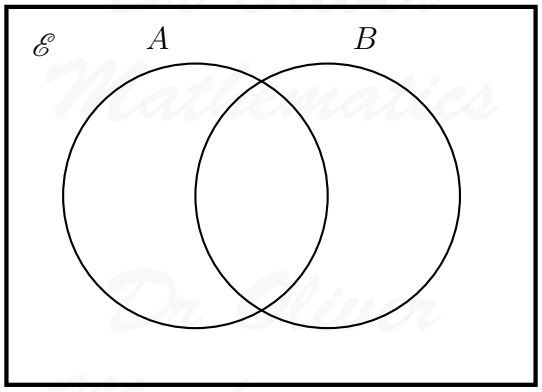
(i) the inverse matrix $\mathbf{C}^{-1}$, (2)

(ii) the matrix $\mathbf{X}$ such that (2)

$$\mathbf{CX} = \mathbf{D}.$$

6. (a) Copy the diagram above and shade the region which represents the set (1)

$$A' \cup B.$$



(b) The sets P, Q, and R are such that (2)

$$P \cap Q = \varnothing \text{ and } P \cup Q \subset R.$$

Draw a Venn diagram showing the sets P, Q, and R.

(c) In a group of 50 students, (3)

- F denotes the set of students who speak French and
- S denotes the set of students who speak Spanish.

It is given that

- $\mathrm{n}(F) = 24$,
- $\mathrm{n}(S) = 18$,
- $\mathrm{n}(F \cap S) = x$, and
- $\mathrm{n}(F' \cap S') = 3x$.

Write down an equation in x and hence find the number of students in the group who speak neither French nor Spanish.

7. The line (7)

$$y = 2x - 6$$

meets the curve

$$4x^2 + 2xy - y^2 = 124$$

at the points A and B.

Find the length of the line AB.

8. (a) Show that (1)

$$(5 + 3\sqrt{2})^2 = 43 + 30\sqrt{2}.$$

Hence find, **without using a calculator**, the positive square root of

(b) $86 + 60\sqrt{2}$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers, (2)

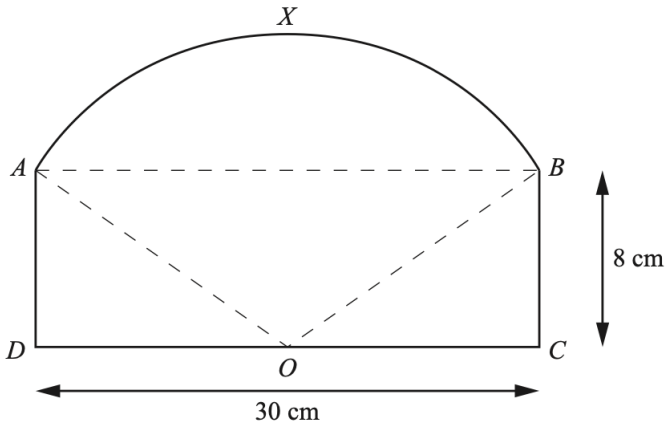
(c) $43 - 30\sqrt{2}$, giving your answer in the form $c + d\sqrt{2}$, where c and d are integers, (1)

(d) $\dfrac{1}{43 + 30\sqrt{2}}$, giving your answer in the form (3)

$$\frac{f + g\sqrt{2}}{h},$$

where f, g, and h are integers.

9. The diagram shows a rectangle $ABCD$ and an arc AXB of a circle with centre at O, the mid-point of DC.



The lengths of DC and BC are 30 cm and 8 cm respectively.

Find

(a) the length of OA, (2)

(b) the angle AOB, in radians, (2)

(c) the perimeter of figure $ADOCBXA$, (2)

(d) the area of figure $ADOCBXA$. (2)

10. The equation of a curve is (9)

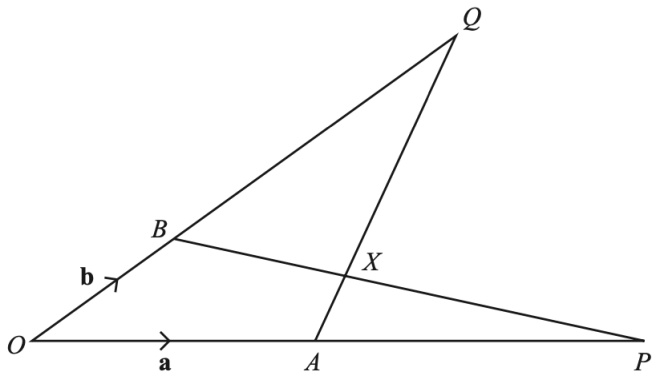
$$y = x^2 \mathrm{e}^x.$$

- The tangent to the curve at the point $P(1, \mathrm{e})$ meets the y-axis at the point A.
- The normal to the curve at P meets the x-axis at the point B.

Find the area of the triangle OAB, where O is the origin.

11. In the diagram,

- $\overrightarrow{OA} = \mathbf{a}$,
- $\overrightarrow{OB} = \mathbf{b}$,
- $\overrightarrow{OP} = 2\mathbf{a}$, and
- $\overrightarrow{OQ} = 3\mathbf{b}$.



(a) Given that (3)

$$\overrightarrow{AX} = \mu\overrightarrow{AQ},$$

express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$, and $\mathbf{b}$.

(b) Given that (3)

$$\overrightarrow{BX} = \lambda\overrightarrow{BP},$$

express $\overrightarrow{OX}$ in terms of λ, $\mathbf{a}$, and $\mathbf{b}$.

(c) Hence find the value of μ and of λ. (3)

EITHER

12. The table shows values of the variables v and p which are related by the equation

$$p = \frac{a}{v^2} + \frac{b}{v},$$

where a and b are constants.

v	2	4	6	8
p	6.22	2.84	1.83	1.35

(a) Using graph paper, plot v^2p on the y-axis against v on the x-axis and draw a straight line graph. (2)

(b) Use your graph to estimate the value of a and of b. (4)

In another method of finding a and b from a straight line graph, $\frac{1}{v}$ is plotted along the x-axis.

In this case, and without drawing a second graph,

(c) state the variable that should be plotted on the y-axis, (2)

(d) explain how the values of a and of b could be obtained. (2)

OR

13. The table shows experimental values of two variables r and t.

t	2	8	24	54
r	22	134	560	1 608

(a) Using the y-axis for $\ln r$ and the x-axis for $\ln t$, plot $\ln r$ against $\ln t$ to obtain a straight line graph. (2)

(b) Find the gradient and the intercept on the y-axis of this graph and express r in terms of t. (6)

Another method of finding the relationship between r and t from a straight line graph is to plot $\log_{10} r$ on the y-axis and $\log_{10} t$ on the x-axis.

(c) Without drawing this second graph, find the value of the gradient and of the intercept on the y-axis for this graph. (2)