

Dr Oliver Mathematics

Divisibility

In this note, we will investigate divisibility of number 2, 3, and so on up to 20.

2

The number ends in a 0, 2, 4, 6, or 8.

3

Sum the digits. The result must be divisible by 3.

4

Check the last two digits of the number. If they are divisible by 4, the entire number is divisible by 4.

5

The number ends in a 0 or 5.

6

Check if it is a multiple of 2 and 3.

7

Remove the last digit, double it, subtract it from the truncated original number and continue doing this until only one digit remains. If this is 0 or 7, then the original number is divisible by 7. Take 1 778.

$$1\ 778 \rightarrow 177 - 2 \times 8$$

$$\rightarrow 161$$

$$\rightarrow 16 - 2 \times 1$$

$$\rightarrow 14$$

$$\rightarrow 1 - 2 \times 4$$

$$\rightarrow -7,$$

and 1 778 is divisible by 7.

8

Check the last three digits of the number. If they are divisible by 8, the entire number is divisible by 8.

9

The *digital root* is 9. Take 8 244. First, sum up the separate digits:

$$8 + 2 + 4 + 4 = 18.$$

If you get a number that is less than 10, stop here; on the other hand, if you get a number that is greater than or equal to 10, then repeat:

$$1 + 8 = 9;$$

this is the *digital root* of the number. If the digital root is 9, then we know that the original number is divisible by 9; if not, no.

10

The number ends in a 0.

11

Read from left to right, take the *alternating sum* of the digits in the number. If that is divisible by 11, so is the original number. Take 5 192.

$$5 - 1 + 9 - 2 = (5 + 9) - (1 + 2) = 11,$$

and so 5 192 is a divisible by 11.

12

Check if it is a multiple of 3 and 4.

13

Form the alternating sum of blocks of three from right to left. If that is divisible by 13, so is the original number. Take 3 534 011:

$$\begin{aligned} 3 - 534 + 011 &= -520 \\ &= 13 \times (-40), \end{aligned}$$

and so it is divisible by 13.

14

Check if it is a multiple of 2 and 7.

15

Check if it is a multiple of 3 and 5.

16

Check the last four digits of the number. If they are divisible by 16, the entire number is divisible by 16.

17

Subtract 5 times the last digit from the of the truncated number. Take 9316.

$$\begin{aligned} 9316 &\rightarrow 931 - 5 \times 6 \\ &\rightarrow 901 \\ &\rightarrow 90 - 5 \times 1 \\ &\rightarrow 85 \\ &\rightarrow 8 - 5 \times 5 \\ &\rightarrow -17, \end{aligned}$$

and so it is divisible by 17.

18

Check if it is a multiple of 2 and 9.

19

Add twice the last digit from the of the truncated number. Take 703:

$$\begin{aligned} 703 &\rightarrow 70 + 2 \times 3 \\ &\rightarrow 76 \\ &\rightarrow 19 \times 4, \end{aligned}$$

and so it is divisible by 19.

20

Check if it is a multiple of 2 and 10.