Dr Oliver Mathematics
Applied Mathematics: Mechanics or Statistics
Section B
2015 Paper
1 hour

The total number of marks available is 32.
You must write down all the stages in your working.

1. Given that (3)

$$y = \mathrm{e}^{5x} \tan 2x,$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

2. (a) Given matrix (2)

$$\mathbf{A} = \begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix},$$

find $\mathbf{A}^2$ and show that the inverse of $\mathbf{A}^2$ exists.

(b) Hence, or otherwise, find matrix $\mathbf{B}$ such that (3)

$$\mathbf{A}^2\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}.$$

3. A curve is defined by (5)

$$y = \frac{\sin x}{2 - \cos x} \text{ for } 0 \leqslant x \leqslant \pi.$$

Find the exact values of the coordinates of the stationary point of this curve.

4. (a) Express (1)

$$\log_a 2 + \log_a 4 + \log_a 8$$

in the form

$$p \log_a 2,$$

where p is a constant.

(b) Hence evaluate (3)

$$\sum_{r=1}^{100} \log_a 2^r,$$

giving your answer in the form

$$q \log_a 2,$$

where q is a constant.

5. Find the general solution, in the form $y = \mathrm{f}(x)$, of the differential equation (6)

$$\frac{1}{\cos x}\frac{\mathrm{d}y}{\mathrm{d}x} + y\tan x = \tan x,\ 0 < x < \pi.$$

6. (a) Express (3)

$$\frac{1}{1-y^2}$$

in partial fractions.

(b) Use the substitution $u = \sqrt{1-x}$ to obtain (6)

$$\int \frac{1}{x\sqrt{1-x}}\,\mathrm{d}x,\ 0 < x < 1.$$