Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2008 June Paper 1: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Express (3)

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}$$

in the form

$$a+b\sqrt{2},$$

where a and b are integers.

Solution

Well,

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = \left(\frac{8-3\sqrt{2}}{4+3\sqrt{2}}\right) \times \left(\frac{4-3\sqrt{2}}{4-3\sqrt{2}}\right)$$

$\times$	8	$-3\sqrt{2}$
4	32	$-12\sqrt{2}$
$-3\sqrt{2}$	$-24\sqrt{2}$	$+18$

$\times$	4	$+3\sqrt{2}$
4	16	$+12\sqrt{2}$
$-3\sqrt{2}$	$-12\sqrt{2}$	-18

$$= \frac{50-36\sqrt{2}}{-2}$$

$$= \underline{\underline{-25+18\sqrt{2}}};$$

so, $\underline{\underline{a = -25}}$ and $\underline{\underline{b = 18}}$.

2. A committee of 5 people is to be selected from 6 men and 4 women.

Find

(a) the number of different ways in which the committee can be selected, (1)

Solution

$$\binom{10}{5} = \underline{\underline{252}}.$$

(b) the number of these selections with more women than men. (4)

Solution

Now,

$$\begin{aligned} \text{P(more women than men)} &= \text{P(3 women, 2 men)} + \text{P(4 women, 1 man)} \\ &= \left[\binom{4}{3} \times \binom{6}{2}\right] + \binom{6}{1} \\ &= 60 + 6 \\ &= \underline{\underline{66}}. \end{aligned}$$

3. The line

$$y = 3x + k$$

is a tangent to the curve

$$x^2 + xy + 16 = 0.$$

(a) Find the possible values of k. (3)

Solution

Now,

$$\begin{aligned} x^2 + xy + 16 = 0 &\Rightarrow x^2 + x(3x + k) + 16 = 0 \\ &\Rightarrow x^2 + 3x^2 + kx + 16 = 0 \\ &\Rightarrow 4x^2 + kx + 16 = 0. \end{aligned}$$

Next, if $y = 3x + k$ is a tangent,

$$\begin{aligned} b^2 - 4ac = 0 &\Rightarrow k^2 - 4(4)(16) = 0 \\ &\Rightarrow k^2 = 256 \\ &\Rightarrow \underline{\underline{k = \pm 16}}. \end{aligned}$$

(b) For each of these values of k, find the coordinates of the point of contact of the tangent with the curve. (2)

Solution

$\underline{k = 16}$:

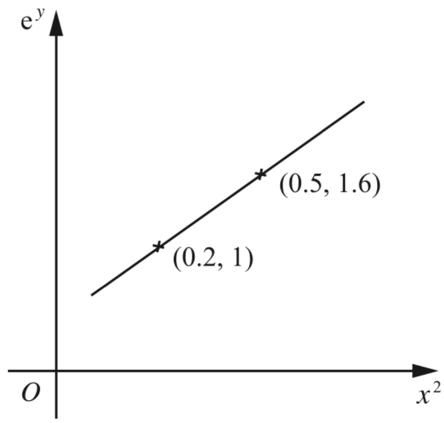
$$\begin{aligned} 4x^2 + 16x + 16 = 0 &\Rightarrow 4(x^2 + 4x + 4) = 0 \\ &\Rightarrow 4(x+2)^2 = 0 \\ &\Rightarrow x + 2 = 0 \\ &\Rightarrow x = -2 \\ &\Rightarrow y = 10. \end{aligned}$$

$\underline{k = -16}$:

$$\begin{aligned} 4x^2 - 16x + 16 = 0 &\Rightarrow 4(x^2 - 4x + 4) = 0 \\ &\Rightarrow 4(x-2)^2 = 0 \\ &\Rightarrow x - 2 = 0 \\ &\Rightarrow x = 2 \\ &\Rightarrow y = -10. \end{aligned}$$

Hence, the two points are $\underline{\underline{(-2, 10)}}$ and $\underline{\underline{(2, -10)}}$.

4. Variables x and y are such that, when e^y is plotted against x^2, a straight line graph passing through the points $(0.2, 1)$ and $(0.5, 1.6)$ is obtained.



(a) Find the value of e^y when $x = 0$. (2)

Solution
Well,

$$m = \frac{1.6 - 1}{0.5 - 0.2} = \frac{0.6}{0.3} = 2$$

and the equation is

$$e^y - 1.6 = 2(x^2 - 0.5) \Rightarrow e^y - 1.6 = 2x^2 - 1 \Rightarrow e^y = 2x^2 + 0.6;$$

so,

$$x = 0 \Rightarrow \underline{\underline{e^y = 0.6}}.$$

(b) Express y in terms of x. (3)

Solution

$$e^y = 2x^2 + 0.6 \Rightarrow \underline{\underline{y = \ln(2x^2 + 0.6)}}.$$

5. Variables x and y are connected by the equation (5)

$$y = \frac{x}{\tan x}.$$

Given that x is increasing at the rate of 2 units per second, find the rate of increase of y when $x = \frac{1}{4}\pi$.

Solution

Well,

$$u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1$$
$$v = \tan x \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = \sec^2 x$$

and

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{(\tan x)(1) - (x)(\sec^2 x)}{(\tan x)} \\ &= \frac{\tan x - x\sec^2 x}{\tan^2 x}.\end{aligned}$$

Next,

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}t} &= \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} \\ &= 2\left(\frac{\tan x - x\sec^2 x}{\tan^2 x}\right)\end{aligned}$$

and

$$\begin{aligned}x = \tfrac{1}{4}\pi \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} &= 2\left(\frac{1 - \frac{1}{4}\pi(\sqrt{2})^2}{1^2}\right) \\ \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} &= 2(1 - \tfrac{1}{2}\pi) \\ \Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}t} = 2 - \pi}}.\end{aligned}$$

6. Solve the equation (6)

$$x^2(2x + 3) = 17x - 12.$$

Solution

Well,

$$x^2(2x+3) = 17x - 12 \Rightarrow 2x^3 + 3x^2 = 17x - 12$$
$$\Rightarrow 2x^3 + 3x^2 - 17x + 12 = 0.$$

Let

$$\mathrm{f}(x) = 2x^3 + 3x^2 - 17x + 12.$$

Then,

$$\mathrm{f}(1) = 2 + 3 - 17 + 12 = 0$$

and $(x-1)$ is a factor.

We use synthetic division:

$$\begin{array}{c|cccc} 1 & 2 & 3 & -17 & 12 \\ & \downarrow & 2 & 5 & -12 \\ \hline & 2 & 5 & -12 & 0 \end{array}$$

$$2x^3 + 3x^2 - 17x + 12 = 0 \Rightarrow (x-1)(2x^2 + 5x - 12) = 0$$

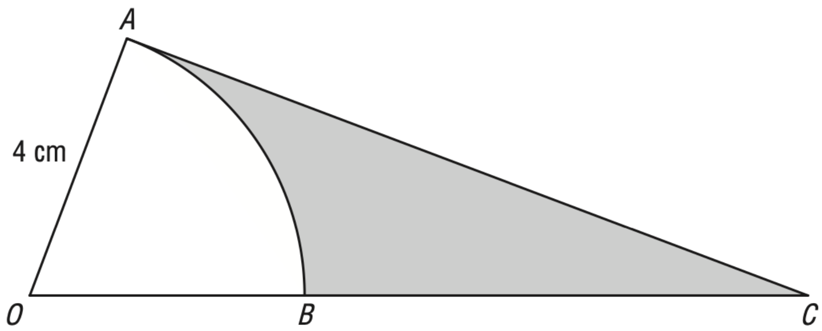
$$\left.\begin{array}{ll} \text{add to:} & +5 \\ \text{multiply to:} & (+2) \times (-12) = -24 \end{array}\right\} -3,\ +8$$

e.g.,

$$\Rightarrow (x-1)[2x^2 + 8x - 3x - 12] = 0$$
$$\Rightarrow (x-1)[2x(x+4) - 3(x+4)] = 0$$
$$\Rightarrow (x-1)(2x-3)(x+4) = 0$$
$$\Rightarrow \underline{\underline{x = -4,\ x = 1,\ \text{or } x = 1\tfrac{1}{2}}}.$$

7. The diagram shows a sector OAB of a circle, centre O, radius 4 cm.

The tangent to the circle at A meets the line OB extended at C.



Given that the area of the sector OAB is 10 cm^2, calculate

(a) the angle AOB in radians, (2)

Solution

Well,

$$\tfrac{1}{2} \times 4^2 \times \angle AOB = 10 \Rightarrow 8 \times \angle AOB = 10$$
$$\Rightarrow \underline{\underline{\angle AOB = 1\tfrac{1}{4} \text{ radians.}}}$$

(b) the perimeter of the shaded region. (4)

Solution

Well,

$$\begin{aligned}\text{curved part of } AB &= 4 \times (1\tfrac{1}{4}) \\ &= 5.\end{aligned}$$

Now,

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan(1\tfrac{1}{4}) = \frac{AC}{4}$$
$$\Rightarrow AC = 4\tan(1\tfrac{1}{4}).$$

Next,

$$\begin{aligned}BC &= OC - OB \\ &= OC - 4\end{aligned}$$

$\cos = \frac{\text{adj}}{\text{hyp}}$:

$$\begin{aligned} &= \frac{4}{\cos(1\frac{1}{4})} - 4 \\ &= 8.685\,430\,775 \text{ (FCD)}. \end{aligned}$$

Finally,

$$\begin{aligned} \text{perimeter} &= 5 + 4\tan(1\tfrac{1}{4}) + 8.685\ldots \\ &= 25.723\,709\,47 \text{ (FCD)} \\ &= \underline{\underline{25.7 \text{ cm (3 sf)}}}. \end{aligned}$$

8. (a) Given that (1)

$$\log_9 x = a \log_3 x,$$

find a.

Solution

Well, we use our change of base:

$$\begin{aligned} \log_9 x &= \frac{\log_3 x}{\log_3 9} \\ &= \frac{\log_3 x}{\log_3 3^2} \\ &= \frac{\log_3 x}{2\log_3 3} \\ &= \underline{\underline{\tfrac{1}{2}\log_3 x}}. \end{aligned}$$

(b) Given that (1)

$$\log_{27} y = b \log_3 y,$$

find b.

Solution

$$\begin{aligned}\log_{27} y &= \frac{\log_3 y}{\log_3 27} \\ &= \frac{\log_3 y}{\log_3 3^3} \\ &= \frac{\log_3 y}{3 \log_3 3} \\ &= \underline{\underline{\tfrac{1}{3} \log_3 y}}.\end{aligned}$$

(c) Hence solve, for x and y, the simultaneous equations (4)

$$\begin{aligned}6 \log_9 x + 3 \log_{27} y &= 8 \\ \log_3 x + 2 \log_9 y &= 2.\end{aligned}$$

Solution

$$\begin{aligned}6 \log_9 x + 3 \log_{27} y &= 8 \quad (1) \\ \log_3 x + 2 \log_9 y &= 2 \quad (2)\end{aligned}$$

From (1),

$$\begin{aligned}6 \log_9 x + 3 \log_{27} y = 8 &\Rightarrow 6(\tfrac{1}{2} \log_3 x) + 3(\tfrac{1}{3} \log_3 y) = 8 \\ &\Rightarrow 3 \log_3 x + \log_3 y = 8 \quad (3).\end{aligned}$$

From (2),

$$\begin{aligned}\log_3 x + 2 \log_9 y = 2 &\Rightarrow \log_3 x + 2 \left(\frac{\log_3 y}{\log_3 9} \right) = 2 \\ &\Rightarrow \log_3 x + 2 \left(\frac{\log_3 y}{\log_3 3^2} \right) = 2 \\ &\Rightarrow \log_3 x + 2 \left(\frac{\log_3 y}{3 \log_3 3} \right) = 2 \\ &\Rightarrow \log_3 x + \log_3 y = 2 \quad (4).\end{aligned}$$

Do (3) − (4):

$$\begin{aligned}2 \log_3 x = 6 &\Rightarrow \log_3 x = 3 \\ &\Rightarrow x = 3^3 \\ &\Rightarrow \underline{\underline{x = 27}}.\end{aligned}$$

Substitute into (4):

$$\begin{aligned}\log_3 x + \log_3 y = 2 &\Rightarrow \log_3 27 + \log_3 y = 2 \\ &\Rightarrow \log_3 3^3 + \log_3 y = 2 \\ &\Rightarrow 3 + \log_3 y = 2 \\ &\Rightarrow \log_3 y = -1 \\ &\Rightarrow y = 3^{-1} \\ &\Rightarrow \underline{\underline{y = \tfrac{1}{3}}}.\end{aligned}$$

9. A curve is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos(2x - \tfrac{1}{2}\pi).$$

The curve passes through the point $(\frac{1}{2}\pi, 3)$.

(a) Find the equation of the curve. (4)

Solution

Integrate:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos(2x - \tfrac{1}{2}\pi) \Rightarrow y = \sin(2x - \tfrac{1}{2}\pi) + c,$$

where c is some constant. Now,

$$\begin{aligned}x = \tfrac{1}{2}\pi, y = 3 &\Rightarrow 3 = \sin(\pi - \tfrac{1}{2}\pi) + c \\ &\Rightarrow 3 = \sin(\tfrac{1}{2}\pi) + c \\ &\Rightarrow 3 = 1 + c \\ &\Rightarrow c = 2;\end{aligned}$$

hence, the equation of the curve is

$$\underline{\underline{y = \sin(2x - \tfrac{1}{2}\pi) + 2}}.$$

(b) Find the equation of the normal to the curve at the point where $x = \frac{3}{4}\pi$. (4)

Solution

Now,

$$x = \tfrac{3}{4}\pi \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -2$$

and

$$m_{\text{normal}} = \tfrac{1}{2}.$$

Next,

$$x = \tfrac{3}{4}\pi \Rightarrow y = 2$$

and the equation of the normal is

$$\begin{aligned} y - 2 = \tfrac{1}{2}(x - \tfrac{3}{4}\pi) &\Rightarrow y - 2 = \tfrac{1}{2}x - \tfrac{3}{8}\pi \\ &\Rightarrow \underline{\underline{y = \tfrac{1}{2}x + 2 - \tfrac{3}{8}\pi}}. \end{aligned}$$

10. In this question, **i** is a unit vector due east and **j** is a unit vector due north.

At 0900 hours a ship sails from the point P with position vector $(2\mathbf{i} + 3\mathbf{j})$ km relative to an origin O.

The ship sails north-east with a speed of $15\sqrt{2}$ km h^{-1}.

(a) Find, in terms of **i** and **j**, the velocity of the ship. (2)

Solution
Let x be the horizontal and vertical components of the ship's velocity. Then,

$$\begin{aligned} x^2 + x^2 = (15\sqrt{2})^2 &\Rightarrow 2x^2 = 450 \\ &\Rightarrow x^2 = 225 \\ &\Rightarrow x = 15; \end{aligned}$$

hence, the velocity of the ship is $\underline{\underline{(15\mathbf{i} + 15\mathbf{j})\ \text{km h}^{-1}}}$.

(b) Show that the ship will be at the point with position vector $(24.5\mathbf{i} + 25.5\mathbf{j})$ km at 1030 hours. (1)

Solution
Let R denote the new point. Then,

$$\begin{aligned} \overrightarrow{OR} &= (2\mathbf{i} + 3\mathbf{j}) + 1.5(15\mathbf{i} + 15\mathbf{j}) \\ &= (2\mathbf{i} + 3\mathbf{j}) + (22.5\mathbf{i} + 22.5\mathbf{j}) \\ &= \underline{\underline{(24.5\mathbf{i} + 25.5\mathbf{j})\ \text{km}}}, \end{aligned}$$

as required.

(c) Find, in terms of $\mathbf{i}$, $\mathbf{j}$, and t, the position of the ship t hours after leaving P. (2)

Solution

$$\underline{\underline{(2\mathbf{i} + 3\mathbf{j}) + t(15\mathbf{i} + 15\mathbf{j}) \text{ km}}}.$$

At the same time as the ship leaves P, a submarine leaves the point Q with position vector $(47\mathbf{i} - 27\mathbf{j})$ km.

The submarine proceeds with a speed of 25 km h^{-1} due north to meet the ship.

(d) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the velocity of the ship relative to the submarine. (2)

Solution

$$(15\mathbf{i} + 15\mathbf{j}) - 25\mathbf{j} = \underline{\underline{(15\mathbf{i} - 10\mathbf{j}) \text{ km h}^{-1}}}.$$

(e) Find the position vector of the point where the submarine meets the ship. (2)

Solution

Now,

$$\begin{aligned} & (2\mathbf{i} + 3\mathbf{j}) + t(15\mathbf{i} + 15\mathbf{j}) = (47\mathbf{i} - 27\mathbf{j}) + 25t\mathbf{j} \\ \Rightarrow\ & (2 + 15t)\mathbf{i} + (3 + 15t)\mathbf{j} = 47\mathbf{i} + (25t - 27)\mathbf{j} \end{aligned}$$

Next, the **i**-component:

$$\begin{aligned} 2 + 15t = 47 &\Rightarrow 15t = 45 \\ &\Rightarrow t = 3 \end{aligned}$$

and, finally, the submarine meets the ship at

$$\underline{\underline{(47\mathbf{i} + 48\mathbf{j})}}.$$

11. Solve the equation

(a) (3)

$$3 \sin x + 5 \cos x = 0,$$

for $0^\circ < x < 360^\circ$,

Solution

$$\begin{aligned}
3\sin x + 5\cos x = 0 &\Rightarrow 3\sin x = -5\cos x \\
&\Rightarrow \tan x = -\tfrac{5}{3} \\
&\Rightarrow x = 120.963\,756\,5,\ 300.963\,756\,5 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{x = 121,\ 301 \text{ (3 sf)}}}.
\end{aligned}$$

(b) (5)

$$3\tan^2 y - \sec y - 1 = 0,$$

for $0^\circ < y < 360^\circ$,

Solution

$$\begin{aligned}
3\tan^2 y - \sec y - 1 = 0 &\Rightarrow 3(\sec^2 y - 1) - \sec y - 1 = 0 \\
&\Rightarrow 3\sec^2 y - 3 - \sec y - 1 = 0 \\
&\Rightarrow 3\sec^2 y - \sec y - 4 = 0
\end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -1 \\ \text{multiply to:} & (+3)\times(-4) = -12 \end{array}\right\} = -4,\ +3$$

e.g.,

$$\begin{aligned}
&\Rightarrow 3\sec^2 y - 4\sec y + 3\sec y - 4 = 0 \\
&\Rightarrow \sec y(3\sec y - 4) + (3\sec y - 4) = 0 \\
&\Rightarrow (3\sec y - 4)(\sec y + 1) = 0 \\
&\Rightarrow 3\sec y - 4 = 0 \text{ or } \sec y + 1 = 0 \\
&\Rightarrow \sec y = \tfrac{4}{3} \text{ or } \sec y = -1 \\
&\Rightarrow \cos y = \tfrac{3}{4} \text{ or } \cos y = -1.
\end{aligned}$$

$\underline{\cos y = \tfrac{3}{4}:}$

$$\begin{aligned}
\cos y = \tfrac{3}{4} &\Rightarrow y = 41.409\,622\,11,\ 318.590\,377\,9 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{y = 41.4,\ 319 \text{ (3 sf)}}}.
\end{aligned}$$

$\underline{\cos y = 1:}$

$$\cos y = 1 \Rightarrow \underline{\underline{y = 180}}.$$

(c) (4)

$$\sin(2z - 0.6) = 0.8,$$

for $0 < z < 3$ radians.

Solution

Now,

$$\begin{aligned} 0 < z < 3 &\Rightarrow 0 < 2z < 6 \\ &\Rightarrow -0.6 < 2z - 0.6 < 5.4 \end{aligned}$$

and

$$\begin{aligned} \sin(2z - 0.6) = 0.8 &\Rightarrow 2z - 0.6 = 0.927\,295\,218,\ 2.214\,297\,436 \text{ (FCD)} \\ &\Rightarrow 2z = 1.527\,295\,218,\ 2.814\,297\,436 \text{ (FCD)} \\ &\Rightarrow z = 0.763\,647\,609,\ 1.407\,148\,718 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{z = 0.764,\ 1.41 \text{ (3 sf)}}}. \end{aligned}$$

EITHER

12. A curve has equation

$$y = (x^2 - 3)e^{-x}.$$

(a) Find the coordinates of the points of intersection of the curve with the x-axis. (2)

Solution

Because $e^{-x} > 0$,

$$\begin{aligned} x^2 - 3 = 0 &\Rightarrow x^2 = 3 \\ &\Rightarrow x = \pm\sqrt{3}. \end{aligned}$$

Hence, the intersections are $\underline{\underline{(\pm\sqrt{3}, 0)}}$.

(b) Find the coordinates of the stationary points of the curve. (5)

Solution

Product rule:

$$\begin{aligned} u = x^2 - 3 &\Rightarrow \frac{du}{dx} = 2x \\ v = e^{-x} &\Rightarrow \frac{dv}{dx} = -e^{-x} \end{aligned}$$

and

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= (x^2-3)(-\mathrm{e}^{-x}) + (2x)(\mathrm{e}^{-x}) \\ &= \mathrm{e}^{-x}[-(x^2-3)+2x] \\ &= \mathrm{e}^{-x}(3+2x-x^2).\end{aligned}$$

Because $\mathrm{e}^{-x} > 0$,

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = 0 &\Rightarrow 3+2x-x^2 = 0 \\ &\Rightarrow x^2-2x-3=0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -2 \\ \text{multiply to:} & -3\end{array}\right\} -3,\ +1$$

$$\begin{aligned}&\Rightarrow (x-3)(x+1)=0 \\ &\Rightarrow x=3 \text{ or } x=-1 \\ &\Rightarrow y = 6\mathrm{e}^{-3} \text{ or } y=-2\mathrm{e};\end{aligned}$$

hence, the stationary points of the curve are $\underline{\underline{(3, 6\mathrm{e}^{-3})}}$ and $\underline{\underline{(-1, 2\mathrm{e})}}$.

(c) Determine the nature of these stationary points. (3)

Solution

Product rule:

$$\begin{aligned}u = 3+2x-x^2 &\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2-2x \\ v = \mathrm{e}^{-x} &\Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = -\mathrm{e}^{-x}\end{aligned}$$

and

$$\begin{aligned}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} &= (3+2x-x^2)(-\mathrm{e}^{-x}) + (2-2x)(\mathrm{e}^{-x}) \\ &= \mathrm{e}^{-x}\left[(2-2x)-(3+2x-x^2)\right] \\ &= \mathrm{e}^{-x}(2-2x-3-2x+x^2) \\ &= \mathrm{e}^{-x}(x^2-4x-1)\end{aligned}$$

Next,

$$x = -1 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 13.591\ldots \text{ so it is a } \underline{\underline{\text{maximum point}}}$$
$$x = 3 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -0.199\ldots \text{ so it is a } \underline{\underline{\text{minimum point}}}.$$

OR

13. A particle moves in a straight line such that its displacement, s m, from a fixed point O at a time t s, is given by

$$s = \ln(t+1), \text{ for } 0 \leqslant t \leqslant 3,$$
$$s = \tfrac{1}{2}\ln(t-2) - \ln(t+1) + \ln 16, \text{ for } t > 3.$$

Find

(a) the initial velocity of the particle, (2)

Solution

$$s = \ln(t+1) \Rightarrow v = \frac{1}{t+1}$$

and

$$t = 0 \Rightarrow \underline{\underline{v = 1 \text{ ms}^{-1}}}.$$

(b) the velocity of the particle when $t = 4$, (2)

Solution

$$s = \tfrac{1}{2}\ln(t-2) - \ln(t+1) + \ln 16 \Rightarrow v = \frac{1}{2(t-2)} - \frac{1}{t+1}$$

and

$$t = 0 \Rightarrow \underline{\underline{v = \tfrac{1}{20} \text{ ms}^{-1}}}.$$

(c) the acceleration of the particle when $t = 4$, (2)

Solution

$$v = \frac{1}{2(t-2)} - \frac{1}{t+1} \Rightarrow v = \tfrac{1}{2}(t-2)^{-1} - (t+1)^{-1}$$
$$\Rightarrow a = -\tfrac{1}{2}(t-2)^{-2} + (t+1)^{-2}$$

and

$$t = 4 \Rightarrow \underline{\underline{a = -0.085 \text{ ms}^{-2}}}.$$

(d) the value of t when the particle is instantaneously at rest, (2)

Solution

$$\begin{aligned} v = 0 &\Rightarrow \frac{1}{2(t-2)} - \frac{1}{t+1} = 0 \\ &\Rightarrow \frac{1}{2(t-2)} = \frac{1}{t+1} \\ &\Rightarrow 2(t-2) = t+1 \\ &\Rightarrow 2t - 4 = t + 1 \\ &\Rightarrow \underline{\underline{t = 5 \text{ seconds}}}. \end{aligned}$$

(e) the distance travelled by the particle in the 4th second. (2)

Solution

Well,

$$\begin{aligned} & \text{distance travelled} \\ = \;& s(4) - s(3) \\ = \;& \left[\tfrac{1}{2}\ln(4-2) - \ln(4+1) + \ln 16\right] - \left[\tfrac{1}{2}\ln(3-2) - \ln(3+1) + \ln 16\right] \\ = \;& \left[\tfrac{1}{2}\ln 2 - \ln 5 + \ln 16\right] - \left[\tfrac{1}{2}\ln 1 - \ln 4 + \ln 16\right] \\ = \;& \underline{\underline{\tfrac{1}{2}\ln 2 - \ln 5 + \ln 4}} \\ = \;& 0.123\,430\,039 \text{ (FCD)} \\ = \;& \underline{\underline{0.123 \text{ (3 sf)}}}. \end{aligned}$$