

Dr Oliver Mathematics
GCSE Mathematics
2008 November Paper 4H: Calculator
1 hour 45 minutes

The total number of marks available is 100.

You must write down all the stages in your working.

1. Here are the ingredients for making cheese pie for 6 people.

Cheese pie for 6 people

180 g flour
240 g cheese
80 g butter
4 eggs
160 ml milk

Bill makes a cheese pie for 3 people.

- (a) Work out how much flour he needs.

(2)

Solution

$$\frac{1}{2} \times 180 = \underline{\underline{90 \text{ g}}}.$$

Jenny makes a cheese pie for 15 people.

- (b) Work out how much milk she needs.

(2)

Solution

$$\frac{5}{2} \times 160 = \underline{\underline{400 \text{ ml}}}.$$

2. Use a calculator to work out

$$\sqrt{\frac{21.6 \times 15.8}{3.8}}.$$

- (a) Write down all the figures on your calculator display.

(2)

Solution

$$\begin{aligned}\sqrt{\frac{21.6 \times 15.8}{3.8}} &= \sqrt{\frac{341.28}{3.8}} \\ &= \sqrt{89\frac{77}{95}} \\ &= \underline{\underline{9.476\ 841\ 579}} \text{ (FCD)}.\end{aligned}$$

- (b) Give your answer to part (a) correct to 3 significant figures. (1)

Solution

The answer is 9.48 (3 sf).

3. The cost of a radio is the list price plus VAT at $17\frac{1}{2}\%$. (3)
The list price of a radio is £240.
Work out the cost of the radio.

Solution

$$240 \times 1.175 = \underline{\underline{£282}}.$$

4. (a) Expand $4(x - 3)$. (1)

Solution

$$4(x - 3) = \underline{\underline{4x - 12}}.$$

- (b) Solve $4t + 1 = 19$. (2)

Solution

$$\begin{aligned}4t + 1 = 19 &\Rightarrow 4t = 18 \\ &\Rightarrow t = \underline{\underline{4\frac{1}{2}}}.\end{aligned}$$

5. The n th term of a sequence is $n^2 + 4$. (2)
Alex says, "The n th term of the sequence is always a prime number when n is an odd number."

Alex is **wrong**.

Give an example to show that Alex is wrong.

Solution

E.g., $n = 9 : 9^2 + 4 = 85 = 5 \times 17$.

6. A circle has a radius of 6 cm.
A square has a side of length 12 cm.

(4)

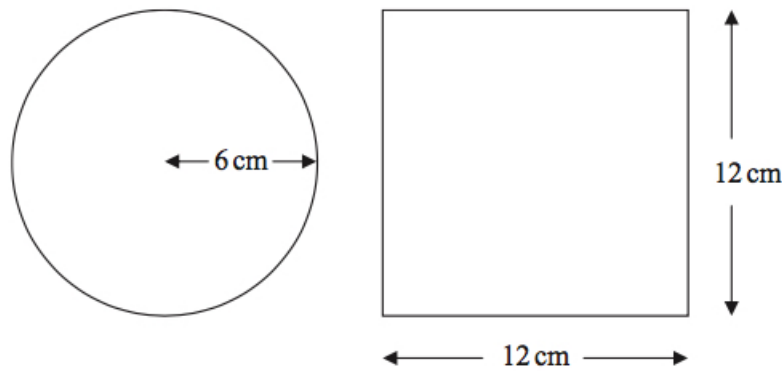


Diagram **NOT**
accurately drawn

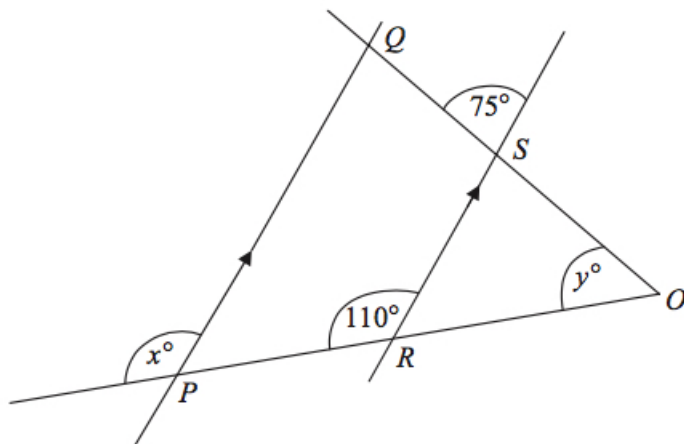
Work out the difference between the area of the circle and the area of the square.
Give your answer correct to one decimal place.

Solution

$$\begin{aligned} \text{Difference} &= 12^2 - \pi \times 6^2 \\ &= 144 - 36\pi \\ &= 30.902\,664\,47 \text{ (FCD)} \\ &= \underline{\underline{30.9 \text{ cm}^2}} \text{ (1 dp)}. \end{aligned}$$

7. PQ is parallel to RS .

Diagram **NOT**
accurately drawn



OSQ and ORP are straight lines.

- (a) (i) Write down the value of x .

(2)

Solution

110° .

- (ii) Give a reason for your answer.

Solution

Corresponding angles.

- (b) Work out the value of y .

(2)

Solution

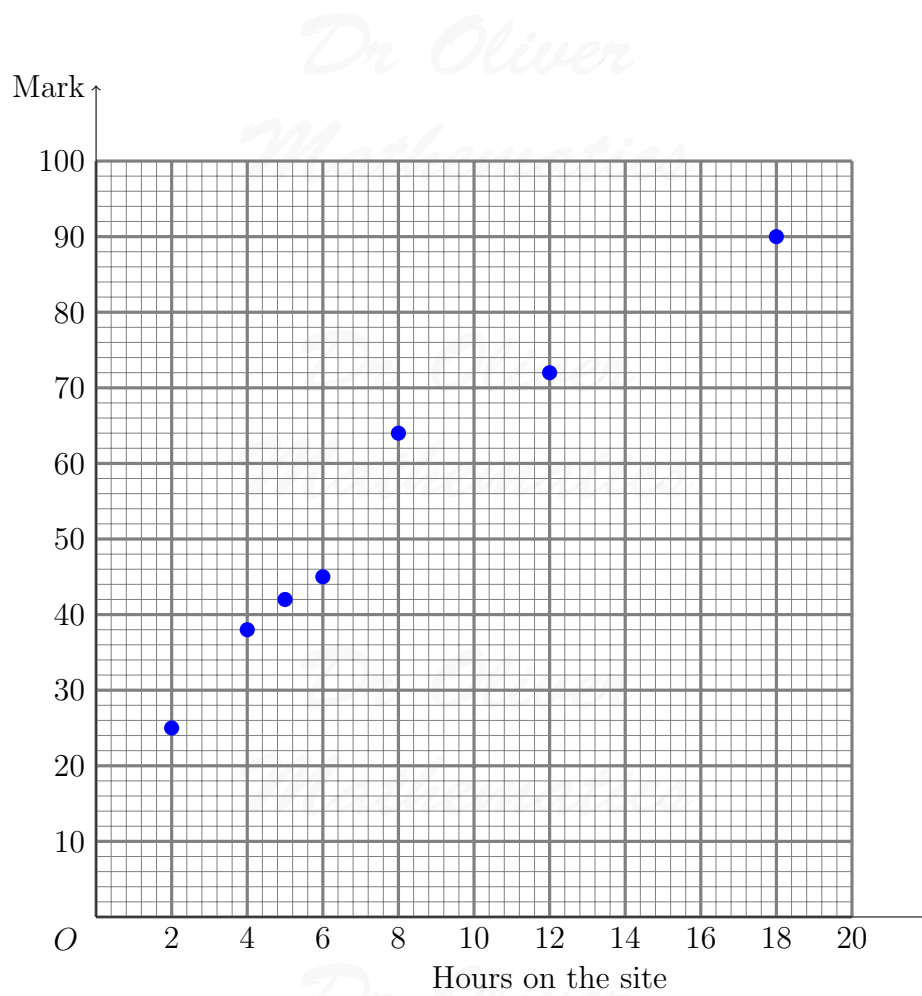
$\angle ORS = 180 - 110 = 70^\circ$ (supplementary angles), $\angle OSR = 75^\circ$ (alternate angles), and

$$\angle ROS = 180 - 70 - 75 = \underline{\underline{35^\circ}}.$$

8. Some students revised for a mathematics exam.

They used an internet revision site.

The scatter graph shows the times seven students spent on the internet revision site and the marks the students got in the mathematics exam.



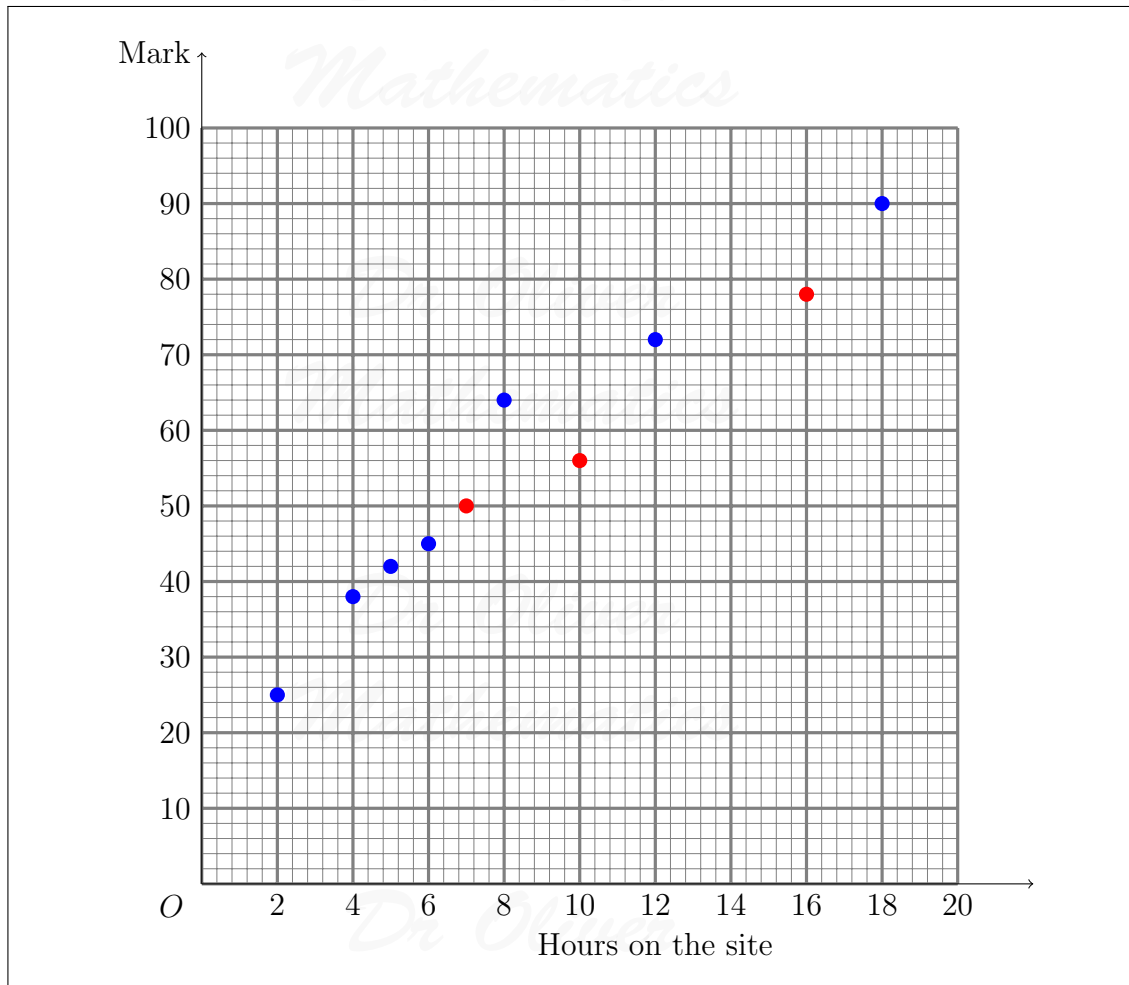
Here is the information for 3 more students.

Hours on the site	7	10	16
Mark	50	56	78

(a) Plot this information on the scatter graph.

(1)

Solution

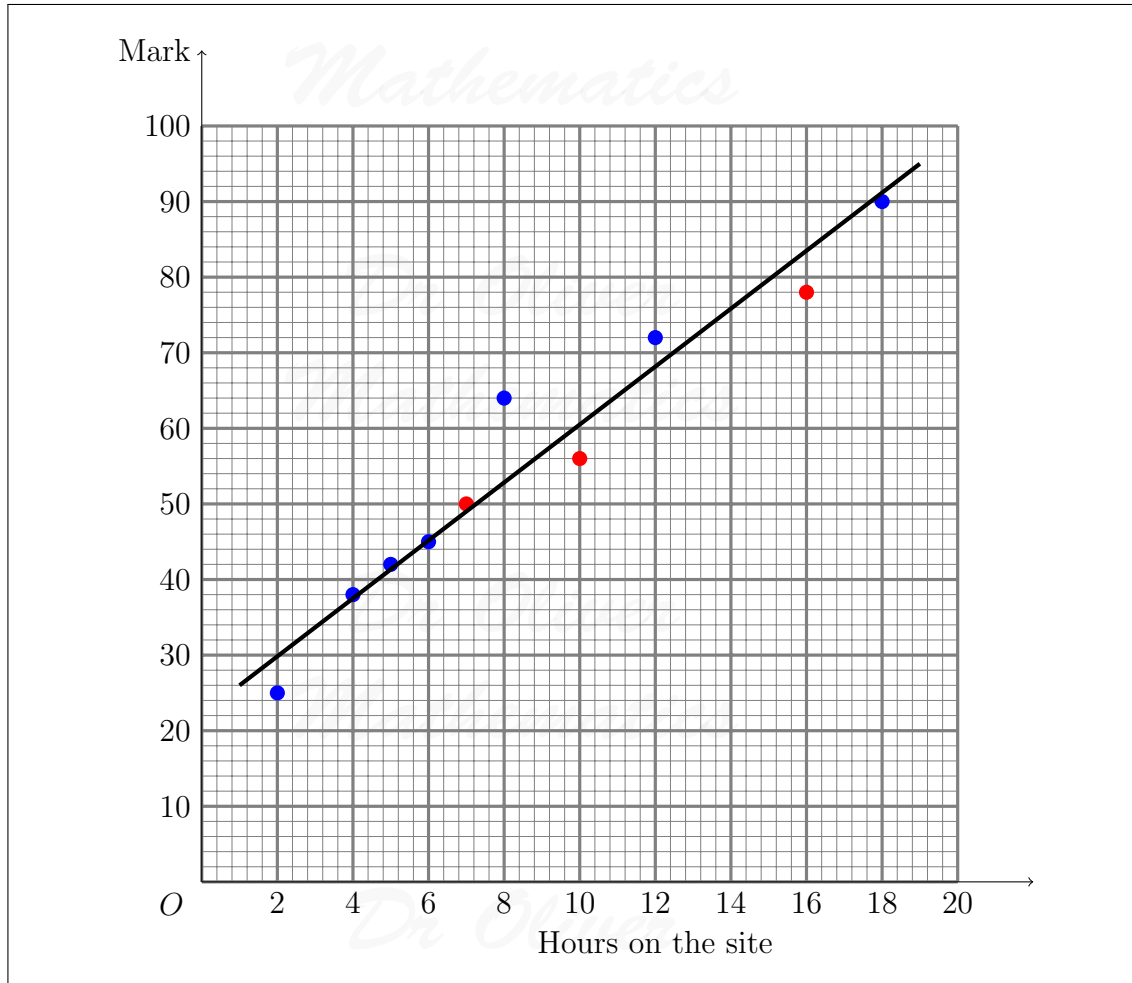


(b) What type of correlation does this scatter graph show? (1)

Solution
Positive correlation, e.g., the more hours they put in on the site, the higher the mark is.

(c) Draw a line of best fit on the scatter graph. (1)

Solution

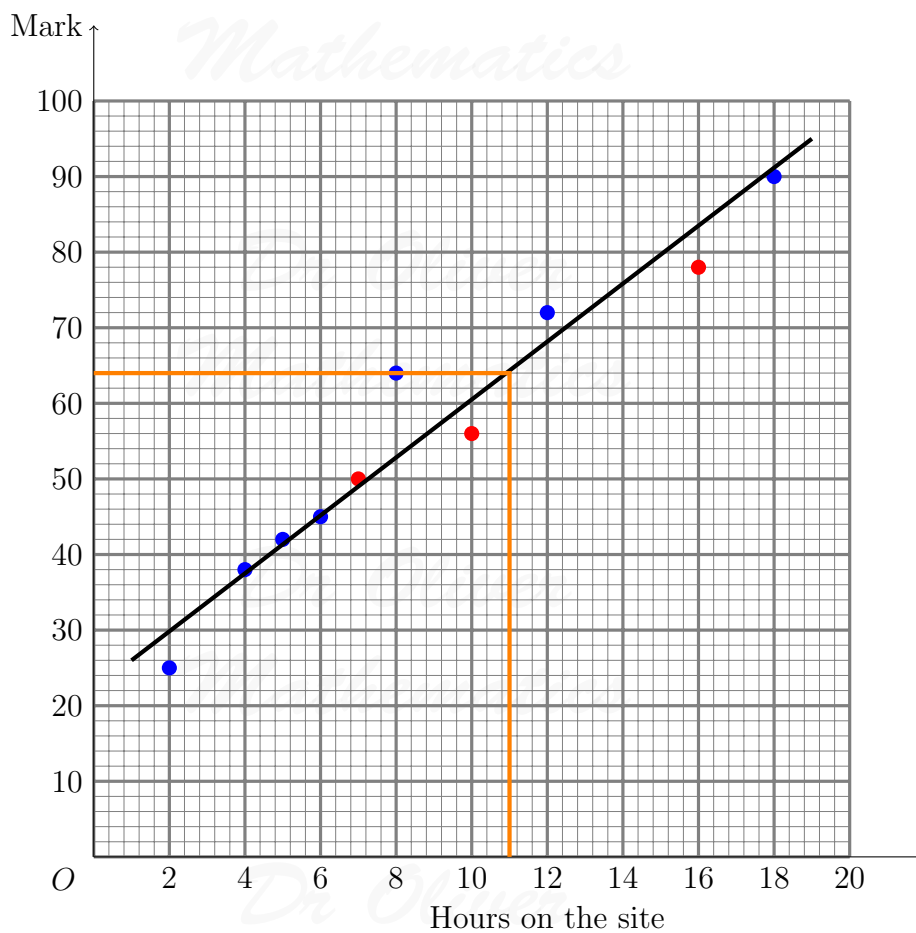


A student spent 11 hours on the internet revision site.

(d) Use the line of best fit to estimate this student's mathematics exam mark.

(1)

Solution



Correct read-off: approximately 64 marks.

9. Jack invests £3000 for 2 years at 4% per annum compound interest.
Work out the value of the investment at the end of 2 years.

(3)

Solution

$$3000 \times 1.04^2 = \underline{\underline{\pounds 3244.80}}$$

10. Jason collected some information about the heights of 19 plants.
This information is shown in the stem and leaf diagram.

(2)

1	1	2	3	3		
2	3	3	5	9	9	
3	0	2	2	6	6	7
4	1	1	4	8		

Key 4|8 means 48 mm

Find the median.

Solution

The median is the

$$\frac{19 + 1}{2} = 10\text{th value}$$

and is it 30 mm.

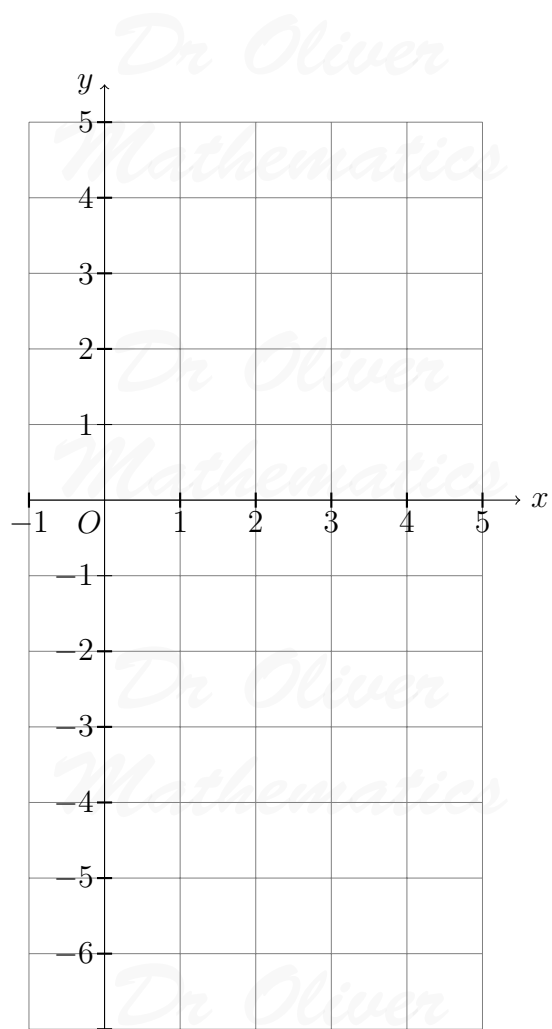
11. (a) Complete the table of values for $y = x^2 - 4x - 2$. (2)

x	-1	0	1	2	3	4	5
y		-2	-5			-2	3

Solution

x	-1	0	1	2	3	4	5
y	<u>3</u>	-2	-5	<u>-6</u>	<u>-5</u>	-2	3

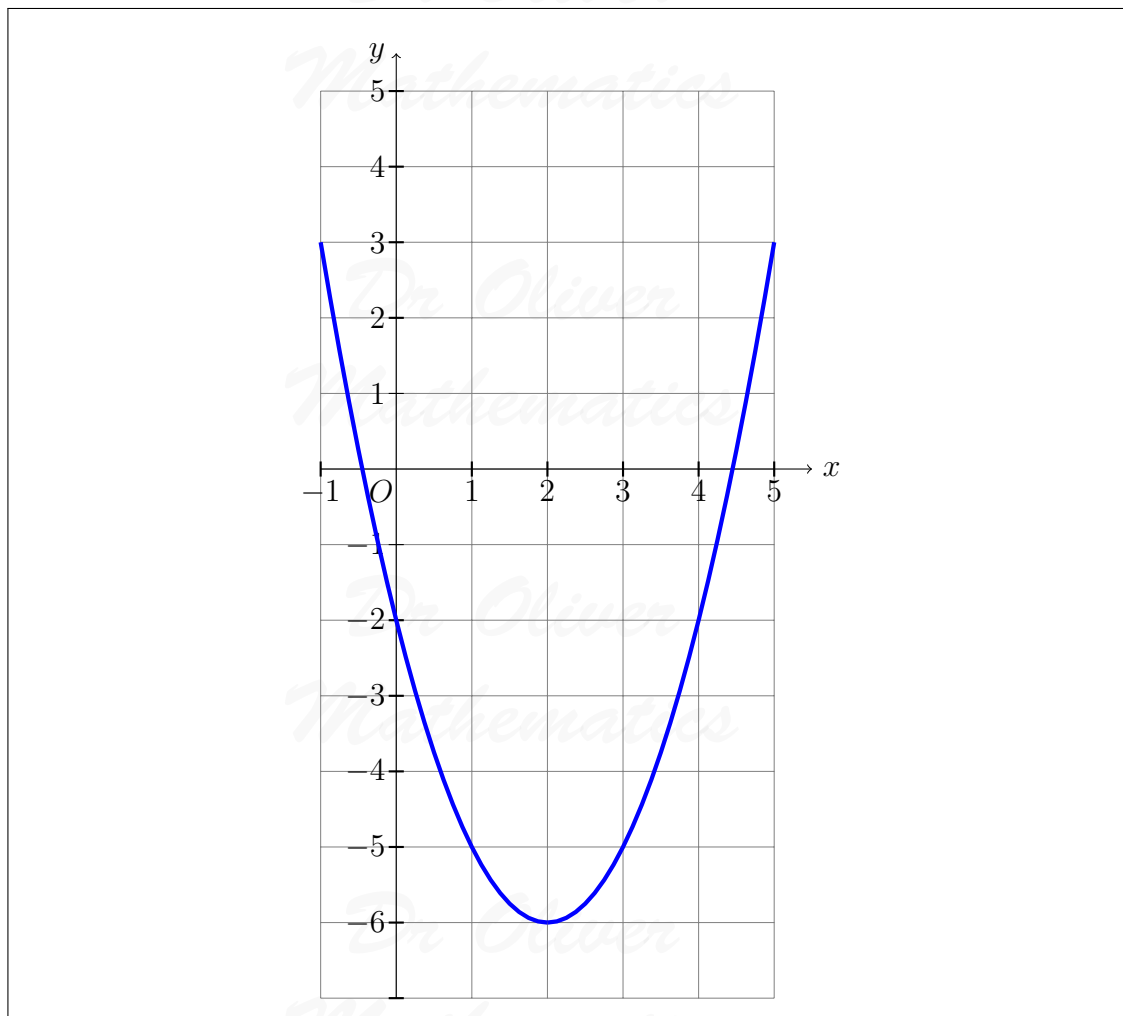
- (b) On the grid, draw the graph of $y = x^2 - 4x - 2$. (2)



Solution

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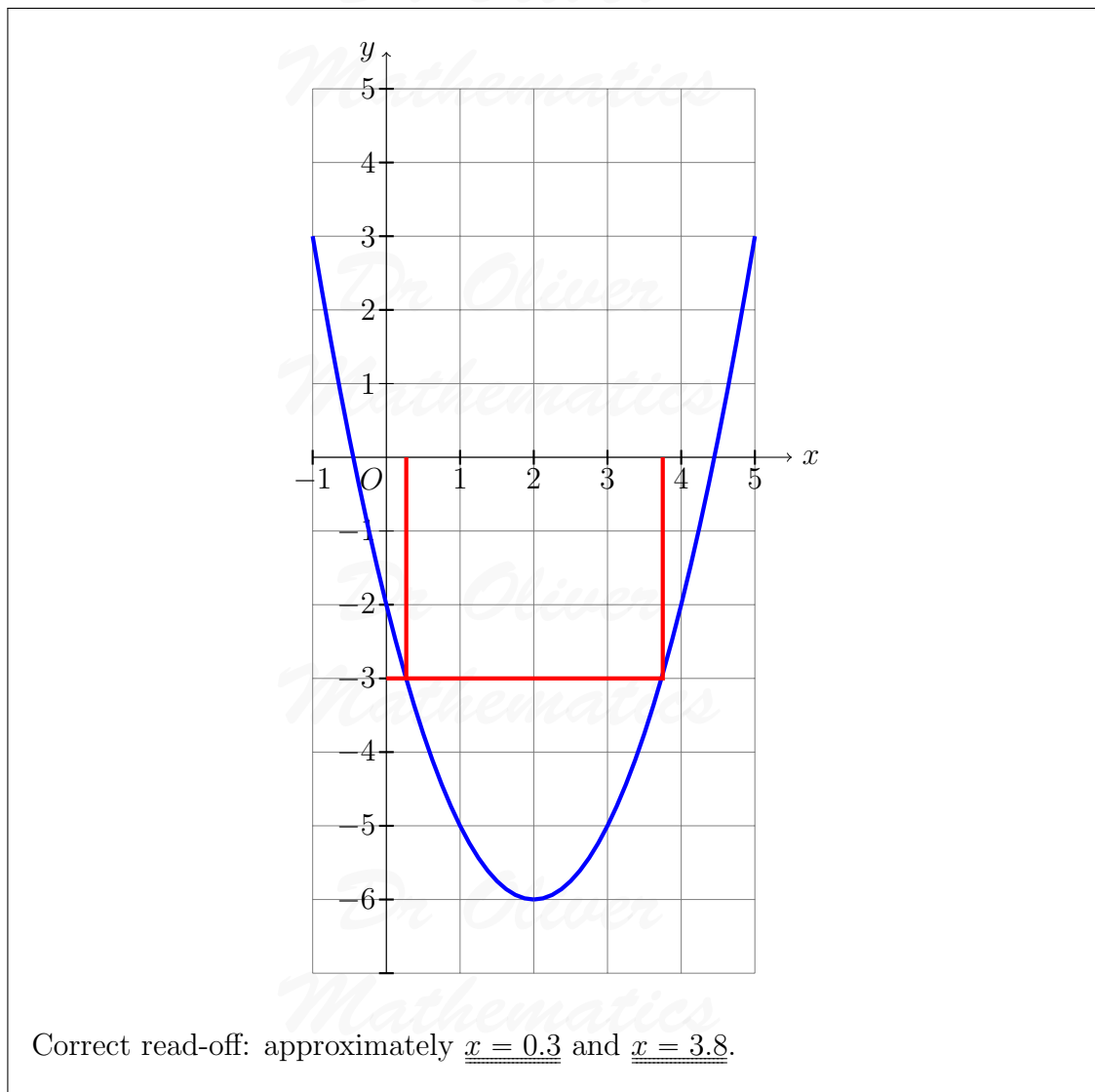
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(c) Use your graph to estimate the values of x when $y = -3$.

(2)

Solution

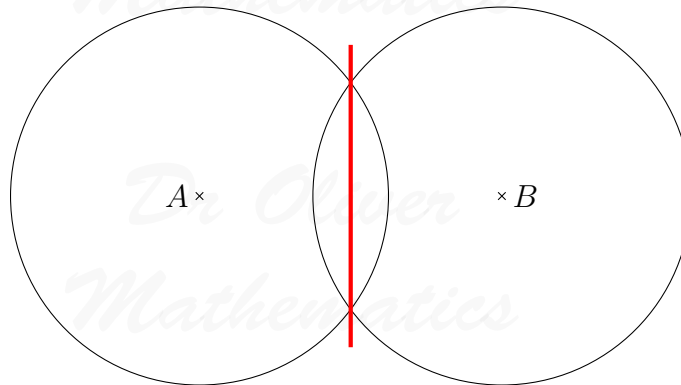


12. (a) Draw the locus of all points which are equidistant from the points A and B . (2)

$A \times$

$\times B$

Solution



(b) Draw the locus of all points that are exactly 3 cm from the line PQ . (2)



Solution



13. Find the Lowest Common Multiple (LCM) of 24 and 36. (2)

Solution

$$\begin{array}{r|l} & 24 \\ 2 & 12 \\ 2 & 6 \\ 2 & 3 \\ 3 & 1 \end{array}$$

So

$$28 = 2^3 \times 3.$$

$$\begin{array}{r|l}
 & 36 \\
 2 & 18 \\
 \hline
 2 & 9 \\
 \hline
 3 & 3 \\
 \hline
 3 & 1 \\
 \hline
 \end{array}$$

So

$$36 = 2^2 \times 3^2.$$

Hence, the LCM is

$$2^3 \times 3^2 = \underline{\underline{72}}.$$

14. (a) Expand and simplify $3(x + 4) + 5(2x + 1)$. (2)

Solution

$$\begin{aligned}
 3(x + 4) + 5(2x + 1) &= 3x + 12 + 10x + 5 \\
 &= \underline{\underline{13x + 17}}.
 \end{aligned}$$

- (b) Simplify $t^4 \times t^6$. (1)

Solution

$$t^4 \times t^6 = \underline{\underline{t^{10}}}.$$

- (c) Simplify $p^8 \div p^5$. (1)

Solution

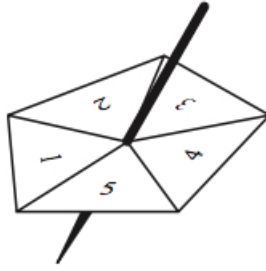
$$p^8 \div p^5 = \underline{\underline{p^3}}.$$

- (d) Simplify $(x^4)^3$. (1)

Solution

$$(x^4)^3 = \underline{\underline{x^{12}}}.$$

15. Here is a 5-sided spinner. (2)



The sides of the spinner are labelled 1, 2, 3, 4 and 5.

The spinner is biased.

The probability that the spinner will land on each of the numbers 1, 2, 3, and 4 is given in the table.

Number	1	2	3	4	5
Probability	0.15	0.05	0.2	0.25	x

Work out the value of x .

Solution

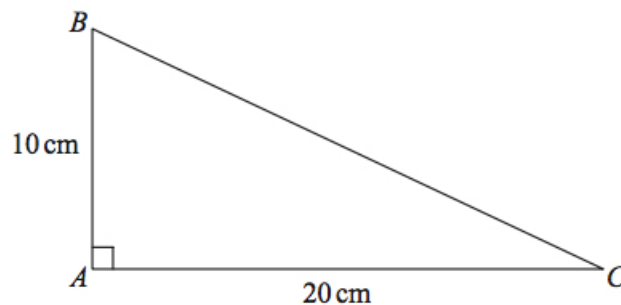
$$0.15 + 0.05 + 0.2 + 0.25 + x = 1 \Rightarrow 0.65 + x = 1$$

$$\Rightarrow \underline{\underline{x = 0.35}}$$

16. In triangle ABC , $AB = 10$ cm, $AC = 20$ cm, and angle $BAC = 90^\circ$.

(4)

**Diagram NOT
accurately drawn**



Work out the length of BC .

Give your answer correct to 3 significant figures.

You must state the units in your answer.

Solution

$$\begin{aligned} BC &= \sqrt{10^2 + 20^2} \\ &= 22.360\ 679\ 77 \text{ (FCD)} \\ &= \underline{\underline{22.4 \text{ cm (3 sf)}}}. \end{aligned}$$

17. Majid carried out a survey of the number of school dinners 32 students had in one week. The table shows this information. (3)

Number of school dinners	Frequency
0	0
1	8
2	12
3	6
4	4
5	2

Calculate the mean.

Solution

Number of school dinners	Frequency	Freq \times number
0	0	0
1	8	8
2	12	24
3	6	18
4	4	16
5	2	10
Total	32	76

So the mean is

$$\frac{76}{32} = \underline{\underline{2\frac{3}{8}}}.$$

18. The value of a car depreciates by 35% each year. (3)
At the end of 2007, the value of the car was £5 460.
Work out the value of the car at the end of 2006.

Solution

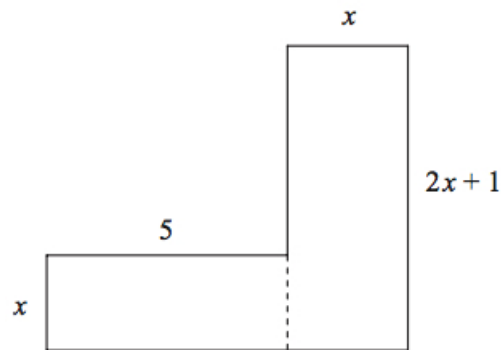
$$1 - 0.35 = 0.65$$

and so the value of the car at the end of 2006 is

$$\frac{5\,460}{0.65} = \underline{\underline{\pounds 8\,400.}}$$

19. The diagram below shows a 6-sided shape.
All the corners are right angles.
All the measurements are given in centimetres.

**Diagram NOT
accurately drawn**



The area of the shape is 95 cm^2 .

- (a) Show that $2x^2 + 6x - 95 = 0$. (3)

Solution

$$\begin{aligned} x(2x + 1) + 5x &= 95 \Rightarrow 2x^2 + 6x = 95 \\ &\Rightarrow \underline{\underline{2x^2 + 6x - 95 = 0}}, \end{aligned}$$

as required.

- (b) Solve the equation $2x^2 + 6x - 95 = 0$. (3)
Give your solutions correct to 3 significant figures.

Solution

$a = 2$, $b = 6$, and $c = -95$:

$$\begin{aligned}x &= \frac{-6 \pm \sqrt{6^2 - 4 \times 2 \times (-95)}}{2 \times 2} \\&= \frac{-6 \pm \sqrt{796}}{4} \\&= -8.553\ 367\ 99 \text{ or } 5.553\ 367\ 99 \text{ (FCD)} \\&= \underline{\underline{-8.55 \text{ or } 5.55 \text{ (3 sf)}}}.\end{aligned}$$

20. The n th even number is $2n$. (1)
The next even number after $2n$ is $2n + 2$.
(a) Explain why. (1)

Solution

Add on 2.

- (b) Write down an expression, in terms of n , for the next even number after $2n + 2$. (1)

Solution

$2n + 4$.

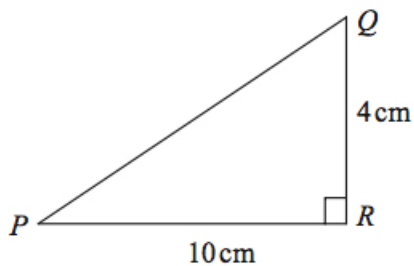
- (c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6. (3)

Solution

$$\begin{aligned}2n + (2n + 2) + (2n + 4) &= 6n + 6 \\&= \underline{\underline{6(n + 1)}}.\end{aligned}$$

21. PQR is a right-angled triangle. (3)

Diagram **NOT**
accurately drawn



$$QR = 4 \text{ cm.}$$

$$PR = 10 \text{ cm.}$$

Work out the size of angle RPQ .

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} \tan \angle RPQ &= \frac{4}{10} \Rightarrow \angle RPQ = 21.801\,409\,49 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\angle RPQ = 21.8^\circ \text{ (3 sf)}}}. \end{aligned}$$

22. D is proportional to S^2 .

$$D = 900 \text{ when } S = 20.$$

Calculate the value of D when $S = 25$.

(4)

Solution

$D = kS^2$ for some constant k . Now,

$$900 = k \times 20^2 \Rightarrow k = 2.25$$

and so

$$D = 2.25S^2.$$

Finally,

$$D = 2.25 \times 25^2 = \underline{\underline{1\,406.25}}.$$

23. A ball is thrown vertically upwards with a speed V metres per second.
The height, H metres, to which it rises is given by

$$H = \frac{V^2}{2g},$$

(3)

where $g \text{ m/s}^2$ is the acceleration due to gravity.

$V = 24.4$ correct to 3 significant figures.

$g = 9.8$ correct to 2 significant figures.

- (a) Write down the lower bound of g .

Solution

9.75.

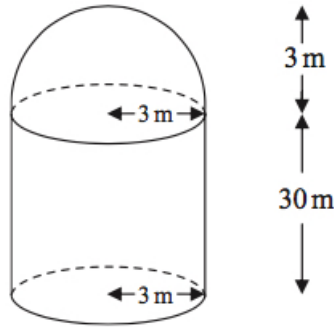
- (b) Calculate the upper bound of H .

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} H &= \frac{24.45^2}{2 \times 9.75} \\ &= 30.656\ 538\ 46 \text{ (FCD)} \\ &= \underline{\underline{30.7 \text{ m (3 sf)}}}. \end{aligned}$$

24. The diagram shows a storage tank.



The storage tank consists of a hemisphere on top of a cylinder.

The height of the cylinder is 30 metres.

The radius of the cylinder is 3 metres.

The radius of the hemisphere is 3 metres.

- (a) Calculate the total volume of the storage tank.

Give your answer correct to 3 significant figures.

(3)

Solution

$$\begin{aligned}\text{Volume} &= \pi r^2 h + \frac{2}{3} \pi r^3 \\ &= (\pi \times 3^2 \times 30) + \left(\frac{2}{3} \times \pi \times 3^3\right) \\ &= 288\pi \\ &= 904.778\,684\,2 \text{ (FCD)} \\ &= \underline{\underline{905 \text{ m}^3 \text{ (3 sf)}}}.\end{aligned}$$

A sphere has a volume of 500 m^3 .

(b) Calculate the radius of the sphere.

(3)

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}500 &= \frac{4}{3} \pi r^3 \Rightarrow r^3 = \frac{375}{\pi} \\ \Rightarrow r &= \sqrt[3]{\frac{375}{\pi}} \\ \Rightarrow r &= 4.923\,725\,109 \text{ (FCD)} \\ \Rightarrow r &= \underline{\underline{4.92 \text{ m (3 sf)}}}.\end{aligned}$$

25. The table gives information about the numbers of students in the two years of a college course.

(3)

	Male	Female
First year	399	602
Second year	252	198

Anna wants to interview some of these students.

She takes a random sample of 70 students stratified by year and by gender.

Work out the number of students in the sample who are male and in the first year.

Solution

The total number of students is

$$399 + 602 + 252 + 198 = 1451$$

and so the number of students in the sample who are male and in the first year is

$$70 \times \frac{399}{1451} = 19.28\dots;$$

hence, 19 students.

26. Here is a diagram.

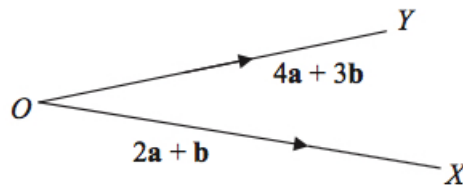


Diagram **NOT**
accurately drawn

$$\overrightarrow{OX} = 2\mathbf{a} + \mathbf{b}.$$

$$\overrightarrow{OY} = 4\mathbf{a} + 3\mathbf{b}.$$

- (a) Express the vector \overrightarrow{XY} in terms of \mathbf{a} and \mathbf{b} .
Give your answer in its simplest form.

(2)

Solution

$$\begin{aligned}\overrightarrow{XY} &= \overrightarrow{XO} + \overrightarrow{OY} \\ &= -(2\mathbf{a} + \mathbf{b}) + (4\mathbf{a} + 3\mathbf{b}) \\ &= \underline{2\mathbf{a} + 2\mathbf{b}}.\end{aligned}$$

Here is another diagram.

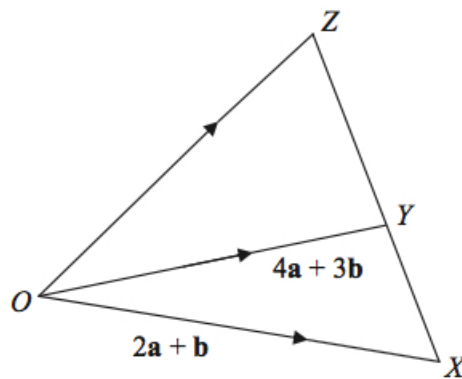


Diagram NOT accurately drawn

XYZ is a straight line.

$XY : YZ = 2 : 3$.

- (b) Express the vector \overrightarrow{OZ} in terms of \mathbf{a} and \mathbf{b} .
Give your answer in its simplest form.

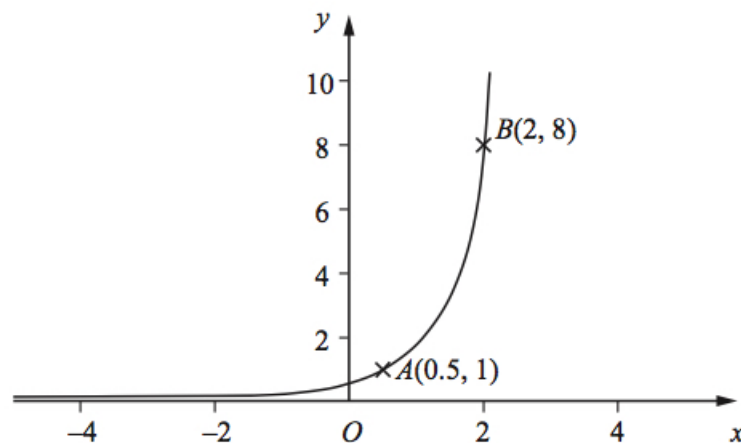
(3)

Solution

$$\begin{aligned}\overrightarrow{OZ} &= \overrightarrow{OX} + \overrightarrow{XZ} \\ &= \overrightarrow{OX} + \frac{5}{2}\overrightarrow{XY} \\ &= (2\mathbf{a} + \mathbf{b}) + \frac{5}{2}(2\mathbf{a} + \mathbf{b}) \\ &= \underline{7\mathbf{a} + 6\mathbf{b}}.\end{aligned}$$

27. The diagram shows a sketch of the graph $y = ab^x$.

(4)



The curve passes through the points $A(0.5, 1)$ and $B(2, 8)$.
The point $C(-0.5, k)$ lies on the curve.
Find the value of k .

Solution

$A(0.5, 1)$:

$$1 = ab^{0.5} \quad (1)$$

$B(2, 8)$:

$$8 = ab^2 \quad (2)$$

Divide:

$$\frac{ab^2}{ab^{0.5}} = \frac{8}{1} \Rightarrow b^{1.5} = 8$$

$$\Rightarrow b = 8^{\frac{2}{3}}$$

$$\Rightarrow b = 4$$

$$\Rightarrow a = \frac{1}{2}$$

$C(-0.5, k)$:

$$k = \frac{1}{2} \times 4^{-0.5} = \underline{\underline{\frac{1}{4}}}$$