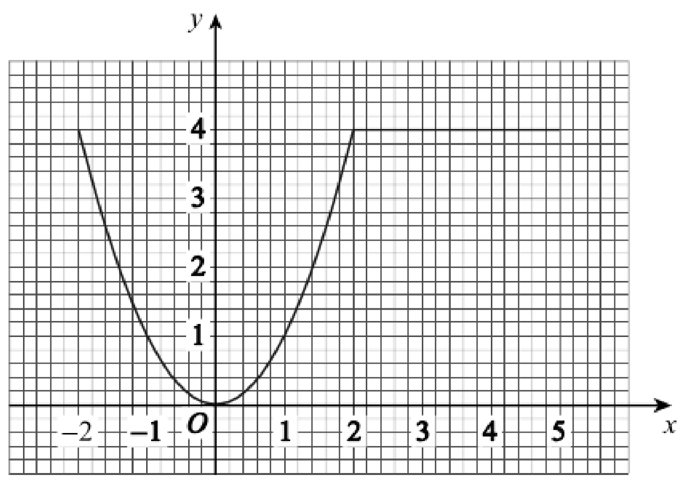
Dr Oliver Mathematics
AQA Further Maths Level 2
June 2014 Paper 2
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.

1. The graph of $y = \mathrm{f}(x)$ for the full domain is shown. (3)
   The graph consists of a quadratic curve and a straight line.



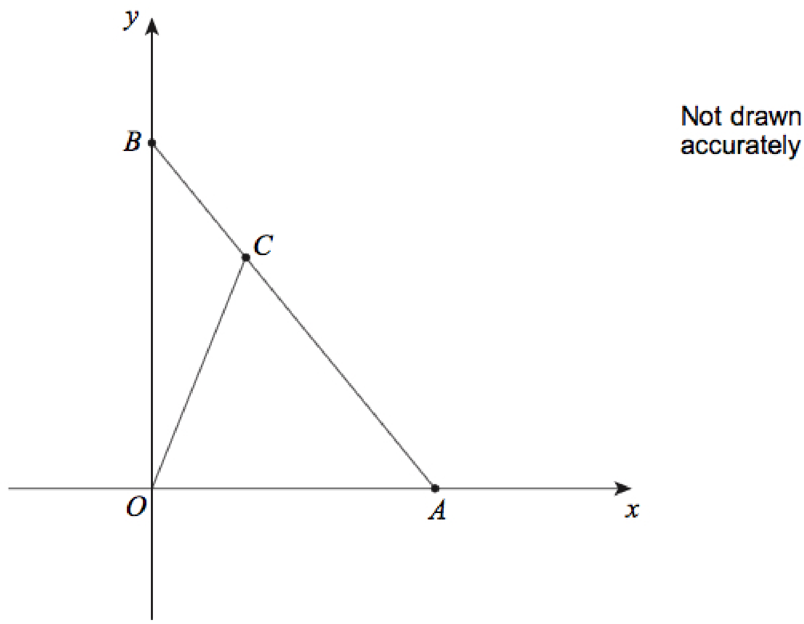
Complete the boxes to describe $\mathrm{f}(x)$:

$$\mathrm{f}(x) = \begin{cases} \quad , & -2 \leqslant x \leqslant 2, \\ \quad , & 2 < x \leqslant \end{cases}$$

Solution

$$\mathrm{f}(x) = \begin{cases} \underline{\underline{x^2}}, & -2 \leqslant x \leqslant 2, \\ \underline{\underline{4}}, & 2 < x \leqslant \underline{\underline{5}}. \end{cases}$$

2. The equation of line AB is $y = 12 - 2x$. (5)
   The area of triangle OCA is 24 square units.



Work out the coordinates of C.

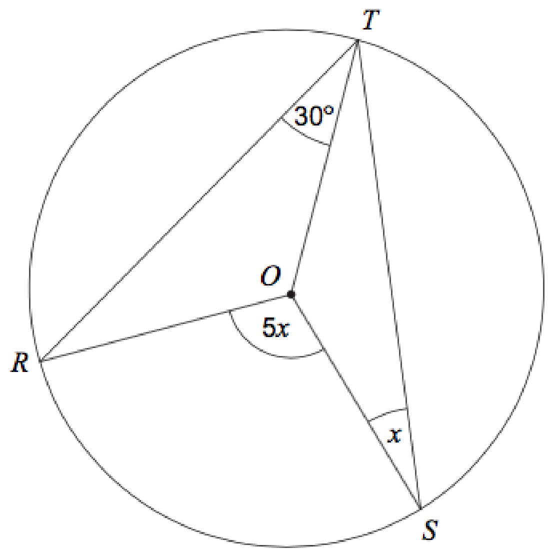
Solution

Well, $A(6,0)$ and $B(0,12)$. Now,

$$\begin{aligned} \text{area} = 24 &\Rightarrow \tfrac{1}{2} \times 6 \times C_y = 24 \\ &\Rightarrow C_y = 8 \\ &\Rightarrow C_x = 2; \end{aligned}$$

hence, the coordinates of $\underline{\underline{C(2,8)}}$.

3. R, S, and T are on the circumference of a circle, centre O.



Not drawn accurately

(a) Give a reason why angle $OTS = x$. (1)

Solution
Base angles in an isosceles triangle.

(b) Work out the value of x. (3)

Solution
The angle at the centre is twice the angle at the circumference:

$$2(x + 30) = 5x \Rightarrow 2x + 60 = 5x$$
$$\Rightarrow 3x = 60$$
$$\Rightarrow x = 20^\circ.$$

4. (a) Expand (2)

$$x^2(x - 2).$$

Solution

$$x^2(x - 2) = x^3 - 2x^2.$$

A curve has equation

$$y = x^2(x - 2).$$

(b) Work out the gradient of the curve at the point $(3, 9)$. (3)

Solution

$$\begin{aligned} y = x^2(x-2) &\Rightarrow y = x^3 - 2x^2 \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x. \end{aligned}$$

Finally,

$$\begin{aligned} x = 3 &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3(3^2) - 4(3) \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 27 - 12 \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 15}}. \end{aligned}$$

Line L is the tangent to the curve

$$y = x^2(x - 2)$$

at the point $(3, 9)$.

(c) Work out the equation of L. Give your answer in the form $y = mx + c$. (2)

Solution

$$\begin{aligned} y - 9 = 15(x - 3) &\Rightarrow y - 9 = 15x - 45 \\ &\Rightarrow \underline{\underline{y = 15x - 36}}. \end{aligned}$$

5. Solve (4)

$$\frac{4c + 3}{2} + \frac{c - 8}{5} = 1.$$

Solution

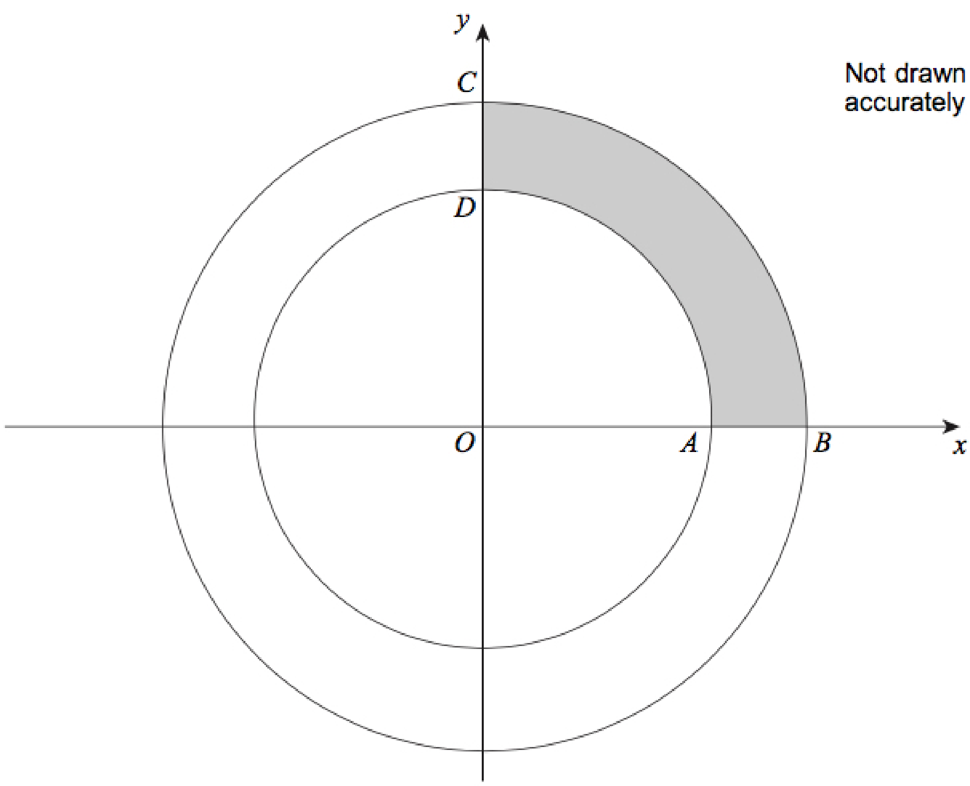
Multiply by 10:

$$\frac{4c+3}{2} + \frac{c-8}{5} = 1 \Rightarrow 5(4c+3) + 2(c-8) = 10$$
$$\Rightarrow (20c+15) + (2c-16) = 10$$
$$\Rightarrow 22c = 11$$
$$\Rightarrow \underline{\underline{c = \tfrac{1}{2}}}.$$

6. Two circles, each with centre O, are shown. (5)

The equations of the circles are

$$x^2 + y^2 = 289 \text{ and } x^2 + y^2 = 121.$$



Work out the **perimeter** of the shaded section $ABCD$.

Solution

Well,

$$\sqrt{289} = 17 \text{ and } \sqrt{121} = 11$$

so horizontal and vertical components are

$$17 - 11 = 6.$$

Finally,

$$\begin{aligned} \text{perimeter} &= \tfrac{1}{4}(2 \times \pi \times 17 + 2 \times \pi \times 11) + 6 + 6 \\ &= \tfrac{1}{4}(34\pi + 22\pi) + 12 \\ &= \tfrac{1}{4}(56\pi) + 12 \\ &= \underline{\underline{14\pi + 12 \text{ or } 56.0 \ \ (3 \text{ sf})}}. \end{aligned}$$

7. (a) Simplify (2)

$$\sqrt{x^5 \times x^9}.$$

Give your answer in the form x^p where p is an integer.

Solution

$$\begin{aligned} \sqrt{x^5 \times x^9} &= \sqrt{x^{14}} \\ &= \underline{\underline{x^7}}. \end{aligned}$$

(b) Solve (2)

$$y^{-3} = 125.$$

Solution

$$\begin{aligned} y^{-3} = 125 &\Rightarrow y^3 = \tfrac{1}{125} \\ &\Rightarrow \underline{\underline{y = \tfrac{1}{5}}}. \end{aligned}$$

8. (4)

$$\mathbf{M} = \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}.$$

Show that

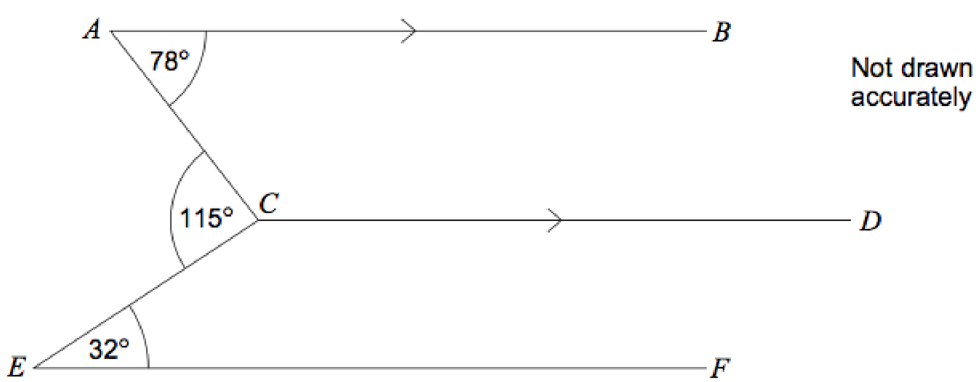
$$\mathbf{M}^3 = \mathbf{I}.$$

Solution

$$\begin{aligned}\mathbf{M}^3 &= \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \underline{\underline{\mathbf{I}}},\end{aligned}$$

as required.

9. AB is parallel to CD. (3)



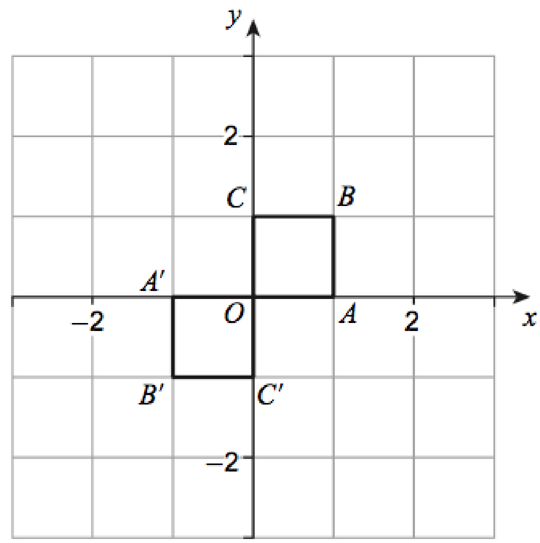
Is EF parallel to CD?
You **must** show your working.

Solution

$\angle ACD = 180 - 78 = 102°$ (interior angles)
$\angle ECD = 360 - 115 - 102 = 143°$ (completing the circle)
But $\angle CEF + \angle ECD = 32 + 143 = 175°$.
EF is <u>not parallel</u> to CD.

10. The unit square $OABC$ has vertices $O(0,0)$, $A(1,0)$, $B(1,1)$, and $C(0,1)$.

(a) $OABC$ is mapped to $OA'B'C'$ under transformation matrix $\mathbf{M}$. (2)



Work out matrix $\mathbf{M}$.

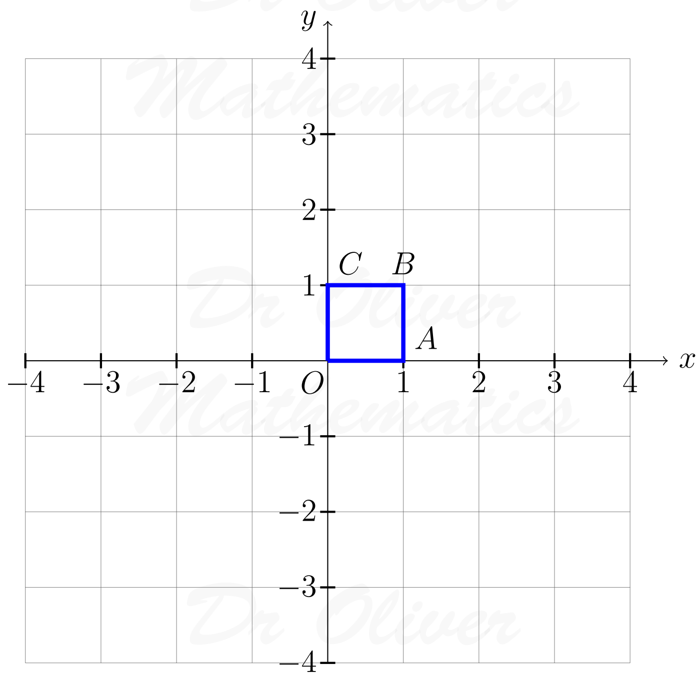
Solution

It is a rotation, centre O, half turn and the matrix is

$$\mathbf{M} = \underline{\underline{\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}}}.$$

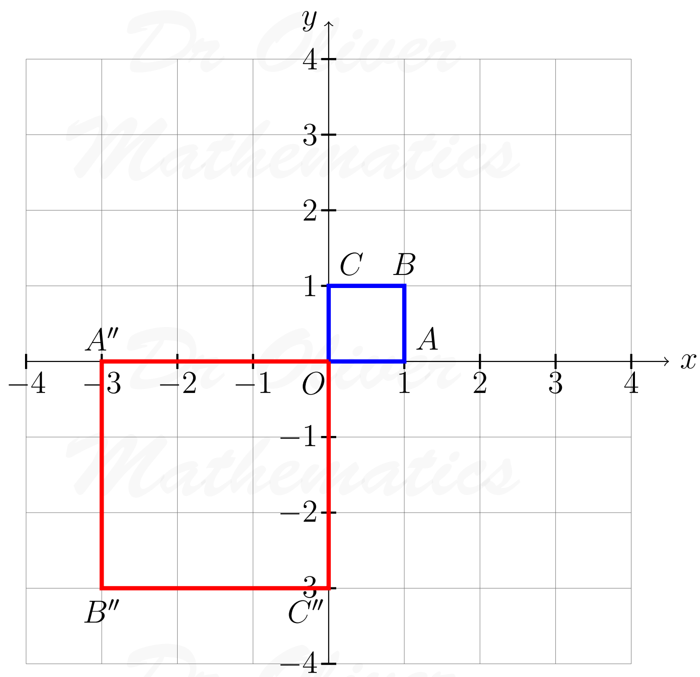
(b) $OABC$ is mapped to $OA''B''C''$ under transformation matrix (3)

$$\mathbf{M} = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}.$$



Draw and label $OA''B''C''$ on the diagram below.

Solution



11. (a) Simplify fully (3)

$$\frac{8c^7}{15d^6} \div \frac{6c^2}{5d^3}.$$

Solution

$$\begin{aligned}\frac{8c^7}{15d^6} \div \frac{6c^2}{5d^3} &= \frac{8c^7}{15d^6} \times \frac{5d^3}{6c^2}\\ &= \frac{4c^5}{3d^3} \times \frac{1}{3}\\ &= \underline{\underline{\frac{4c^5}{9d^3}}}.\end{aligned}$$

(b) Write as a single fraction (4)

$$\frac{5}{m+1} + \frac{6}{m-4}.$$

Give your answer in its simplest form.

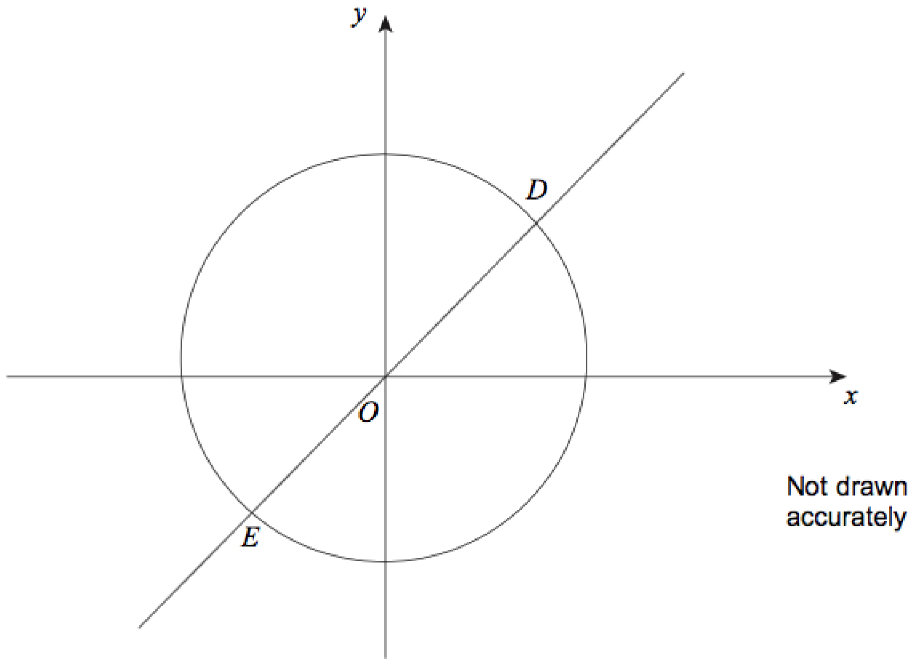
Solution

$$\begin{aligned}\frac{5}{m+1} + \frac{6}{m-4} &= \frac{5(m-4)}{(m+1)(m-4)} + \frac{6(m+1)}{(m+1)(m-4)}\\ &= \frac{5(m-4)+6(m+1)}{(m+1)(m-4)}\\ &= \frac{(5m-20)+(6m+6)}{(m+1)(m-4)}\\ &= \underline{\underline{\frac{11m-14}{(m+1)(m-4)}}}.\end{aligned}$$

12. The circle (5)

$$x^2 + y^2 = 20 \text{ and the line } y = 2x$$

intersect at points D and E.



Work out the coordinates of D and E.
Do **not** use trial and improvement.
You **must** show your working.

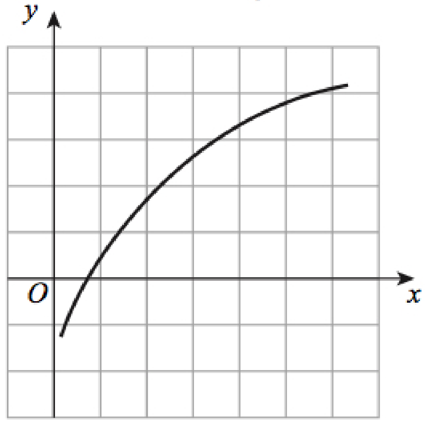
Solution

$$\begin{aligned} x^2 + y^2 = 20 &\Rightarrow x^2 + (2x)^2 = 20 \\ &\Rightarrow x^2 + 4x^2 = 20 \\ &\Rightarrow 5x^2 = 20 \\ &\Rightarrow x^2 = 4 \\ &\Rightarrow x = \pm 2. \qquad \Rightarrow y = \pm 4; \end{aligned}$$

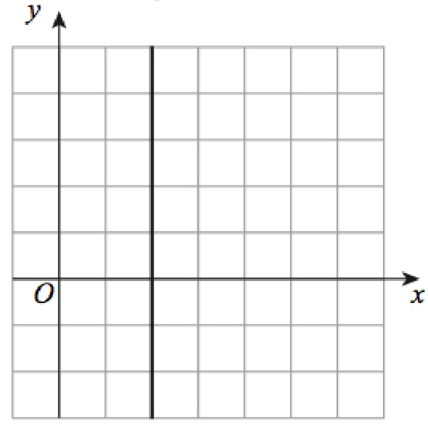
hence, $\underline{\underline{D(2,4)}}$ and $\underline{\underline{E(-2,-4)}}$ (because $x_D > x_E$).

13. Here are five graphs.

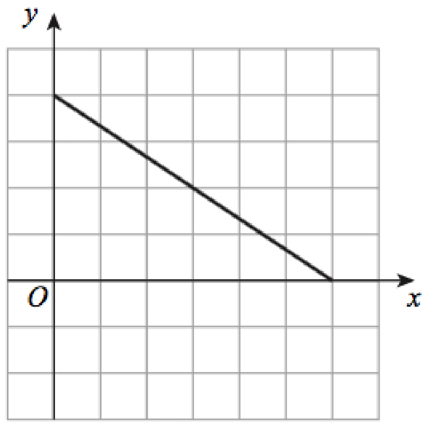
Graph *A*



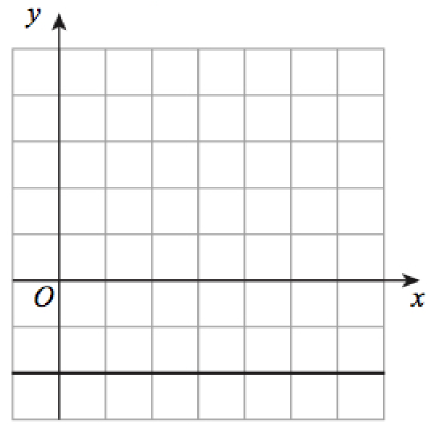
Graph *B*



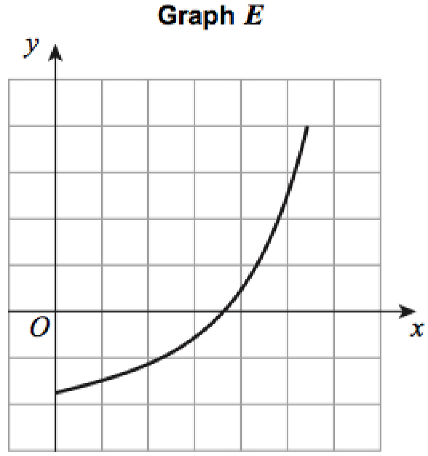
Graph *C*



Graph *D*



Graph *E*



For each of the following statements, decide which graph is being described.

Circle your answer each time.

(a) The rate of change of y with respect to x is always negative. (1)

$A \quad B \quad C \quad D \quad E$

Solution

$\underline{\underline{C}}$

(b) The rate of change of y with respect to x is always zeroe. (1)

$A \quad B \quad C \quad D \quad E$

Solution

$\underline{\underline{D}}$

(c) The rate of change of y with respect to x is always decreases. (1)

$A \quad B \quad C \quad D \quad E$

Solution

$\underline{\underline{A}}$

14. Rearrange (4)

$$x = \frac{2w+1}{5-3w}$$

to make w the subject.

Solution

$$\begin{aligned} x = \frac{2w+1}{5-3w} &\Rightarrow x(5-3w) = 2w+1 \\ &\Rightarrow 5x - 3wx = 2w + 1 \\ &\Rightarrow 5x - 1 = 2w + 3wx \\ &\Rightarrow 5x - 1 = w(2 + 3x) \\ &\Rightarrow \underline{\underline{w = \frac{5x-1}{2+3x}}}. \end{aligned}$$

15. The nth term of a sequence is

$$n^2 + 12n + 27.$$

(a) By factorising, or otherwise, show that the 20th term can be written as the product of two prime numbers. (2)

Solution

$$\begin{aligned} 20\text{th term} &= 20^2 + 12(20) + 27 \\ &= 400 + 240 + 27 \\ &= 667 \\ &= 23 \times 29; \end{aligned}$$

hence, the 20th term can be written as the product of $\underline{\underline{\text{two prime numbers}}}$.

The nth term of a different sequence is

$$n^2 - 6n + 14.$$

(b) By completing the square, or otherwise, show that every term is positive. (3)

Solution

$$\begin{aligned} n^2 - 6n + 14 &= (n^2 - 6n + 9) + 5 \\ &= (n-3)^2 + 5 \\ &\geqslant 5; \end{aligned}$$

hence, every term is $\underline{\underline{\text{positive}}}$.

16. (a) Simplify (1)

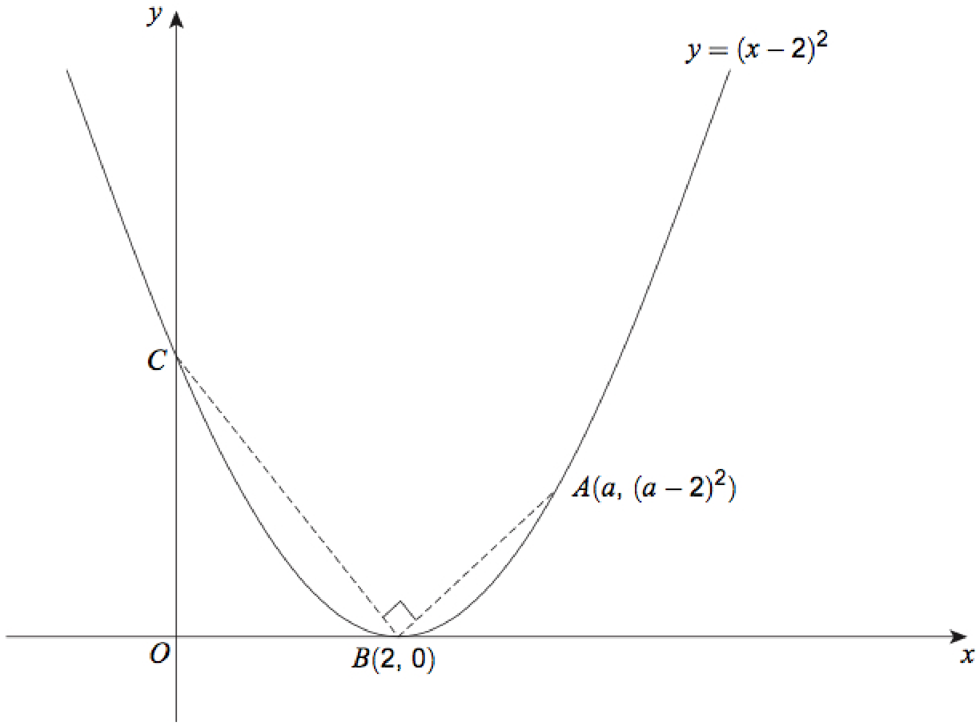
$$\frac{(a-2)^2}{a-2}.$$

Solution

$$\frac{(a-2)^2}{a-2} = \underline{\underline{a-2}}.$$

Here is a sketch of the curve

$$y = (x - 2)^2.$$



- The curve touches the x-axis at B and intersects the y-axis at C.
- Angle ABC is 90°.
- The curve passes through $A(a, (a-2)^2)$.

(b) Work out the value of a. (5)

Solution

Well,

$$x = 0 \Rightarrow y = (0-2)^2 = 4$$

so $C(0, 4)$. Now,

$$\begin{aligned} m_{AB} &= \frac{(a-2)^2 - 0}{a-2} \\ &= \frac{(a-2)^2}{a-2} \\ &= a - 2 \end{aligned}$$

and

$$m_{BC} = -\frac{1}{a-2}.$$

But

$$m_{BC} = -2$$

and

$$-\frac{1}{a-2} = -2 \Rightarrow a - 2 = \tfrac{1}{2}$$
$$\Rightarrow \underline{\underline{a = 2\tfrac{1}{2}}}.$$

17. (a) Factorise fully (2)

$$12c^2d - 9d^2.$$

Solution

$$12c^2d - 9d^2 = \underline{\underline{3d(4c^2 - 9d)}}.$$

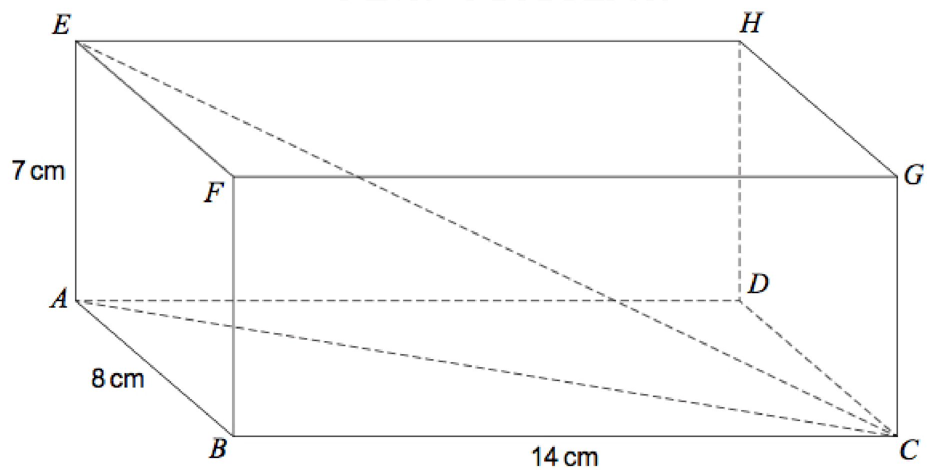
(b) Factorise fully (3)

$$(w+4)^3 - (w+4)^2(w+1).$$

Solution

$$\begin{aligned}(w+4)^3 - (w+4)^2(w+1) &= (w+4)^2\left[(w+4) - (w+1)\right] \\ &= \underline{\underline{3(w+4)^2}}.\end{aligned}$$

18. $ABCDEFGH$ is a cuboid. (3)



Work out the angle between EC and $ABCD$.

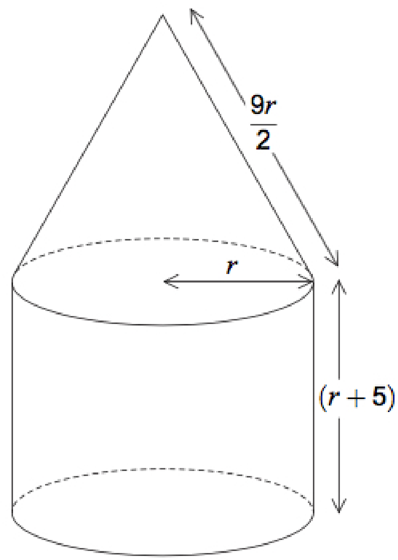
Solution

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{8^2 + 14^2} \\ &= \sqrt{260} \\ &= 2\sqrt{65} \end{aligned}$$

and

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan \text{angle} = \frac{EA}{AC} \\ &\Rightarrow \tan \text{angle} = \tfrac{7}{2\sqrt{65}} \\ &\Rightarrow \text{angle} = 23.466\,704\,72 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\text{angle} = 23.5^\circ \text{ (3 sf)}}}. \end{aligned}$$

19. On this diagram all lengths are given in centimetres.
A cylinder and cone are joined together to make a solid.
The cylinder has radius r and height $(r + 5)$.
The cone has radius r and slant height $\frac{9}{2}r$.



(a) Show that the **total** surface area of the solid, in cm^2, is (4)

$$\tfrac{5}{2}\pi r(3r+4).$$

Solution

$$\begin{aligned}\text{Area} &= \text{circle} + \text{cylinder} + \text{cone}\\ &= \pi r^2 + 2\pi r(r+5) + \pi r(\tfrac{9}{2}r)\\ &= \pi r[r + 2(r+5) + \tfrac{9}{2}r]\\ &= \pi r(\tfrac{15}{2}r + 10)\\ &= \underline{\underline{\tfrac{5}{2}\pi r(3r+4)}},\end{aligned}$$

as required.

The total surface area of the solid is $1\,200\pi\ \text{cm}^2$.

(b) Work out the value of r. (5)

Solution

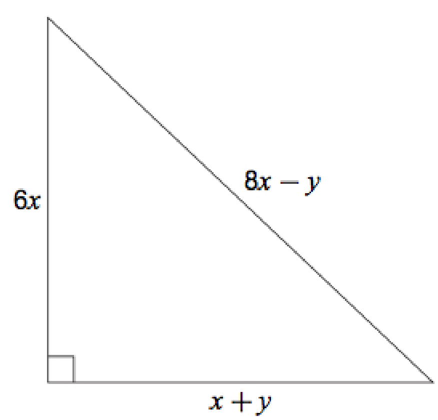
$$\begin{aligned}\text{Area} = 1\,200\pi \Rightarrow \tfrac{5}{2}\pi r(3r+4) &= 1\,200\pi \\ \Rightarrow r(3r+4) &= 480 \\ \Rightarrow 3r^2 + 4r &= 480 \\ \Rightarrow 3r^2 + 4r - 480 &= 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & +4 \\ \text{multiply to:} & (+3)\times(-480) = -1\,440\end{array}\right\} -36,\ +40$$

$$\begin{aligned}&\Rightarrow 3r^2 - 36r + 40r - 480 = 0 \\ &\Rightarrow 3r(r-12) + 40(r-12) = 0 \\ &\Rightarrow (3r+40)(r-12) = 0 \\ &\Rightarrow r = -13\tfrac{1}{3} \text{ or } r = 12;\end{aligned}$$

hence, as $r > 0$, $\underline{\underline{r = 12}}$.

20. The diagram shows a right-angled triangle. (6)



Prove algebraically that

$$x : y = 2 : 3.$$

Solution

$\times$	x	$-y$
x	x^2	$+xy$
$-y$	$+xy$	$+y^2$

$\times$	$8x$	$-y$
$8x$	$64x^2$	$-8xy$
$-y$	$-8xy$	$+y^2$

$$
\begin{aligned}
(8x-y)^2 = (6x)^2 + (x+y)^2 &\Rightarrow 64x^2 - 16xy + y^2 = 36x^2 + (x^2 + 2xy + y^2) \\
&\Rightarrow 27x^2 - 18xy = 0 \\
&\Rightarrow 9x(3x - 2y) = 0 \\
&\Rightarrow x = 0 \text{ or } 3x = 2y.
\end{aligned}
$$

Now, $x > 0$ (or else we do not have a triangle!) and

$$
\begin{aligned}
3x = 2y &\Rightarrow \frac{x}{y} = \frac{2}{3} \\
&\Rightarrow \underline{\underline{x : y = 2 : 3}},
\end{aligned}
$$

as required.

21. Solve (5)

$$16 \sin^2 x = 1$$

for $0^\circ \leqslant x \leqslant 270^\circ$.

Solution

$$
\begin{aligned}
16 \sin^2 x = 1 &\Rightarrow \sin^2 x = \tfrac{1}{16} \\
&\Rightarrow \sin x = \pm\tfrac{1}{4}.
\end{aligned}
$$

$\underline{\sin x = \tfrac{1}{4}}$:

$$
\begin{aligned}
\sin x = \tfrac{1}{4} &\Rightarrow x = 14.477\,512\,19,\ 165.522\,487\,8 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{x = 14.5^\circ,\ 166^\circ \text{ (3 sf)}}}.
\end{aligned}
$$

$\underline{\sin x = -\frac{1}{4}}$:

$$\begin{aligned}\sin x = -\tfrac{1}{4} \Rightarrow x &= -14.4 \text{ (not in range)}, 365.522\,487\,8 \text{ (not in range)} \\ &\quad \text{or } 194.477\,512\,2 \text{ (in range)} \\ \Rightarrow x &= 194^\circ \text{ (3 sf)}.\end{aligned}$$

22. The curve $y = \mathrm{f}(x)$ has (3)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = kx(x-3)^3,$$

where k is a **negative** constant.
There is a stationary point at $x = 3$.
Determine the nature of this stationary point.
You **must** show your working.

Solution

Now,

$$x = 3^- \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} > 0$$

and

$$x = 3^+ \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0.$$

Hence, we have a maximum point.