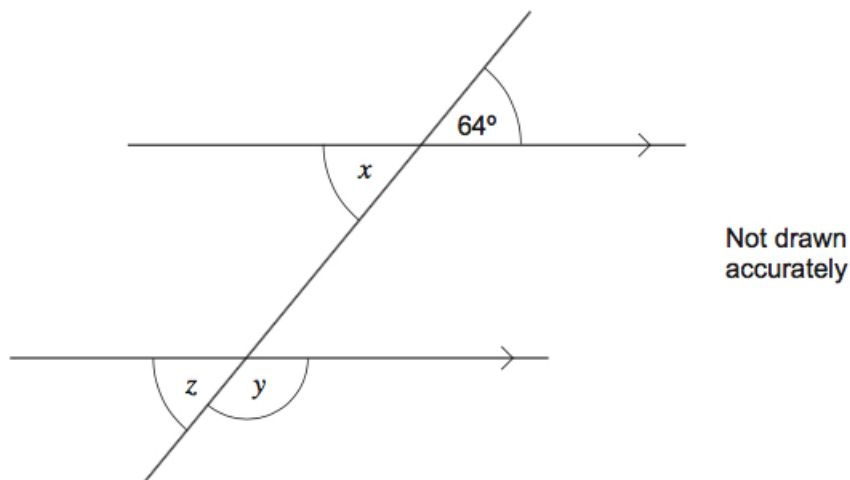


Dr Oliver Mathematics
AQA GCSE Mathematics
2012 November Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

1. Here is a figure.



- (a) Write down the size of angle x . (1)

Solution
 $x = 64^\circ$.

- (b) Work out the size of angle y . (1)

Solution
 $y = 180 - 64 = 116^\circ$.

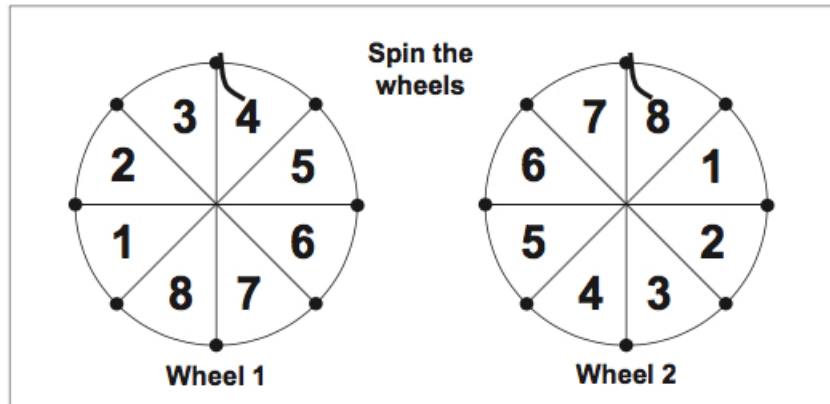
- (c) Choose the correct word from the list to complete the sentence. (1)

opposite alternate corresponding interior

Solution
Corresponding

- 2. In a game, players spin two wheels.
The wheels are fair.

The numbers are added to get a score.
The wheels show a score of $4 + 8 = 12$.



You may use the grid below to help you answer the questions on the next page.

	1	2	3	4	5	6	7	8
1								
2								
3								
4								12
5								
6								
7								
8								

- (a) What is the most likely score?

(2)

Solution

+	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	12	12
5	6	7	8	9	10	12	13	13
6	7	8	9	10	12	13	14	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

Add up the entries that give you 9 $((1 + 8), (2 + 7), \dots, (8 + 1))$: 9.

**Score
2, 3, 15 or 16
to win a prize**

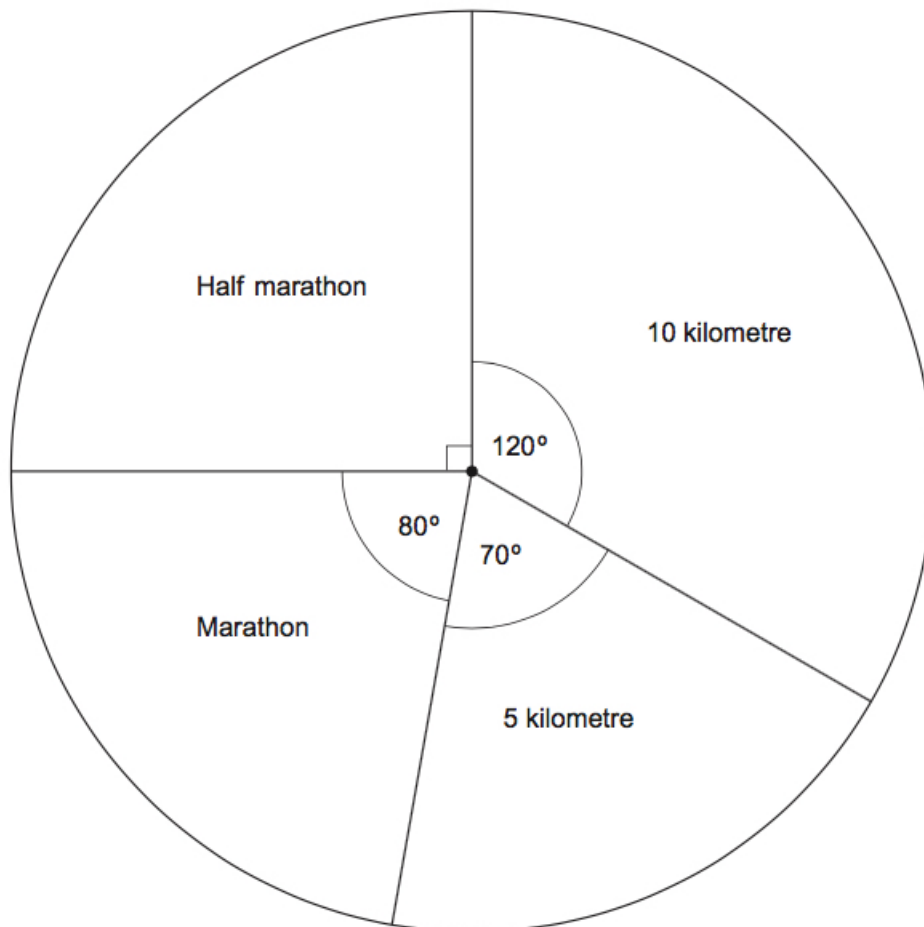
- (b) Work out the probability of winning a prize. (3)

Solution

$$\begin{aligned}
 P(\text{winning a prize}) &= \frac{1 + 1 + 1 + 1 + 1 + 1}{64} \\
 &= \frac{6}{64} \\
 &= \frac{3}{32}
 \end{aligned}$$

3. There are 36 men in a running club. (4)
The pie chart shows information about their favourite races.

Men



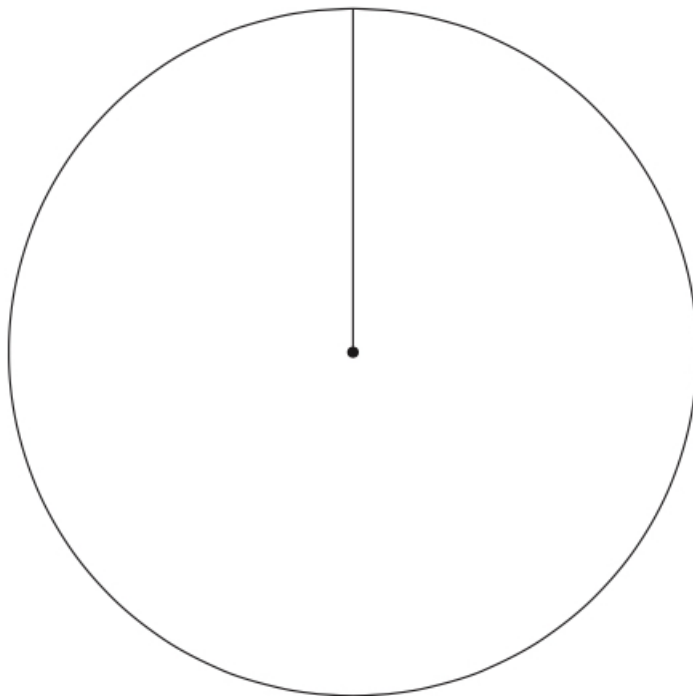
There are 20 women in the running club.

Here is information about their favourite races.

- The same **proportion** of women prefer the half marathon as men.
- The same **number** of women prefer 5 km races as men.
- **Equal** numbers of women prefer 10 km races and the marathon.

Use this information to draw a fully labelled pie chart to show the favourite races of the women.

Women



Solution

So, the number of men:

$$5 \text{ km} : \frac{70}{360} \times 36 = 7$$

$$10 \text{ km} : \frac{120}{360} \times 36 = 12$$

$$\text{Half-marathon} : \frac{90}{360} \times 36 = 9$$

$$\text{Marathon} : \frac{80}{360} \times 36 = 8.$$

How, the same **proportion** of women prefer the half marathon as men: one-quarter of the women:

$$\frac{90}{360} \times 20 = 5.$$

5 km: there are 7 women who like this (The same **number** of women prefer 5 km races as men.)

That takes us to to 12.

Equal numbers of women prefer 10 km races and the marathon: 4 for each 10 km races and the marathon.

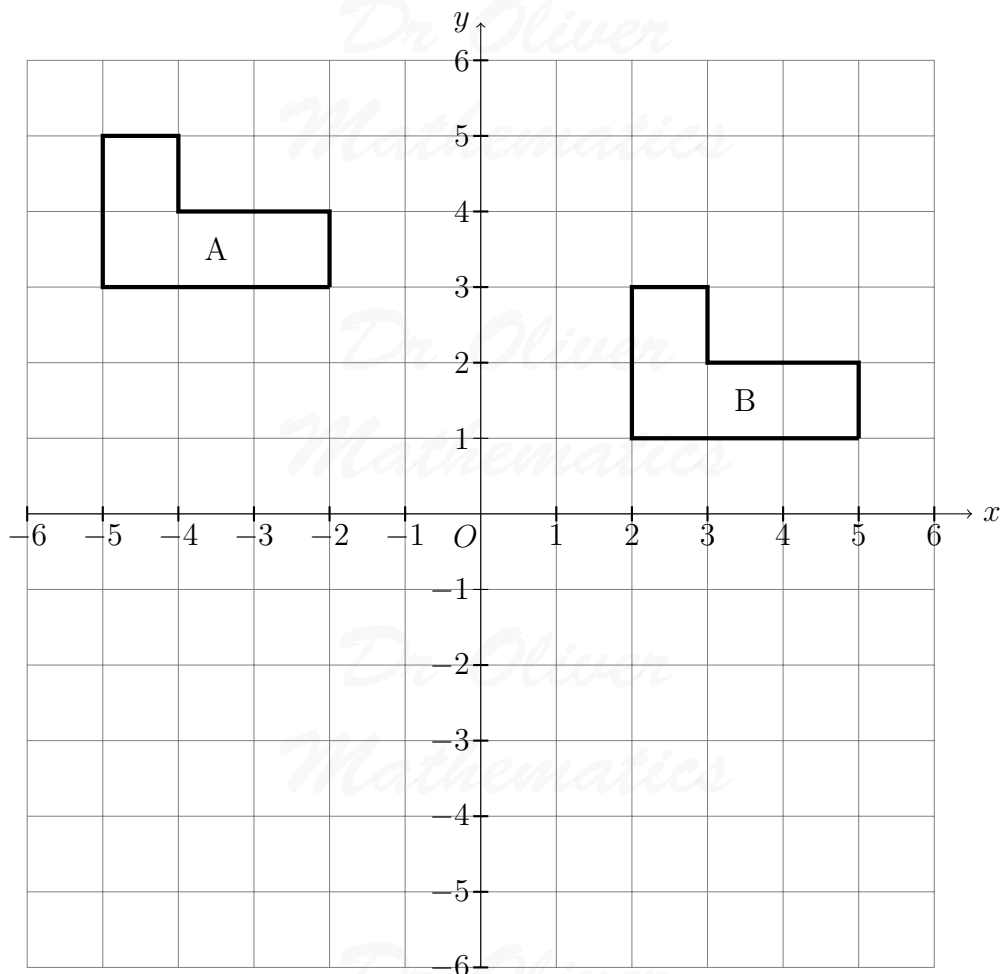
$$\text{Half-marathon : } \frac{5}{20} \times 360 = \underline{\underline{90^\circ}}$$

$$5 \text{ km : } \frac{7}{20} \times 360 = \underline{\underline{126^\circ}}$$

$$10 \text{ km : } \frac{4}{20} \times 360 = \underline{\underline{72^\circ}}$$

$$\text{Marathon : } \frac{4}{20} \times 360 = \underline{\underline{72^\circ}}$$

4. Here is a picture.



(a) Describe fully the **single** transformation that maps shape A to shape B.

(2)

Solution

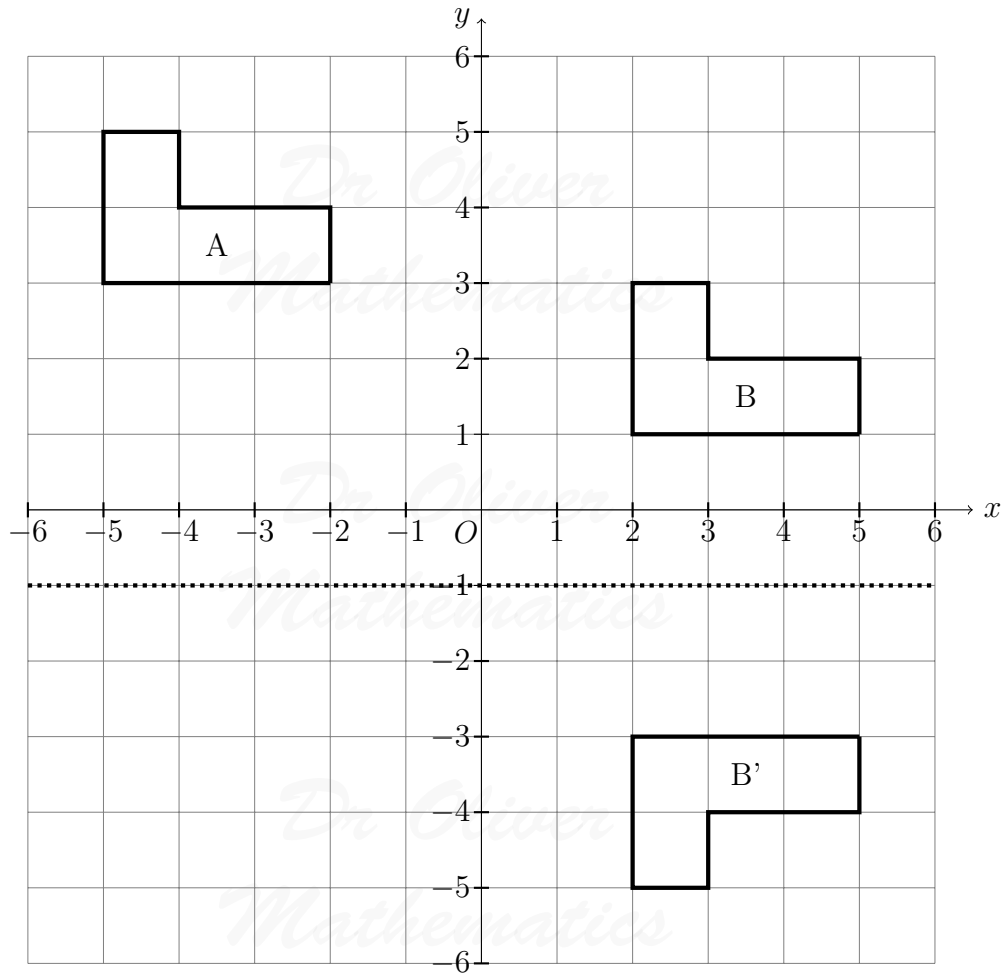
A translation by

$$\underline{\underline{\begin{pmatrix} 7 \\ -2 \end{pmatrix}}}$$

(b) Draw the reflection of shape B in the line $y = -1$.

(2)

Solution



5. Solve

$$9x - 3 = 4x + 17.$$

(3)

Solution

$$9x - 3 = 4x + 17 \Rightarrow 5x = 20 \\ \Rightarrow \underline{\underline{x = 4.}}$$

6. (a) Factorise

$$7x - 21.$$

(1)

Solution

$$7x - 21 = \underline{\underline{7(x - 3)}}.$$

(b) Multiply out

$$4(y + 9).$$

(1)

Solution

$$4(y + 9) = \underline{\underline{4y + 36}}.$$

7. Expand and simplify

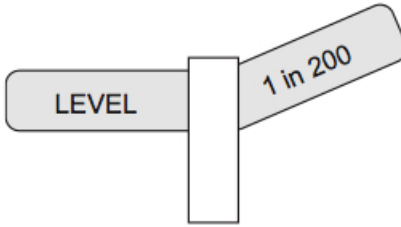
$$5(x - 3) - 2(x - 1).$$

(3)

Solution

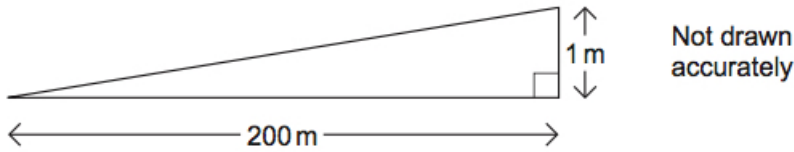
$$5(x - 3) - 2(x - 1) = 5x - 15 - 2x + 2 \\ = \underline{\underline{3x - 13.}}$$

8. The steepness of a railway track is shown by signs like this.

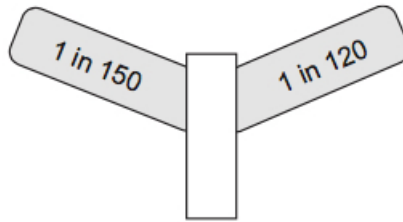


On the left of the sign the track is level.

On the right of the sign the track rises 1 metre for every 200 metres travelled horizontally.



- (a) Which side of this sign shows the steeper track? (2)



Show clearly how you decide.

You may use a diagram to explain your answer.

Solution

1 in 120: the shorter the 'adjacent' side is, the slope's gradient is increased.

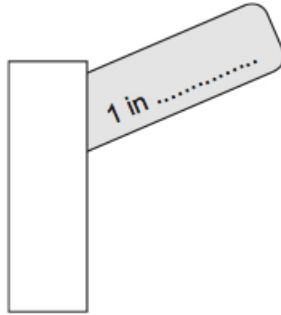
The steepness can also be measured as a percentage.

For example, 1 in 200 would be

$$\frac{1}{200} \times 100 = 0.5\%.$$

The steepest railway line in Britain has a percentage of 2.5%.

- (b) Fill in this sign to show a steepness of 2.5%. (2)



Solution

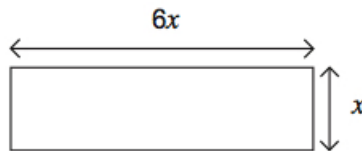
Let x be the steepness. Then

$$\begin{aligned} \frac{1}{x} \times 100 &= 2.5\% \Rightarrow \frac{x}{100} = \frac{1}{2.5} \\ &\Rightarrow \frac{x}{100} = \frac{2}{5} \\ &\Rightarrow x = \frac{2}{5} \times 100 \\ &\Rightarrow x = 40. \end{aligned}$$

So, it reads 1 in 40.

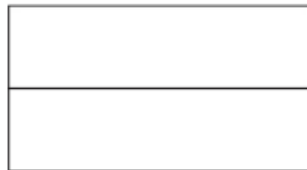
9. The length of this rectangular tile is 6 times the width.

(3)



Not drawn accurately

Two tiles are put together to make this shape.



Not drawn accurately

The perimeter of the new shape is 24 cm.

Work out the width of one tile.

Solution

Well,

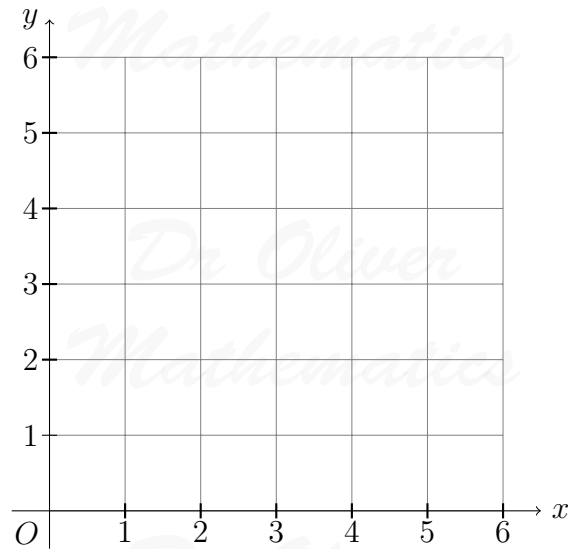
$$6x + 2x + 2x + 6x = 24 \Rightarrow 16x = 24$$
$$\Rightarrow \underline{\underline{x = 1\frac{1}{2}}}.$$

10. On the grid draw lines to show the region satisfied by the three inequalities.

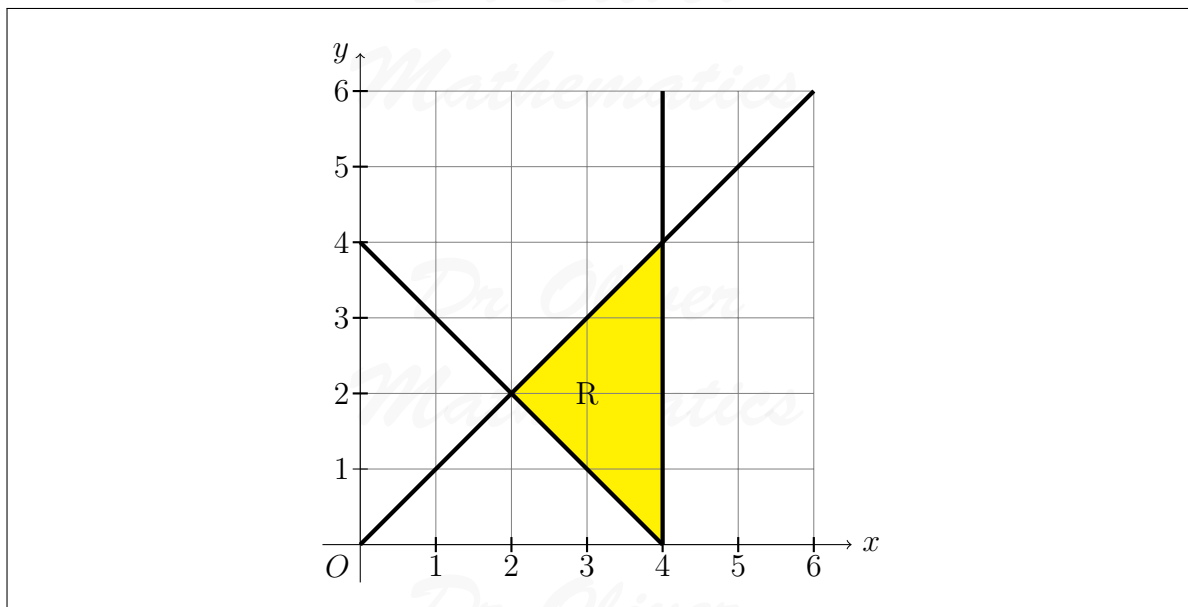
(3)

$$x \leq 4$$
$$y \leq x$$
$$x + y \geq 4.$$

Label the region clearly with the letter R.

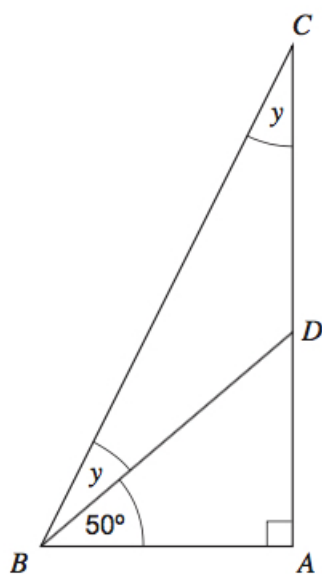


Solution



11. (a) ABC is a right-angled triangle.

(3)



Not drawn
accurately

Work out the size of angle y .

Solution

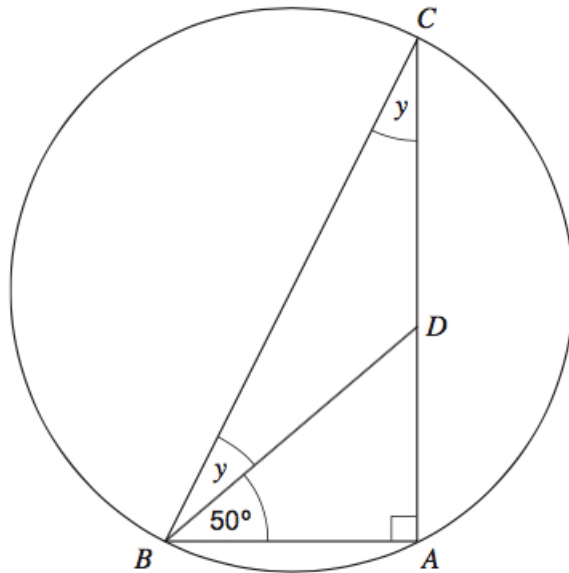
Well, $\angle BDA = 180 - 50 - 90 = 40^\circ$ (angles in a triangle)

$\angle BDC = 180 - 40 = 140^\circ$ (supplementary angles)

$\angle CBD = \angle BCD = \frac{1}{2}(180 - 140) = \frac{1}{2} \times 40 = \underline{\underline{20^\circ}}$ (angles in a triangle)

(b) A circle is drawn around ABC .

(1)



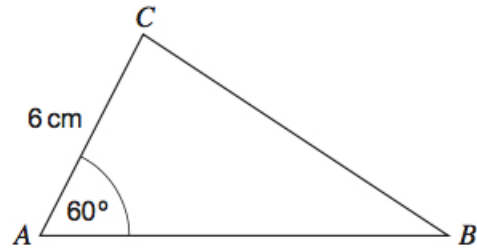
Not drawn
accurately

Give a reason why BC is a diameter of the circle.

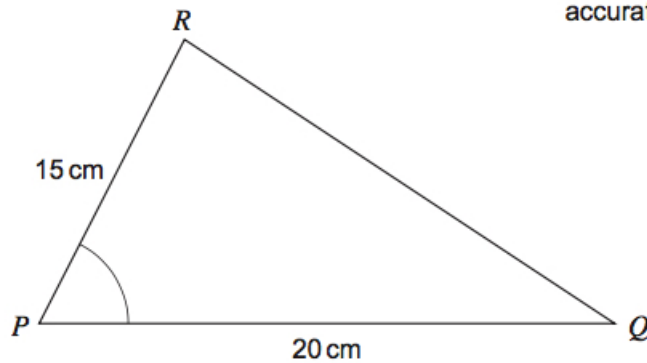
Solution

$BAC = 90^\circ$ (right-angle) which makes BC the diameter of the circle.

12. PQR is an enlargement of ABC .



Not drawn accurately



- (a) Work out the scale factor of the enlargement. (1)

Solution

$$\frac{15}{6} = \underline{\underline{2\frac{1}{2}}}$$

- (b) Write down the size of angle P . (1)

Solution

$$P = \underline{\underline{60^\circ}}$$

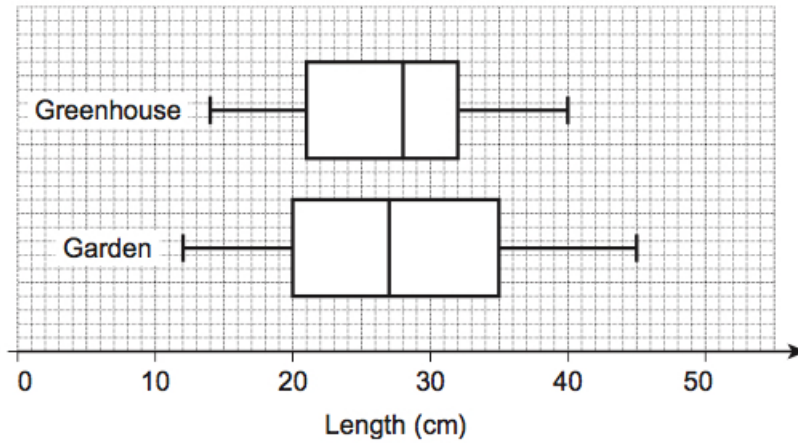
- (c) Work out the length AB . (2)

Solution

$$\begin{aligned} \frac{AB}{AC} &= \frac{PQ}{PR} \Rightarrow \frac{AB}{6} = \frac{20}{15} \\ &\Rightarrow AB = \frac{4}{3} \times 6 \\ &\Rightarrow \underline{\underline{AB = 8\text{ cm}}} \end{aligned}$$

13. Cucumbers are grown in a greenhouse or in a garden.

The box plots show data about their lengths, in centimetres.



- (a) Write down the median length of the cucumbers grown in the garden. (1)

Solution

27 cm.

- (b) Give **two** comparisons between the lengths of cucumbers grown in the greenhouse and cucumbers grown in the garden. (3)

Solution

Average

Since the median for the Greenhouse (27) is higher than the median for the Garden (25), the Greenhouse is longer on average.

Spread

Since the range for the Greenhouse ($40 - 14 = 26$) is smaller than the range for the Garden ($45 - 12 = 33$), the Greenhouse was more consistent.

OR

Since the IQR for the Greenhouse ($32 - 21 = 11$) is smaller than the IQR for the Garden ($35 - 20 = 15$), the Greenhouse was more consistent.

14. (a) Factorise (1)

$$x^2 - 9.$$

Solution

$$\left. \begin{array}{l} \text{add to: } 0 \\ \text{multiply to: } -9 \end{array} \right\} -3, +3$$

Hence,

$$x^2 - 9 = \underline{\underline{(x - 3)(x + 3)}}.$$

(b) Hence, simplify fully

$$\frac{x^2 - 9}{2x^2 - 5x - 3}.$$

(3)

Solution

$$\left. \begin{array}{l} \text{add to: } -5 \\ \text{multiply to: } (+2) \times (-3) = -6 \end{array} \right\} -6, +1$$

E.g.,

$$\begin{aligned} \frac{x^2 - 9}{2x^2 - 5x - 3} &= \frac{(x - 3)(x + 3)}{2x^2 - 6x + x - 3} \\ &= \frac{(x - 3)(x + 3)}{2x(x - 3) + 1(x - 3)} \\ &= \frac{(x - 3)(x + 3)}{(2x + 1)(x - 3)} \\ &= \frac{x + 3}{\underline{\underline{2x + 1}}}. \end{aligned}$$

15. The area of a trapezium is given by the formula

$$\text{area of trapezium} = \frac{1}{2}(a + b)h.$$

- (a) For a trapezium, $a = 5$ cm, $b = 8$ cm, and $h = 6$ cm.
All measurements are given to the nearest centimetre.
Work out the minimum possible area.

(3)

Solution

$4.5 \leq a < 5.5$, $7.5 \leq b < 8.5$, and $5.5 \leq h < 6.5$:

$$\begin{aligned}\text{Minimum possible area} &= \frac{1}{2} \times (4.5 + 7.5) \times 5.5 \\ &= \frac{1}{2} \times 12 \times 5.5 \\ &= 6 \times 5.5 \\ &= \underline{\underline{33 \text{ cm}^2}}.\end{aligned}$$

(b) Rearrange

$$A = \frac{1}{2}(a + b)h$$

to make h the subject.

Solution

$$\begin{aligned}A = \frac{1}{2}(a + b)h &\Rightarrow h = \frac{A}{\frac{1}{2}(a + b)} \\ &\Rightarrow h = \underline{\underline{\frac{2A}{a + b}}}.\end{aligned}$$

16. The manager of a company wants to survey his employees.

He decides to sample 20% of them, stratified by the type of job they do.

This table shows the number of employees.

Office Staff	Drivers	Mechanics	Total
12	24	4	40

Fill in the table below to show how many of each group he should survey.

Office Staff	Drivers	Mechanics

Solution

He wants to survey

$$40 \times 0.2 = 8$$

of them.

$$\text{Office Staff : } \frac{12}{40} \times 8 = 2.4$$

$$\text{Drivers : } \frac{24}{40} \times 8 = 4.8$$

$$\text{Mechanics : } \frac{4}{40} \times 8 = 0.8.$$

So,

Office Staff	Drivers	Mechanics
<u>2</u>	<u>5</u>	<u>1</u>

17. Write

$$\sqrt{12} + \sqrt{75}$$

(2)

in the form $a\sqrt{3}$, where a is an integer.

Solution

$$\begin{aligned}\sqrt{12} + \sqrt{75} &= \sqrt{4 \times 3} + \sqrt{25 \times 3} \\ &= \sqrt{4} \times \sqrt{3} + \sqrt{25} \times \sqrt{3} \\ &= 2\sqrt{3} + 5\sqrt{3} \\ &= \underline{\underline{7\sqrt{3}}};\end{aligned}$$

hence, $a = 7$.

18. Write these numbers in order of size starting with the smallest.

You **must** show your working.

$$27^{\frac{2}{3}} \quad 64^{\frac{1}{3}} \quad 4^{\frac{3}{2}}$$

(3)

Solution

$$27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$$

$$= 3^2$$

$$= 9$$

$$64^{\frac{1}{3}} = 4$$

$$4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3$$

$$= 2^3$$

$$= 8.$$

The order is $64^{\frac{1}{3}}$, $4^{\frac{3}{2}}$, $27^{\frac{2}{3}}$.

19. $ABCD$ is a triangular based pyramid.

The base BCD is a right-angled triangle.

A is directly above B .

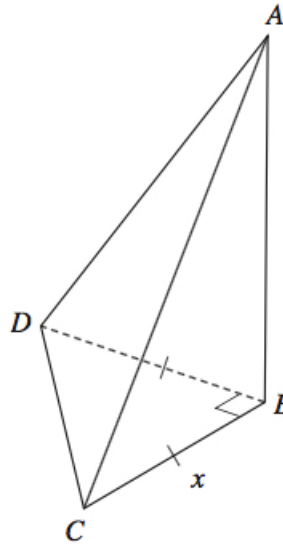
$BC = BD$. $AB = 2 \times BC$.

The volume of the pyramid is 72 cm^3 .

The formula for the volume of a pyramid is

$$\frac{1}{3} \times \text{base area} \times \text{height}.$$

(3)



Calculate the length of BC , labelled x in the diagram.

Solution

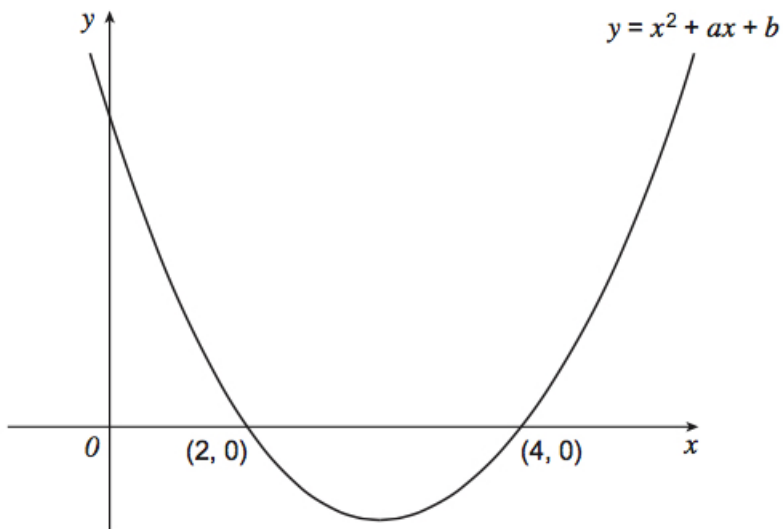
$$\begin{aligned}
 \text{Volume} &= \frac{1}{3} \times \text{base area} \times \text{height} \Rightarrow 72 = \frac{1}{3} \times \left(\frac{1}{2} \times x \times x\right) \times 2x \\
 &\Rightarrow 72 = \frac{1}{3}x^3 \\
 &\Rightarrow x^3 = 216 \\
 &\Rightarrow \underline{\underline{x = 6}}.
 \end{aligned}$$

20. The diagram shows a sketch of the graph of

(4)

$$y = x^2 + ax + b.$$

The graph crosses the x -axis at $(2, 0)$ and $(4, 0)$.



Work out the value of b .
You **must** show your working.

Solution

\times	x	-2
x	x^2	$-2x$
-4	$-4x$	$+8$

Clearly,

$$(x - 2)(x - 4) = x^2 - 6x + 12,$$

so that means

$$\underline{\underline{b = 8.}}$$