

Dr Oliver Mathematics
GCSE Mathematics
2023 November Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

1. Work out

$$6.3 \times 2.4.$$

(3)

Solution

Well,

$$\begin{array}{r} 6^1 3 \\ \times 2 4 \\ \hline 2 5 2 \\ 1 2_1 6 \\ \hline 1 5 1 2 \end{array}$$

After the decimal point: *one* digits in the first number (6.3) and *one* number in the second (2.4), we need *two*. Hence,

$$6.3 \times 2.4 = \underline{\underline{15.12}}.$$

2. (a) (i) Write down the value of 5^0 .

(1)

Solution

$$5^0 = \underline{\underline{1}}.$$

- (ii) Write down the value of 5^{-2} .

(1)

Solution

Well,

$$\begin{aligned} 5^{-2} &= \frac{1}{5^2} \\ &= \frac{1}{\underline{\underline{25}}}. \end{aligned}$$

(b) Write

(2)

$$\frac{2^5 \times 2^4}{2^3}$$

in the form 2^n where n is an integer.

Solution

Now,

$$\begin{aligned} \frac{2^5 \times 2^4}{2^3} &= \frac{2^{5+4}}{2^3} \\ &= \frac{2^9}{2^3} \\ &= 2^{9-3} \\ &= \underline{\underline{2^6}}; \end{aligned}$$

hence, $n = \underline{\underline{6}}$.

3. (a) Write 156 as a product of its prime factors.

(2)

Solution

Well,

	156
2	78
2	39
3	13
13	1

so

$$156 = \underline{\underline{2^2 \times 3 \times 13}}.$$

(b) Find the highest common factor (HCF) of 156 and 130.

(2)

Solution

Now,

$$\begin{array}{r|l} & 130 \\ 2 & 65 \\ 5 & 13 \\ 13 & 1 \end{array}$$

so

$$130 = 2 \times 5 \times 13.$$

Next, we pick the highest power common to each term:

$$2 \times 13 = \underline{\underline{26}}.$$

4. The mean length of 5 sticks is 4.2 cm.

Nawal measured the length of one of the sticks as 7 cm.

- (a) Work out the mean length of the other 4 sticks.

(3)

Solution

Well,

$$\begin{aligned} & \frac{\text{mean length of the other 4 sticks} + 7}{5} = 4.2 \\ \Rightarrow & \text{mean length of the other 4 sticks} + 7 = 21 \\ \Rightarrow & \text{mean length of the other 4 sticks} = 14 \\ \Rightarrow & \text{mean length} = \underline{\underline{3.5 \text{ cm}}}. \end{aligned}$$

Nawal made a mistake.

The stick was not 7 cm long.

It was 17 cm long.

- (b) How does this affect your answer to part (a)?

(1)

Solution

Now,

$$\frac{\text{mean length of the other 4 sticks} + 17}{5} = 4.2$$

$$\Rightarrow \text{mean length of the other 4 sticks} + 17 = 21$$

$$\Rightarrow \text{mean length of the other 4 sticks} = 4$$

$$\Rightarrow \text{mean length} = \underline{\underline{1 \text{ cm}}};$$

hence, part (a) is lower.

5. The point P lies on the line AB .

(2)

Use ruler and compasses to construct an angle of 90° at P .

You must show all your construction lines.



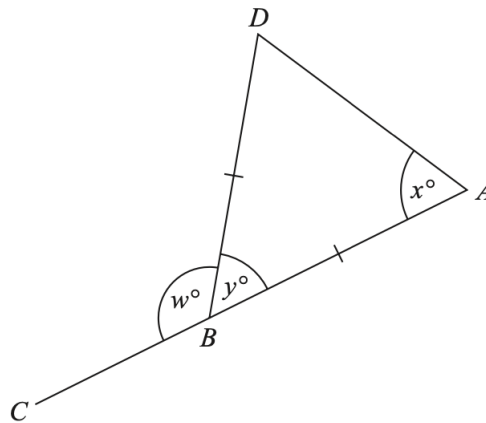
Solution

- Open your pair of compasses about half between the distance between P and the end of A .

- Make a circle about P .
- Label the points where your circle has gone through the straight line C and D .
- Place your compasses at C .
- Draw a light (but visible!) arc from about 1 o'clock down to about 5 o'clock.
- Do not change the separation of the points!
- Place your compasses at D .
- Draw a light (but visible!) arc from about 7 o'clock down to about 11 o'clock.
- Next, take your ruler and line up the two points made, one above the line and the one other below.
- Draw a straight line. And that's it!

6. The diagram shows an isosceles triangle ABD and the straight line ABC .

(4)



- $BA = BD$.
- $x : y = 2 : 1$.

Work out the value of w .

Solution

Well, $\angle BDA = x$ (base angles) and

$$\begin{aligned}x : y = 2 : 1 &\Rightarrow x : (180 - 2x) = 2 : 1 \\&\Rightarrow \frac{x}{180 - 2x} = \frac{2}{1} \\&\Rightarrow x = 2(180 - 2x) \\&\Rightarrow x = 360 - 4x \\&\Rightarrow 5x = 360 \\&\Rightarrow x = 72 \\&\Rightarrow y = 36 \\&\Rightarrow w = 180 - 36 \text{ (supplementary angles)} \\&\Rightarrow \underline{w = 144^\circ}.\end{aligned}$$

7. Mano has three shelves of books.

(5)

- There are x books on shelf A .
- There are $(3x + 1)$ books on shelf B .
- There are $(2x - 5)$ books on shelf C .

There is a total of 44 books on the three shelves.

All the books have the same mass.

The books on shelf B have a total mass of 7 500 g.

Work out the total mass of the books on shelf A .

Solution

Well,

$$\begin{aligned}x + (3x + 1) + (2x - 5) &= 44 \Rightarrow 6x - 4 = 44 \\&\Rightarrow 6x = 48 \\&\Rightarrow x = 8.\end{aligned}$$

So, 8 books on shelf A , 25 books on shelf B , and 11 books on shelf C .

Now, the books on shelf B have a total mass of 7 500 g so the mass of one book equal

$$\frac{7\,500}{25} = 300 \text{ g.}$$

Finally,

$$\begin{aligned}\text{total mass} &= 300 \times 9 \\ &= \underline{\underline{2\,700 \text{ g}}}.\end{aligned}$$

8. The normal price of a mattress is reduced by 40% in a sale.
The price of the mattress in the sale is £660.

(2)

Work out the normal price of the mattress.

Solution

Well,

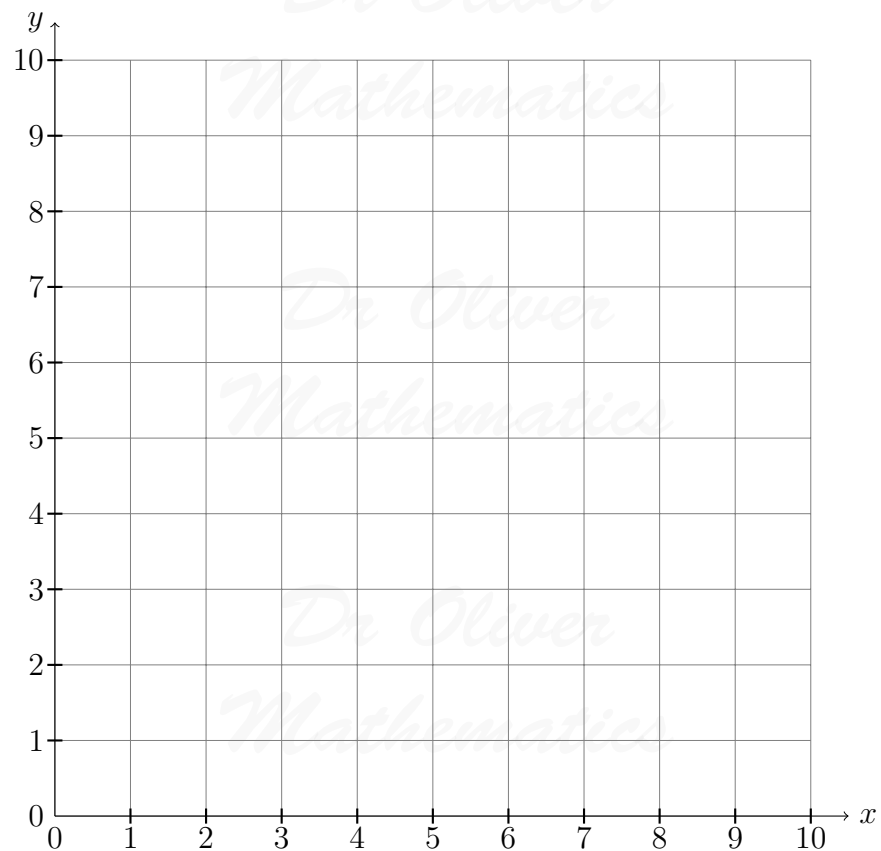
$$\begin{aligned}\text{new price} &= \text{old price} \times \text{discount} \Rightarrow 660 = \text{old price} \times (1 - 0.4) \\ &\Rightarrow \text{old price} = \frac{660}{0.6} \\ &\Rightarrow \text{old price} = \frac{110}{0.1} \\ &\Rightarrow \text{old price} = \underline{\underline{\pounds 1\,100}}.\end{aligned}$$

9. To cook rice,

the number of cups of rice (x) : the number of cups of water (y) = 4 : 5.

- (a) Use this information to draw a graph to show the relationship between the number of cups of rice and the number of cups of water needed to cook rice.

(2)

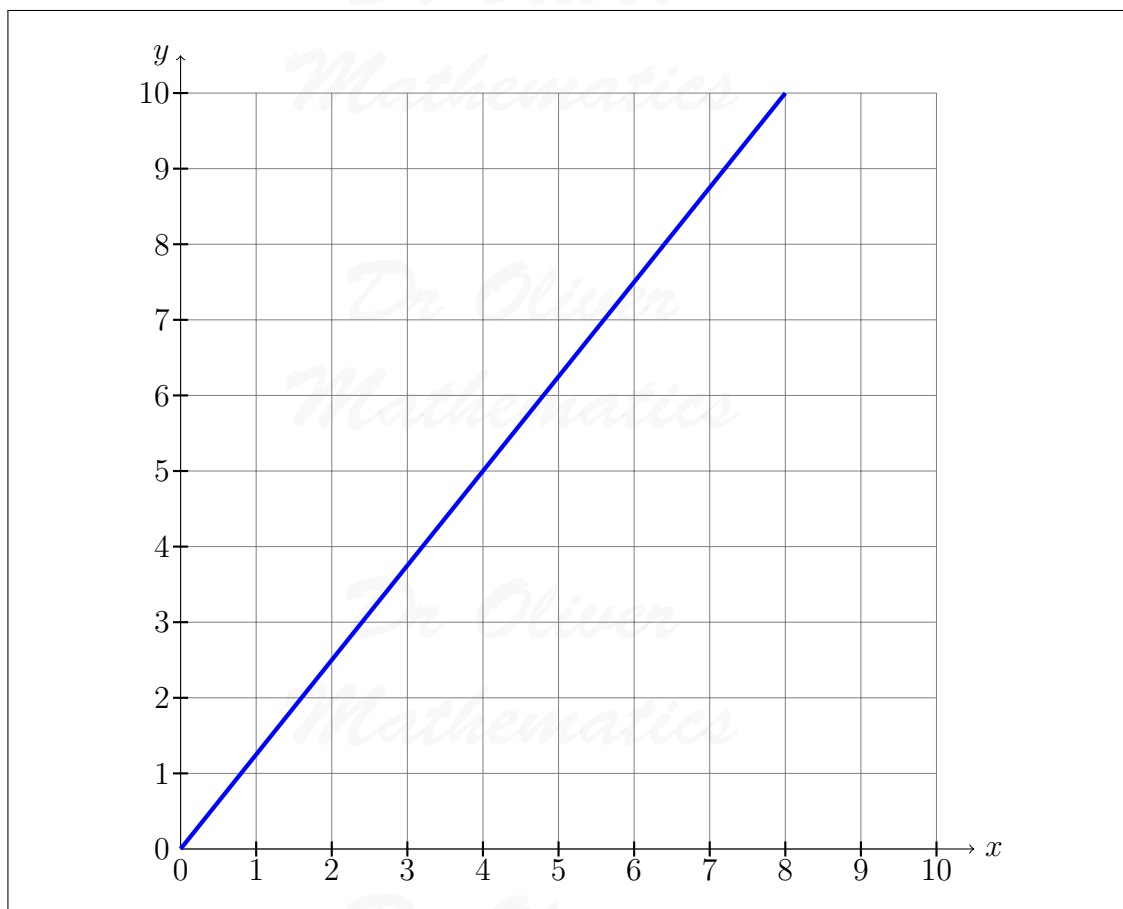


Solution

Well,

$$\begin{aligned}x : y = 4 : 5 &\Rightarrow \frac{y}{x} = \frac{5}{4} \\&\Rightarrow y = \frac{5}{4}x\end{aligned}$$

and so the graph is



- (b) (i) Find the gradient of the line drawn in part (a). (1)

Solution

$$\underline{\underline{\frac{5}{4}}}$$

- (ii) Explain what this gradient represents. (1)

Solution

E.g., for every additional cup of rice, you need to add $\frac{5}{4}$ cups of water.

10. The circumference of a circle is 10 m. (3)

Work out the area of the circle.

Give your answer in terms of π .

Solution

Let r m be the radius of the circle. Then

$$2 \times \pi \times r = 10 \Rightarrow r = \frac{10}{2\pi}$$

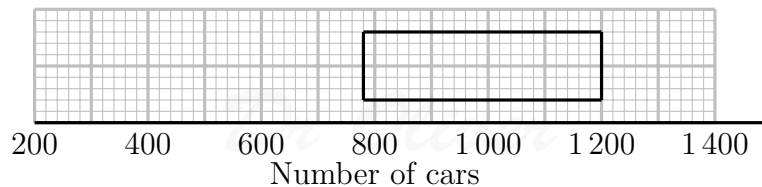
and

$$\begin{aligned} \text{area} &= \pi \times \left(\frac{10}{2\pi} \right)^2 \\ &= \pi \left(\frac{100}{4\pi^2} \right) \\ &= \frac{25}{\pi} \text{ m}^2. \end{aligned}$$

11. Alice recorded the number of cars going into a village on each of 80 days.

The incomplete table and the incomplete box plot give information about her results.

	Number of cars
Least number	300
Lower quartile	
Median	900
Upper quartile	
Range quartile	1 000



- (a) (i) Use the information in the table to complete the box plot.

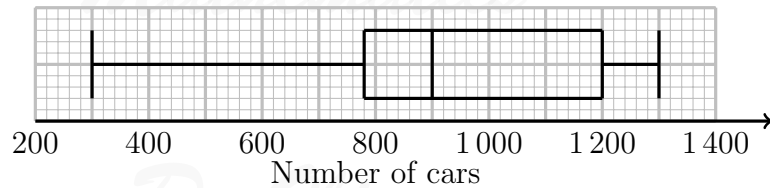
(3)

Solution

Well,

$$\begin{aligned} \text{range} &= \text{highest value} - \text{lowest value} \Rightarrow 1\,000 = \text{highest value} - 300 \\ &\Rightarrow \text{highest value} = 1\,300 \end{aligned}$$

and we complete the box plot:



(ii) Use the information in the box plot to complete the table.

Solution

And we use the box plot to complete the table:

	Number of cars
Least number	300
Lower quartile	<u>780</u>
Median	900
Upper quartile	<u>1 200</u>
Range quartile	1 000

On some of these 80 days Alice saw fewer than 1 200 cars going into the village.

(b) Work out an estimate for the number of days Alice saw fewer than 1 200 cars going into the village. (2)

Solution

I would estimate it about

$$\frac{3}{4} \times 80 = \underline{\underline{60 \text{ days}}}.$$

12. The straight line L has equation (3)

$$2y = 3x - 7.$$

Find an equation of the straight line perpendicular to L that passes through $(6, -5)$.

Solution

Well,

$$2y = 3x - 7 \Rightarrow y = \frac{3}{2}x - \frac{7}{2}$$

and

$$m_{\text{perp}} = -\frac{2}{3}.$$

Now, an equation of the straight line is

$$\begin{aligned}y + 5 &= -\frac{2}{3}(x - 6) \Rightarrow y + 5 = -\frac{2}{3}x + 4 \\&\Rightarrow \underline{\underline{y = -\frac{2}{3}x - 1.}}\end{aligned}$$

13. Solid A and solid B are similar.

(3)

The ratio of the height of solid A to the height of solid B is $2 : 5$.

The volume of solid A is 12 cm^3 .

Work out the volume of solid B .

Solution

Well, the length scale ratio (LSR) is $2 : 5$ and so the volume scale ratio (VSR) is

$$2^3 : 5^3 = 8 : 125.$$

Hence, the volume of solid B is

$$\begin{aligned}\frac{125}{8} \times 12 &= \frac{125}{2} \times 3 \\&= \underline{\underline{187.5.}}\end{aligned}$$

14. Work out the value of

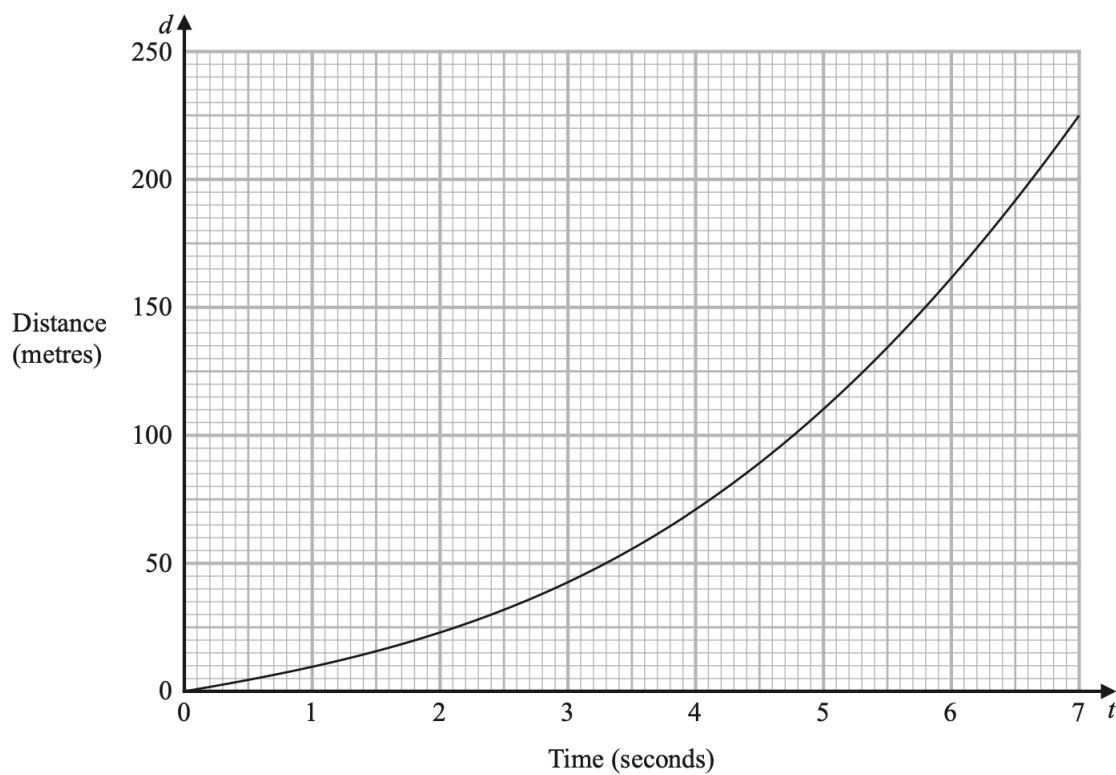
(3)

$$27^{\frac{2}{3}} + \left(\frac{1}{2}\right)^{-3}.$$

Solution

$$\begin{aligned}
 27^{\frac{2}{3}} + \left(\frac{1}{2}\right)^{-3} &= (27^{\frac{1}{3}})^2 + (2)^3 \\
 &= (3)^2 + 8 \\
 &= 9 + 8 \\
 &= \underline{\underline{17}}.
 \end{aligned}$$

15. Here is the distance-time graph for the distance (d metres) fallen by the object t seconds after it starts to fall. (3)



Work out an estimate for the gradient of the graph at $t = 3$.
You must show how you get your answer.

Solution

The gradient goes through (1.4, 0) and (5, 90):

$$\begin{aligned}
 m &= \frac{90 - 0}{5 - 1.4} \\
 &= \frac{90}{3.4} \\
 &= \frac{900}{34} \\
 &= \frac{(34 \times 26) + 16}{34} \\
 &= \underline{\underline{26\frac{8}{17}}}.
 \end{aligned}$$

16. At the start of year n the population of a species is P_n .

At the start of the following year the population of the species is given by

$$P_{n+1} = kP_n,$$

where k is a positive constant.

- The population of the species at the start of Year 1 is 8 million.
 - The population of the species at the start of Year 2 is 6 million.
- (a) Work out the population of the species at the start of Year 3.

(3)

Solution

Well,

$$\begin{aligned}
 P_{n+1} &= kP_n \Rightarrow 6 = 8k \\
 &\Rightarrow k = \frac{3}{4}.
 \end{aligned}$$

Hence,

$$\begin{aligned}
 P_3 &= \frac{3}{4}P_2 \\
 &= \frac{3}{4} \times 6 \\
 &= \underline{\underline{4.5 \text{ million}}}.
 \end{aligned}$$

At the start of Year 5 the value of k is increased by 0.3 to a new constant value.

Louise thinks that from the start of Year 5 the population of the species would increase year on year.

- (b) Is Louise correct? (1)
You must give a reason for your answer.

Solution

Well,

$$\frac{3}{4} + 0.3 = 1.05$$

and this is bigger than 1.

Hence, Louise is correct.

17. (a) Factorise (2)
 $6x^2 - 5x - 4$.

Solution

Now,

$$\left. \begin{array}{l} \text{add to:} \quad -5 \\ \text{multiply to: } (+6) \times (-4) = -24 \end{array} \right\} -8, +3$$

and so, e.g.,

$$\begin{aligned} 6x^2 - 5x - 4 &= 6x^2 - 8x + 3x - 4 \\ &= 2x(3x - 4) + 1(3x - 4) \\ &= \underline{\underline{(2x + 1)(3x - 4)}}. \end{aligned}$$

- (b) Hence, or otherwise, solve (2)
 $6x^2 - 5x - 4 < 0$.

Solution

Well,

$$\begin{aligned} 6x^2 - 5x - 4 = 0 &\Rightarrow (2x + 1)(3x - 4) = 0 \\ &\Rightarrow 2x + 1 = 0 \text{ or } 3x - 4 = 0 \\ &\Rightarrow 2x = -1 \text{ or } 3x = 4 \\ &\Rightarrow x = -\frac{1}{2} \text{ or } x = \frac{4}{3}. \end{aligned}$$

We need a ‘table of signs’:

	$x < -\frac{1}{2}$	$x = -\frac{1}{2}$	$-\frac{1}{2} < x < \frac{4}{3}$	$x = \frac{4}{3}$	$x > \frac{4}{3}$
$2x + 1$	—	0	+	+	+
$3x - 4$	—	—	—	0	+
$(2x + 1)(3x - 4)$	+	0	—	0	+

Hence,

$$\underline{\underline{-\frac{1}{2} < x < \frac{4}{3}}}.$$

18. Spinner A and spinner B are each spun once.

(4)

The probability that spinner A lands on red is $\frac{1}{4}$.

The probability that both spinner A and spinner B land on red is $\frac{1}{24}$.

Work out the probability that one spinner lands on red and the other spinner does **not** land on red.

Solution

Well,

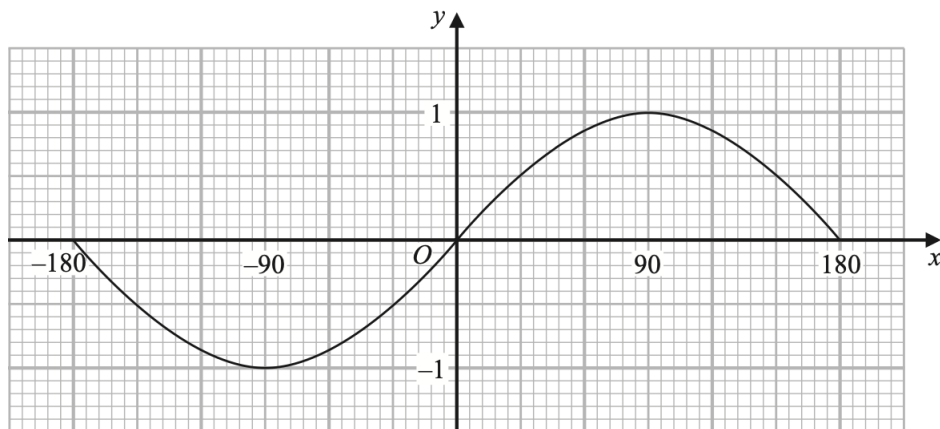
$$\begin{aligned} P(RR) &= \frac{1}{24} \Rightarrow \frac{1}{4} \times P(\text{spinner } B \text{ land on red}) = \frac{1}{24} \\ &\Rightarrow P(\text{spinner } B \text{ land on red}) = \frac{\frac{1}{24}}{\frac{1}{4}} \\ &\Rightarrow P(\text{spinner } B \text{ land on red}) = \frac{1}{6}. \end{aligned}$$

Finally,

$$\begin{aligned} \text{probability} &= \left(\frac{1}{4} \times \frac{5}{6}\right) + \left(\frac{3}{4} \times \frac{1}{6}\right) \\ &= \frac{5}{24} + \frac{3}{24} \\ &= \frac{8}{24} \\ &= \underline{\underline{\frac{1}{3}}}. \end{aligned}$$

19. Here is the graph of

$$y = \sin x^\circ \text{ for } -180 \leq x \leq 180.$$



- (a) Use the graph to find estimates for the solutions of (2)

$$\sin x^\circ = 0.3 \text{ for } -180 \leq x \leq 180.$$

Solution

Draw $y = 0.3$ and read off the intersections: approximately $x = 18$ or $x = 162$.

- (b) Write down a value of x such that (1)

$$\sin(x + 20)^\circ = 0 \text{ for } -180 \leq x \leq 180.$$

Solution

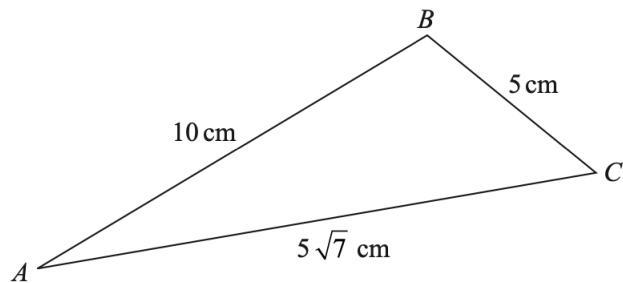
Well,

$$-180 \leq x \leq 180 \Rightarrow -160 \leq x + 20 \leq 200$$

and so

$$\begin{aligned} \sin(x + 20)^\circ = 0 &\Rightarrow x + 20 = 0, 180 \\ &\Rightarrow \underline{\underline{x = -20, 160.}} \end{aligned}$$

20. Here is triangle ABC . (4)



Find the size of angle ABC .
You must show all your working.

Solution

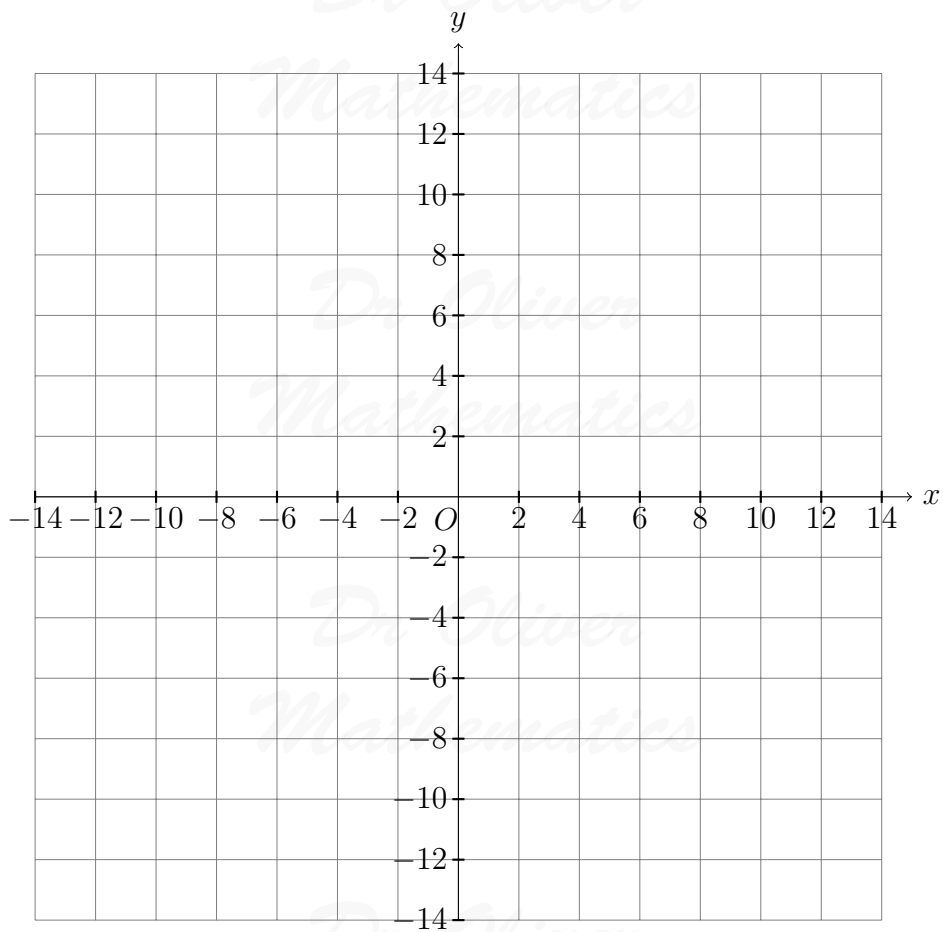
Cosine rule:

$$\begin{aligned}
 AC^2 &= AB^2 + BC^2 - 2 \times AC \times BC \times \cos ABC \\
 \Rightarrow (5\sqrt{7})^2 &= 10^2 + 5^2 - 2 \times 10 \times 5 \times \cos ABC \\
 \Rightarrow 175 &= 125 - 100 \cos ABC \\
 \Rightarrow 100 \cos ABC &= -50 \\
 \Rightarrow \cos ABC &= -\frac{1}{2} \\
 \Rightarrow \underline{\underline{\angle ABC = 120^\circ}}.
 \end{aligned}$$

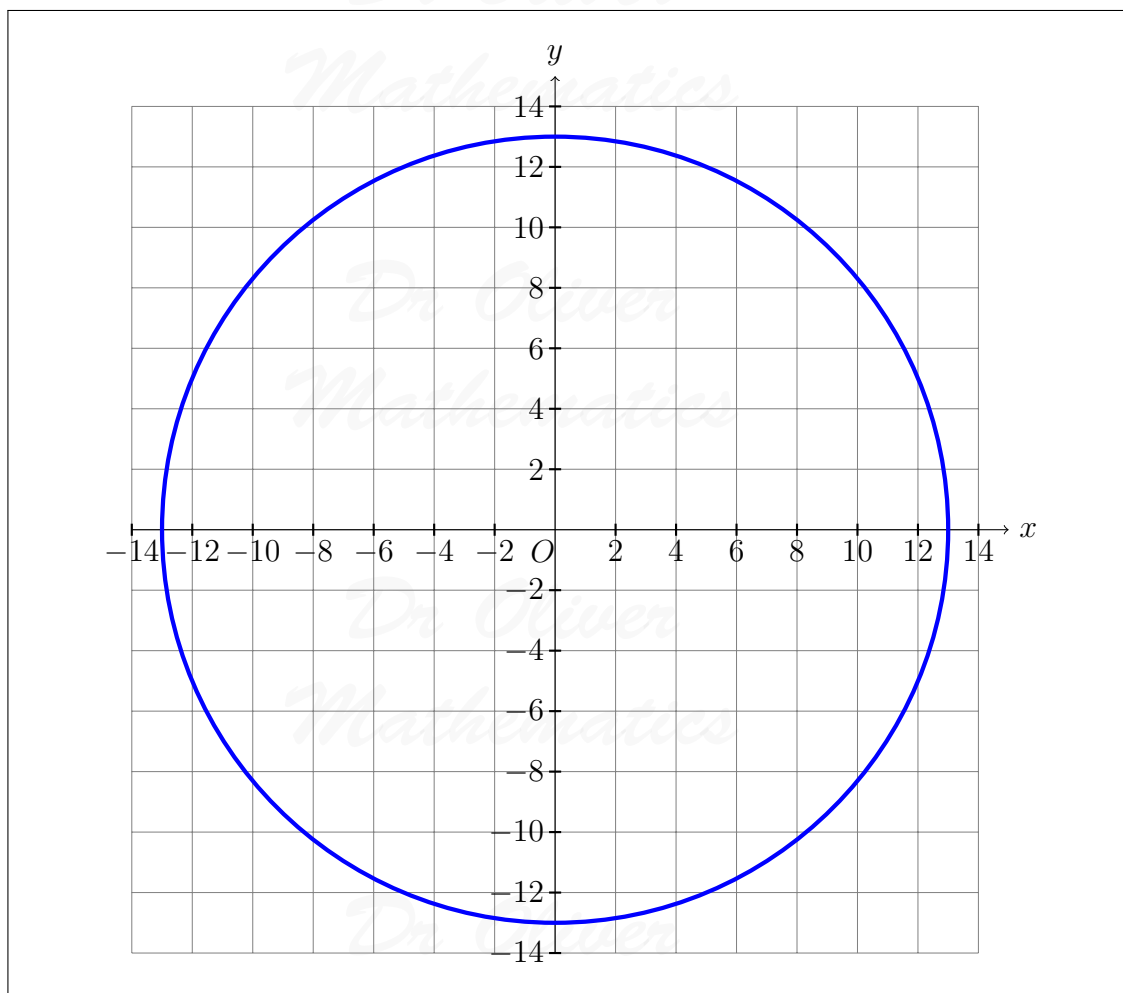
21. (a) On the grid, draw the graph of

$$x^2 + y^2 = 169.$$

(2)



Solution



- (b) Use your graph to find estimates for the solutions of the simultaneous equations (3)

$$x^2 + y^2 = 169$$

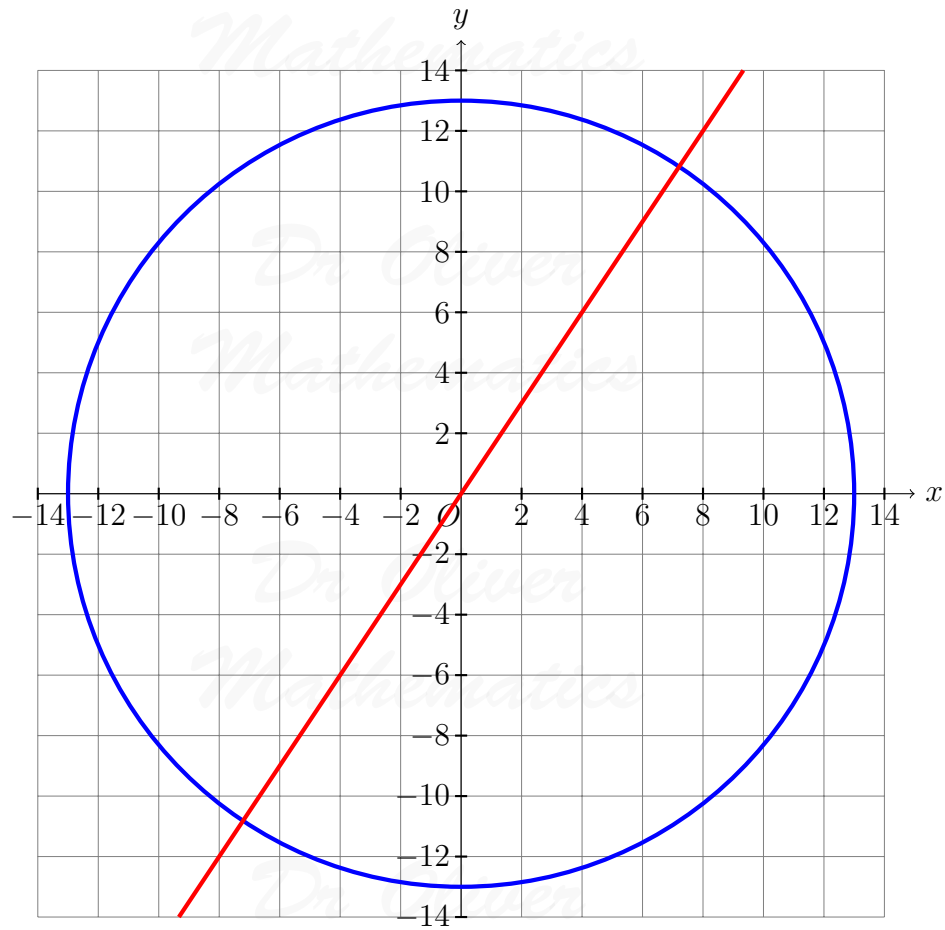
$$2y = 3x.$$

Solution

Well,

$$2y = 3x \Rightarrow y = \frac{3}{2}.$$

Draw it on the graph and find the points of intersection:



approximately

$$\underline{\underline{x = -7.2, y = -10.8 \text{ or } x = 7.2, y = 10.8.}}$$

22. The 2nd term of a geometric sequence is $3 + 2\sqrt{2}$.

(4)

The 3rd term of the sequence is $13 + 9\sqrt{2}$.

Find the value of the common ratio of the sequence.

Give your answer in the form $a + \sqrt{b}$, where a and b are integers.

You must show all your working.

Solution

Well,

$$r = \frac{13 + 9\sqrt{2}}{3 + 2\sqrt{2}}$$

$$= \frac{13 + 9\sqrt{2}}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$$

$\times$	13	$+9\sqrt{2}$
3	39	$+27\sqrt{2}$
$-2\sqrt{2}$	$-26\sqrt{2}$	-36

$\times$	3	$+2\sqrt{2}$
3	9	$+6\sqrt{2}$
$-2\sqrt{2}$	$-6\sqrt{2}$	-8

$$= \frac{3 + \sqrt{2}}{1}$$

$$= \underline{\underline{3 + \sqrt{2}}};$$

hence,

$$\underline{\underline{a = 3 \text{ and } b = 1.}}$$