

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2009 November Paper 2: Calculator
2 hours

The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You must write down all the stages in your working.

1. A function f is defined by

$$f : x \mapsto e^{x-1},$$

where $x > 0$

- (a) State the range of f . (1)
- (b) Find an expression for f^{-1} . (2)
- (c) State the domain of f^{-1} . (1)
2. (a) Find the first four terms, in ascending powers of x , in the expansion of (4)

$$\left(2 - \frac{1}{2}x\right)^6.$$

- (b) Find the coefficient of x^3 in the expansion of (2)

$$(1 + x)^2 \left(2 - \frac{1}{2}x\right)^6.$$

3. The table shows experimental values of the variables x and y which are related by the equation

$$y = \frac{a}{x^2} + \frac{b}{x},$$

where a and b are constants.

x	2	4	6	8	10
y	6.24	2.82	1.79	1.33	1.05

- (a) Using graph paper, plot x^2y against x and draw a straight line graph. (3)
- (b) Use your graph to estimate the value of a and of b . (4)
4. Find the coordinates and the nature of the stationary points of the curve (7)

$$y = x^3 + 3x^2 - 45x + 60.$$

5. Relative to an origin O , the position vectors of points A and B are

$$\begin{pmatrix} 7 \\ 24 \end{pmatrix} \text{ and } \begin{pmatrix} 10 \\ 20 \end{pmatrix},$$

respectively.

Find

(a) the length of \overrightarrow{OA} , (2)

(b) the length of \overrightarrow{AB} . (2)

Given that ABC is a straight line and that the length of \overrightarrow{AC} is equal to the length of \overrightarrow{OA} , find

(c) the position vector of the point C . (3)

6. (a) Given that

$$y = x\sqrt{4x + 12}, \quad (4)$$

show that

$$\frac{dy}{dx} = \frac{k(x + 2)}{\sqrt{4x + 12}},$$

where k is a constant to be found.

(b) Hence evaluate (3)

$$\int_{-2}^6 \frac{3x + 6}{\sqrt{4x + 12}} dx.$$

7. (a) Using graph paper, draw the curve (3)

$$y = \sin 2x, \quad 0^\circ \leq x \leq 360^\circ.$$

In order to solve the equation

$$1 + \sin 2x = 2 \cos x$$

another curve must be added to your diagram.

(b) Write down the equation of this curve and add this curve to your diagram. (3)

(c) State the number of values of x which satisfy the equation (1)

$$1 + \sin 2x = 2 \cos x, \quad 0^\circ \leq x \leq 360^\circ.$$

8. It is given that

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & -3 \\ 2 & 0 \end{pmatrix}, \text{ and } \mathbf{C} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}.$$

Find

- (a) \mathbf{AB} , (2)
- (b) \mathbf{BC} , (2)
- (c) \mathbf{A}^{-1} , and hence find the matrix \mathbf{X} such that $\mathbf{AX} = \mathbf{B}$. (4)

9. A particle moves in a straight line so that, t seconds after passing through a fixed point O , its velocity, $v \text{ ms}^{-1}$, is given by

$$v = \frac{20}{(2t + 4)^2}.$$

Find

- (a) the velocity of the particle at O , (1)
 - (b) the acceleration of the particle when $t = 3$, (3)
 - (c) the distance travelled by the particle in the first 8 seconds. (4)
10. (a) Solve (4)

$$\log_{10}(7x - 3) + 2 \log_{10} 5 = 2 + \log_{10}(x + 3).$$

- (b) Use the substitution (5)

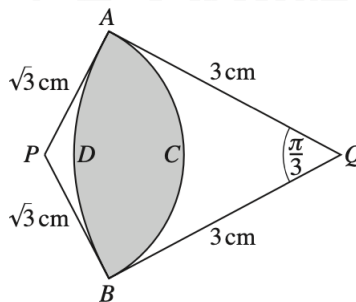
$$u = 3^x$$

to solve the equation

$$3^{x+1} + 3^{2-x} = 28.$$

EITHER

11. In the diagram, ACB is an arc of a circle with centre P , and ADB is an arc of a circle with centre Q .

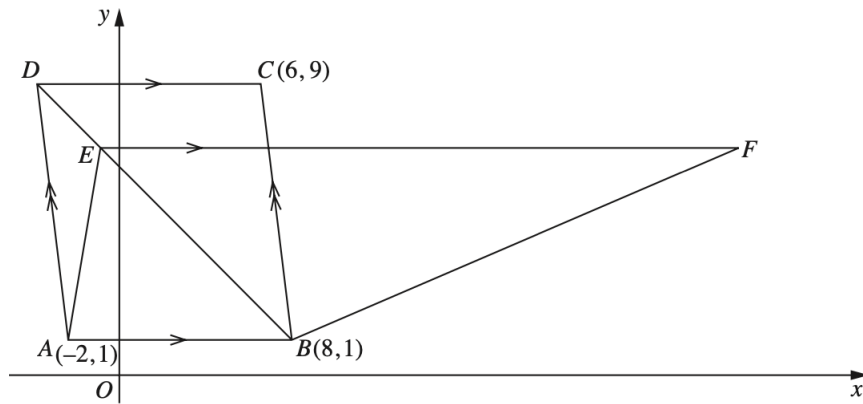


- Angle $AQB = \frac{1}{3}\pi$.

- $AQ = BQ = 3$ cm.
 - $AP = BP = \sqrt{3}$ cm.
- (a) Show that angle $APB = \frac{2}{3}\pi$. (2)
- (b) Find the perimeter of the shaded region. (3)
- (c) Find the area of the shaded region. (5)

OR

12. The diagram shows a parallelogram with vertices $A(-2, 1)$, $B(8, 1)$, $C(6, 9)$, and D .



- (a) Find the coordinates of D . (2)

The point E lies on the diagonal DB such that

$$DE = \frac{1}{4}DB.$$

- (b) Find the coordinates of E . (2)

- The point F is such that EF is parallel to AB .
- The area of trapezium $AEFB$ is

$$\frac{3}{2} \times (\text{the area of parallelogram } ABCD).$$

- (c) Find the coordinates of F . (6)