Dr Oliver Mathematics
AQA Further Maths Level 2
June 2018 Paper 1
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.
You are **not** permitted to use a scientific or graphical calculator in this paper.

1. (2)

$$y = \tfrac{1}{2}x^6 + \tfrac{1}{4}x^4.$$

Work out $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.
Simplify your answer.

Solution

$$y = \tfrac{1}{2}x^6 + \tfrac{1}{4}x^4 \Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^5 + x^3}}.$$

2. P is the point $(-12, b)$. (3)
Q is the point $(a, 4)$.
R is the point $(6, -2)$.
Q is the midpoint of PR.

Work out the values of a and b.

Solution

$$\begin{aligned} a &= \frac{-12+6}{2} \\ &= \frac{-6}{2} \\ &= \underline{\underline{-3}} \end{aligned}$$

and

$$\begin{aligned} 4 = \frac{b+(-2)}{2} &\Rightarrow 8 = b - 2 \\ &\Rightarrow \underline{\underline{b = 10}}. \end{aligned}$$

3. (2)

$$\mathbf{A} = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -2 & 6 \\ 2 & 1 \end{pmatrix}.$$

Work out **AB**.

Solution

$$\begin{aligned} \mathbf{AB} &= \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} -2 & 6 \\ 2 & 1 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} 4 & 16 \\ -8 & 17 \end{pmatrix}}}. \end{aligned}$$

4. $P = 4x$ and $Q = 7x$. (4)
P increases by 25%.
Q decreases by 40%.
Now, P is 28 greater than Q.

Work out the value of x.

Solution

$$\begin{aligned} (1 + 0.25)(4x) = (1 - 0.4)(7x) + 28 &\Rightarrow (1.25)(4x) = (0.6)(7x) + 28 \\ &\Rightarrow 5x = 4.2x + 28 \\ &\Rightarrow 0.8x = 28 \\ &\Rightarrow x = 28 \div 0.8 \\ &\Rightarrow x = 28 \times \tfrac{5}{4} \\ &\Rightarrow x = 7 \times 5 \\ &\Rightarrow \underline{\underline{x = 35}}. \end{aligned}$$

5. In the expansion and simplification of (3)

$$(x - 3)(x^2 + 5x + k),$$

the coefficient of x^2 is equal to the coefficient of x.
k is a constant.

Work out the value of k.

Solution

$\times$	x^2	$+5x$	$+k$
x	x^3	$+5x^2$	$+kx$
-3	$-3x^2$	$-15x$	$-3k$

Finally,

$$5 - 3 = k - 15 \Rightarrow \underline{\underline{k = 17}}.$$

6. A circle has centre $(-1, 2)$ and radius 5. (1)
Which of these is the equation of the circle?
Tick **one** box.

$(x + 1)^2 + (y - 2)^2 = 5$ ☐

$(x - 1)^2 + (y + 2)^2 = 5$ ☐

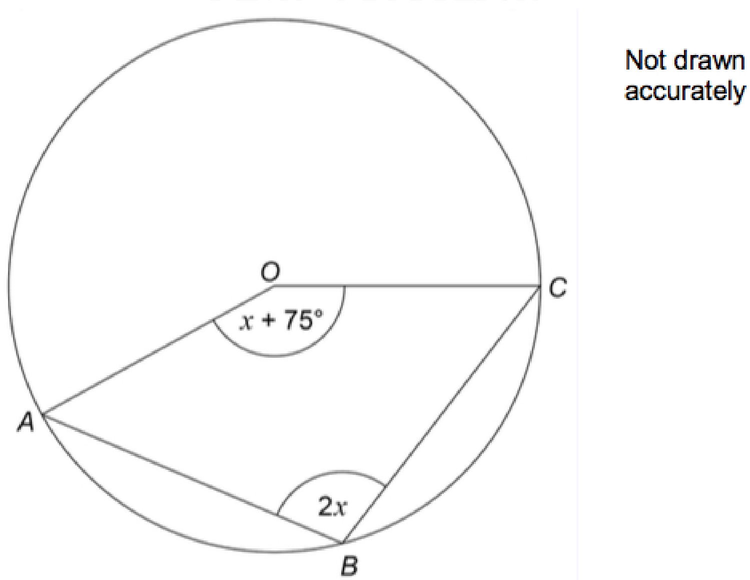
$(x + 1)^2 + (y - 2)^2 = 25$ ☐

$(x - 1)^2 + (y + 2)^2 = 25$ ☐

Solution

The third box: $(x + 1)^2 + (y - 2)^2 = 25$.

7. Points A, B, and C lie on a circle, centre O. (3)
Angle $AOC = (x + 75)^\circ$.
Angle $ABC = (2x)^\circ$.



Work out the value of x.

Solution

Let us make B at the top of the circle.
$\angle ABC = 180 - 2x$ (opposite angles in a cyclic quadrilateral are equal).
Now, the angle at the circumference is twice is angle at the centre:

$$\begin{aligned} 2(180 - 2x) = x + 75 &\Rightarrow 360 - 4x = x + 75 \\ &\Rightarrow 285 = 5x \\ &\Rightarrow \underline{\underline{x = 57}}. \end{aligned}$$

8. Write (2)

$$(1 + 2\sqrt{5})(4 - \sqrt{5})$$

in the form $a + b\sqrt{5}$, where a and b are integers.

Solution

$\times$	1	$+2\sqrt{5}$
4	4	$+8\sqrt{5}$
$-\sqrt{5}$	$-\sqrt{5}$	-10

$$(1 + 2\sqrt{5})(4 - \sqrt{5}) = \underline{\underline{-6 + 7\sqrt{5}}};$$

so, $\underline{\underline{a = -6}}$ and $\underline{\underline{b = 7}}$.

9. (4)

$$f(x) = 14 - x^2,$$

for all real values of x.

Solve

$$f(2x) = 5.$$

You **must** show your working.

Solution

$$\begin{aligned} f(2x) = 5 &\Rightarrow 14 - (2x)^2 = 5 \\ &\Rightarrow 14 - 4x^2 = 5 \\ &\Rightarrow 4x^2 = 9 \\ &\Rightarrow x^2 = \tfrac{9}{4} \\ &\Rightarrow \underline{\underline{x = \pm 1\tfrac{1}{2}}}. \end{aligned}$$

10. Rearrange (3)

$$\frac{1}{xy} = 4 - \frac{3}{y}$$

to make x the subject.

Solution

$$\begin{aligned} \frac{1}{xy} = 4 - \frac{3}{y} &\Rightarrow \frac{1}{xy} = \frac{4y}{y} - \frac{3}{y} \\ &\Rightarrow \frac{1}{xy} = \frac{4y - 3}{y} \\ &\Rightarrow xy = \frac{y}{4y - 3} \\ &\Rightarrow \underline{\underline{x = \frac{1}{4y - 3}}}. \end{aligned}$$

11. A curve has equation (4)

$$y = 2x^2 + 3x - 9.$$

At a point P on the curve, the tangent is parallel to the line

$$y = 4 - 5x.$$

Work out the coordinates of P.
You **must** show your working.

Solution

$$y = 2x^2 + 3x - 9 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4x + 3$$

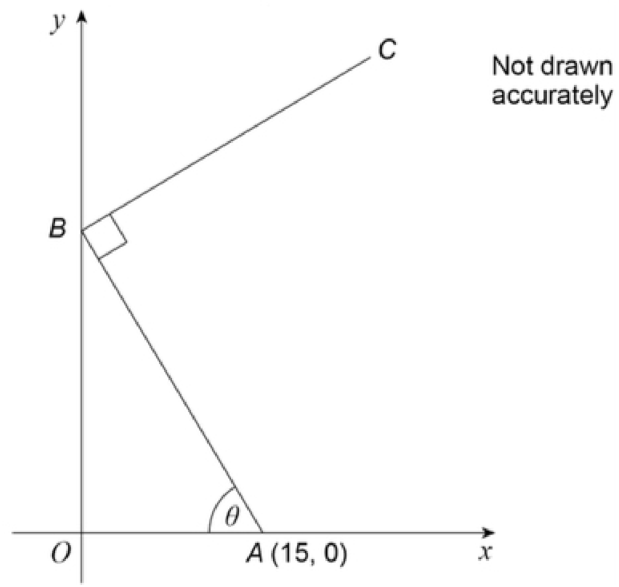
and

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} = -5 &\Rightarrow 4x + 3 = -5 \\ &\Rightarrow 4x = -8 \\ &\Rightarrow x = -2 \\ &\Rightarrow y = 2[(-2)^2] + 3(-2) - 9 \\ &\Rightarrow y = 8 - 6 - 9 \\ &\Rightarrow y = -7; \end{aligned}$$

hence, $\underline{\underline{P(-2, -7)}}$.

12. In the diagram, (4)

- A is the point $(15, 0)$ and B lies on the y-axis.
- Angle $ABC = 90^\circ$ and $\tan\theta = \frac{5}{3}$.



Work out the equation of the line BC.

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan\theta = \frac{OB}{15}$$
$$\Rightarrow \tfrac{5}{3} = \frac{OB}{15}$$
$$\Rightarrow OB = \tfrac{5}{3} \times 15$$
$$\Rightarrow OB = 5 \times 5$$
$$\Rightarrow OB = 25;$$

hence, $B(0, 25)$. Now,

$$\tan\theta = \tfrac{5}{3} \Rightarrow \angle ABO = \tfrac{3}{5}$$

and a point that lies on BC is, for example, $(5, 28)$. Hence, the equation of the line BC is

$$y - 25 = \tfrac{3}{5}(x - 0) \Rightarrow y - 25 = \tfrac{3}{5}x$$
$$\Rightarrow \underline{\underline{y = \tfrac{3}{5}x + 25}}.$$

13. Solve the simultaneous equations (6)

$$xy = 2 \text{ and } y = 3x + 5.$$

Do **not** use trial and improvement.
You **must** show your working.

Solution

$$\begin{aligned} xy = 2 &\Rightarrow x(3x+5) = 2 \\ &\Rightarrow 3x^2 + 5x = 2 \\ &\Rightarrow 3x^2 + 5x - 2 = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & +5 \\ \text{multiply to:} & (+3) \times (-2) = -6 \end{array}\right\} -1,\ +6$$

$$\begin{aligned} &\Rightarrow 3x^2 + 6x - x - 2 = 0 \\ &\Rightarrow 3x(x+2) - (x+2) = 0 \\ &\Rightarrow (3x-1)(x+2) = 0 \\ &\Rightarrow x = \tfrac{1}{3} \text{ or } x = -2 \\ &\Rightarrow y = 6 \text{ or } y = -1; \end{aligned}$$

hence, the solutions are

$$\underline{\underline{x = \tfrac{1}{3},\ y = 6 \text{ or } x = -2,\ y = -1.}}$$

14. Work out the value of (3)

$$\left(3^{\frac{1}{2}} + 3^{\frac{3}{2}}\right)^2.$$

You **must** show your working.

Solution

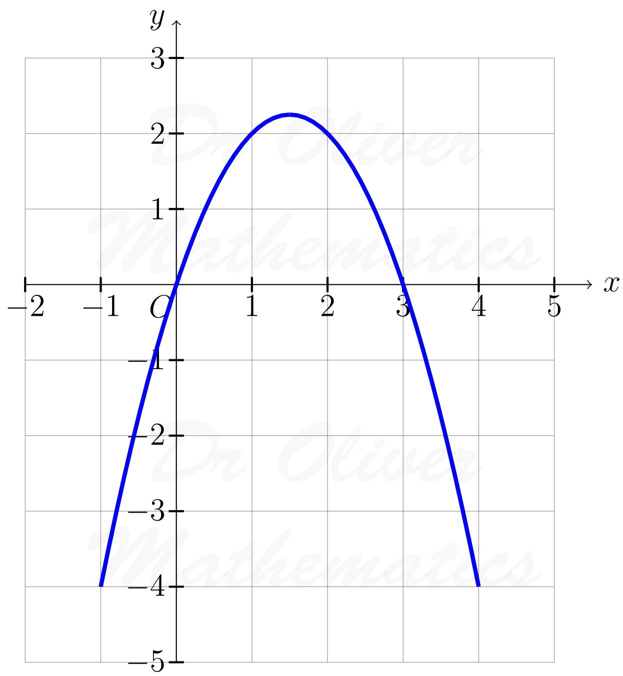
$\times$	$3^{\frac{1}{2}}$	$3^{\frac{3}{2}}$
$3^{\frac{1}{2}}$	3	$+3^2$
$+3^{\frac{3}{2}}$	$+3^2$	$+3^3$

$$\left(3^{\frac{1}{2}} + 3^{\frac{3}{2}}\right)^2 = 3 + 9 + 9 + 27$$
$$= \underline{\underline{48}}.$$

15. Here is the graph of (4)

$$y = 3x - x^2$$

for values of x from -1 to 4.



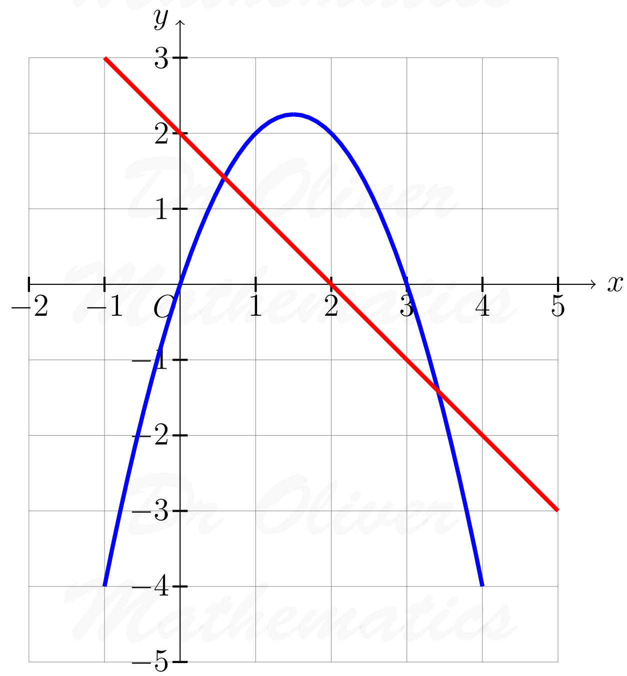
By drawing a suitable **linear** graph on the grid, work out approximate solutions to

$$x^2 - 4x + 2 = 0.$$

Solution

$$\begin{aligned} x^2 - 4x + 2 = 0 &\Rightarrow -4x + 2 = -x^2 \\ &\Rightarrow -x + 2 = 3x - x^2 \end{aligned}$$

so draw in $y = -x + 2$.

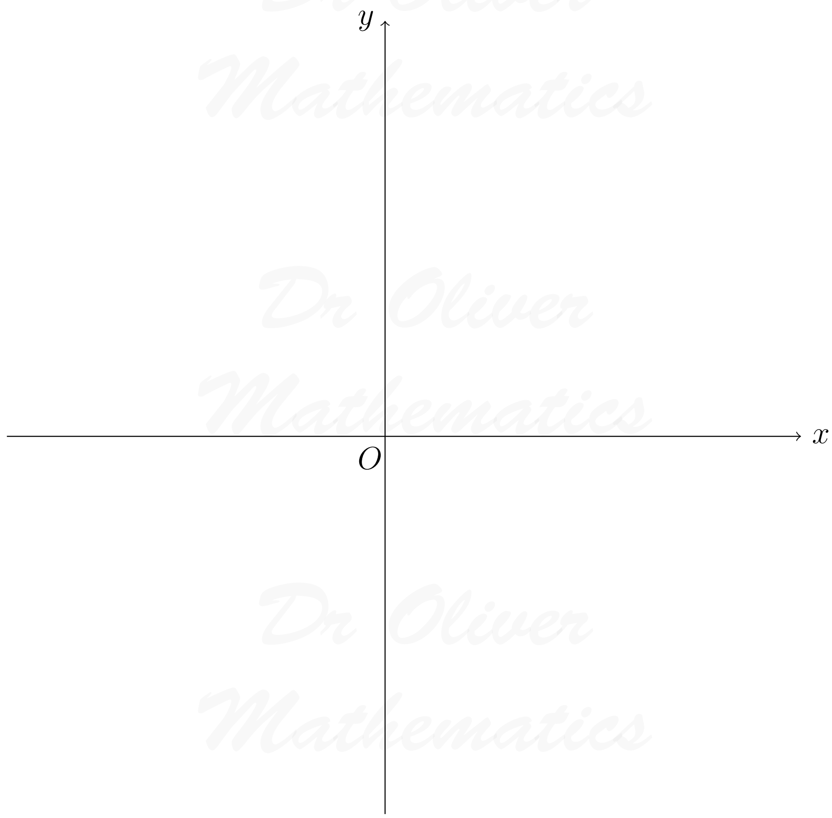


Correct read-off: approximately $\underline{\underline{x = 0.6}}$ or $\underline{\underline{x = 3.4}}$.

16. $y = \mathrm{f}(x)$ is a cubic curve with a maximum and a minimum stationary point. (4)

- $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 2x - 3$.
- The y-coordinate of the minimum point is $2\frac{1}{3}$.
- The y-coordinate of the maximum point is 13
- $(0, 4)$ is a point on the curve.
- The tangent at $(0, 4)$ has a negative gradient.

Sketch the curve on the grid below.
Show the coordinates of the stationary points.

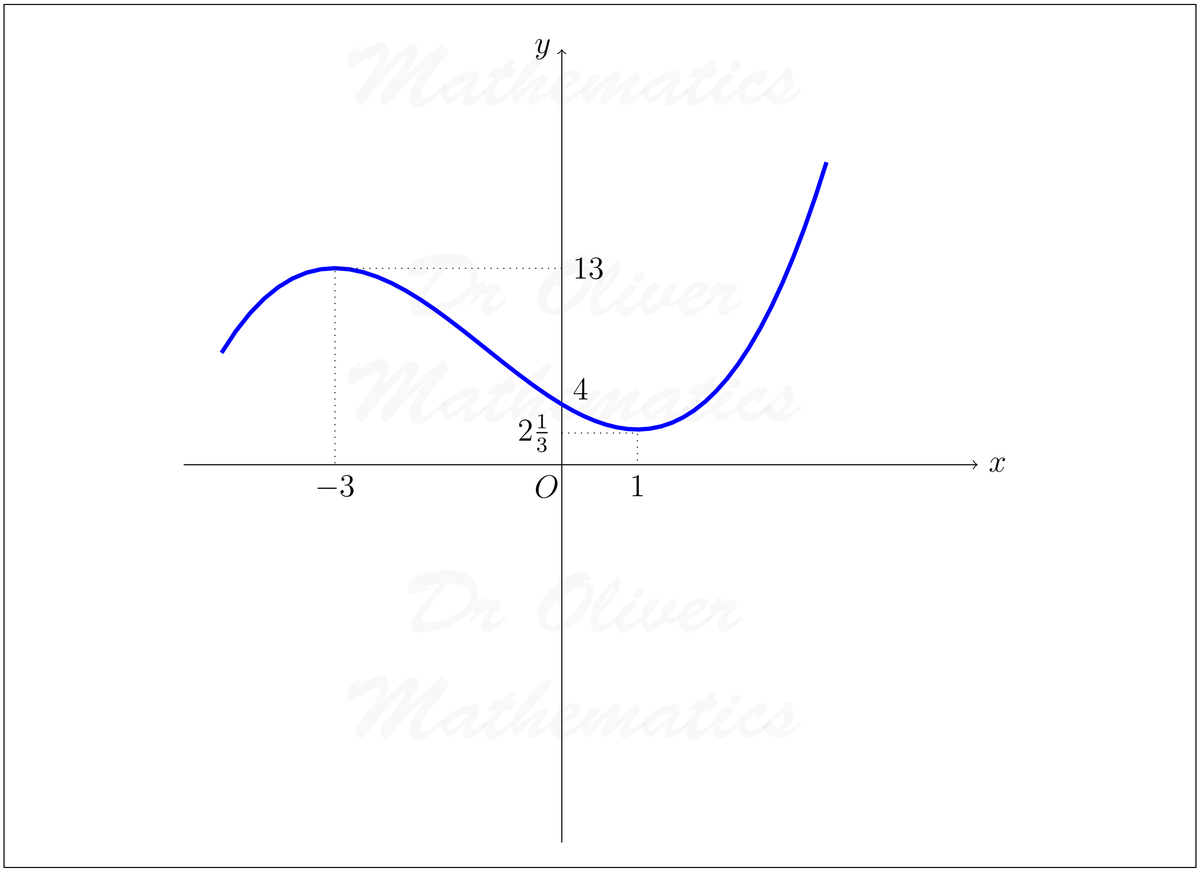


Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow x^2 + 2x - 3 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & +2 \\ \text{multiply to:} & -3 \end{array}\right\} -3,\ +1$$

$$\begin{aligned} &\Rightarrow (x-3)(x+1) = 0 \\ &\Rightarrow x = 3 \text{ or } x = -1. \end{aligned}$$



17. (a) Use the factor theorem to show that $(x-2)$ is a factor of (1)

$$x^3 + 8x^2 + 5x - 50.$$

Solution

Let

$$f(x) = x^3 + 8x^2 + 5x - 50.$$

Now,

$$\begin{aligned} f(2) &= 2^3 + 8(2^2) + 5(2) - 50 \\ &= 8 + 32 + 10 - 50 \\ &= 0. \end{aligned}$$

As their is no remainder, $\underline{\underline{(x-2)}}$ is a factor of

$$x^3 + 8x^2 + 5x - 50.$$

(b) Hence, factorise fully (3)

$$x^3 + 8x^2 + 5x - 50.$$

Solution

We use synthetic division:

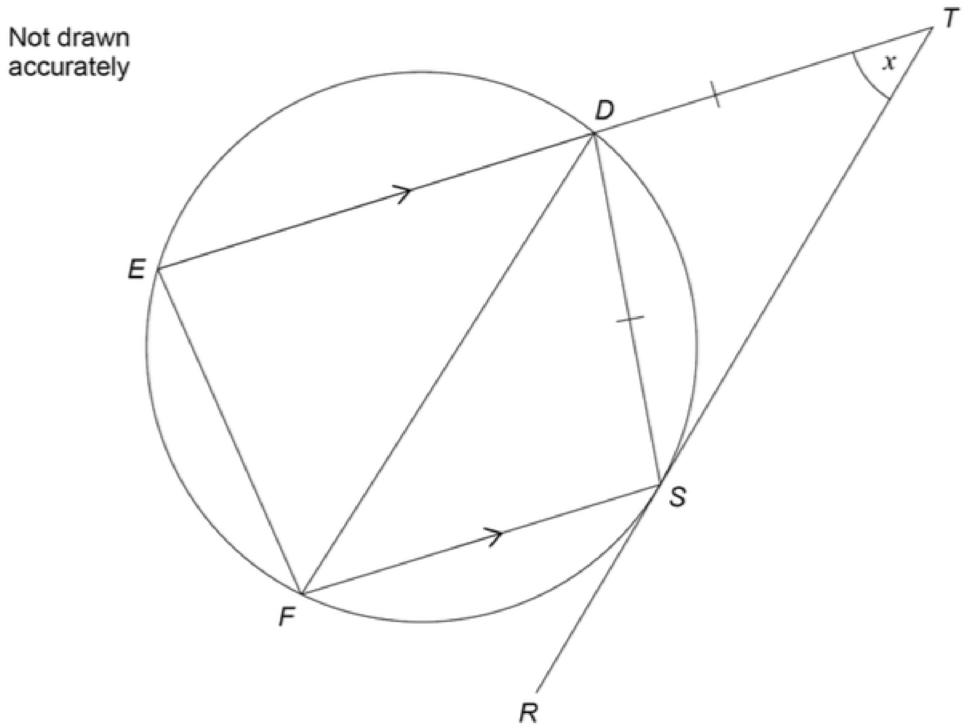
2	1	8	5	−50
	↓	2	20	50
	1	10	25	0

$$x^3 + 8x^2 + 5x - 50 = (x-2)(x^2 + 10x + 25)$$

$$\left.\begin{array}{ll}\text{add to:} & +10 \\ \text{multiply to:} & +25\end{array}\right\} +5,\ +5$$

$$= \underline{\underline{(x-2)(x+5)^2}}.$$

18. D, E, F, and S are points on a circle. (5)
RST is a tangent.
The straight line EDT is parallel to FS.
$DS = DT$.



Prove that FD is parallel to RST.
Use angle DTS as x to help you.

Solution

$\angle DST = x$ (base angles)
$\angle SDT = 180 - 2x$ (angles on a straight line)
$\angle DFS = x$ (alternate segment theorem)
$\angle EDF = x$ (alternate angles)
$\angle FDS = 180 - (180 - 2x) - x = x$ (angles on a straight line)
But $\angle FDS = \angle DST$ which means that FD is parallel to RST.

19. Write (4)

$$2x^2 - 16x + 13$$

in the form

$$a(x + b)^2 + c,$$

where a, b, and c are integers.

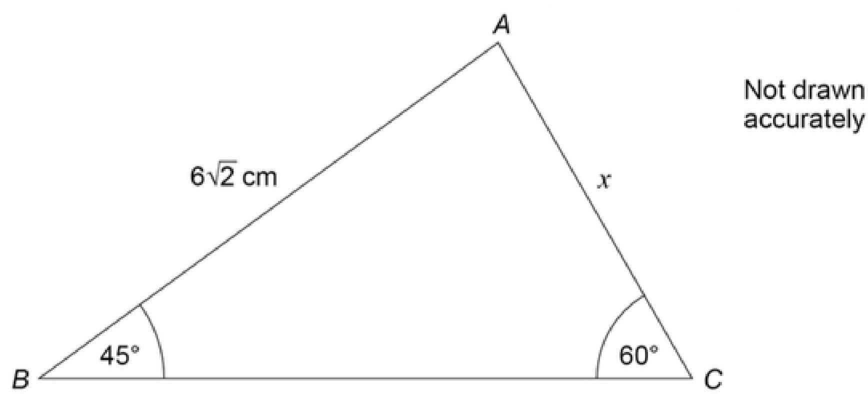
Solution

$$\begin{aligned} 2x^2 - 16x + 13 &= 2(x^2 - 8x) + 13 \\ &= 2[(x^2 - 8x + 16) - 16] + 13 \\ &= 2[(x-4)^2 - 16] + 13 \\ &= 2(x-4)^2 - 32 + 13 \\ &= \underline{\underline{2(x-4)^2 - 19}}; \end{aligned}$$

hence, $\underline{\underline{a = 2}}$, $\underline{\underline{b = -4}}$, and $\underline{\underline{c = -19}}$.

20. In triangle ABC, (5)

- $AB = 6\sqrt{2}$ cm,
- angle $ABC = 45°$, and
- angle $ACB = 60°$.



Work out the value of x.
Give your answer in the form $a\sqrt{b}$, where a and b are integers.
You must show your working.

Solution

We use the sine rule:

$$\frac{AC}{\sin ABC} = \frac{AB}{\sin ACB} \Rightarrow \frac{x}{\sin 45^\circ} = \frac{6\sqrt{2}}{\sin 60^\circ}$$
$$\Rightarrow x = \frac{6\sqrt{2} \cdot \sin 45^\circ}{\sin 60^\circ}$$
$$\Rightarrow x = \frac{6\sqrt{2} \cdot \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}}$$
$$\Rightarrow x = 6 \times \frac{2}{\sqrt{3}}$$
$$\Rightarrow x = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$\Rightarrow x = \frac{12\sqrt{3}}{3}$$
$$\Rightarrow \underline{\underline{x = 4\sqrt{3}}};$$

$\underline{\underline{a = 4}}$ and $\underline{\underline{b = 3}}$.