Dr Oliver Mathematics
Mathematics
Differentiation Part 1
Past Examination Questions

This booklet consists of 34 questions across a variety of examination topics.
The total number of marks available is 131.

1. Given that $y = 5x^3 + 7x + 3$, find

   (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, (3)

   Solution

   $$\begin{aligned} y = 5x^3 + 7x + 3 &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 5 \times 3x^2 + 7 \times 1 + 0 \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 15x^2 + 7}}. \end{aligned}$$

   (b) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. (1)

   Solution

   $$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 15 \times 2x = \underline{\underline{30x}}.$$

2. The curve C has equation $y = 4x^2 + \dfrac{5-x}{x}$, $x \neq 0$.

   The point P on C has x-coordinate 1.

   Show that the value $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at P is 3.

   Solution

   $$\begin{aligned} & y = 4x^2 + \frac{5-x}{x} \\ \Rightarrow\ & y = 4x^2 + 5x^{-1} - 1 \\ \Rightarrow\ & \frac{\mathrm{d}y}{\mathrm{d}x} = 4 \times 2x + 5 \times (-x^{-2}) + 0 \\ \Rightarrow\ & \frac{\mathrm{d}y}{\mathrm{d}x} = 8x - 5x^{-2}. \end{aligned}$$

When $x = 1$,

$$\frac{dy}{dx} = 8 \times 1 - 5 \times 1^{-2} = 8 - 5 = \underline{\underline{3}}.$$

3. Given that $y = 6x - \frac{4}{x^2}$, $x \neq 0$, find $\frac{dy}{dx}$. (2)

Solution

$$\begin{aligned} y = 6x - \frac{4}{x^2} &\Rightarrow y = 6x - 4x^{-2} \\ &\Rightarrow \frac{dy}{dx} = 6 \times 1 - 4 \times (-2x^{-3}) \\ &\Rightarrow \underline{\underline{\frac{dy}{dx} = 6 + 8x^{-3}}}. \end{aligned}$$

4. Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$, find $\frac{dy}{dx}$. (2)

Solution

$$\begin{aligned} y = 2x^2 - \frac{6}{x^3} &\Rightarrow y = 2x^2 - 6x^{-3} \\ &\Rightarrow \frac{dy}{dx} = 2 \times 2x - 6 \times (-3x^{-4}) \\ &\Rightarrow \underline{\underline{\frac{dy}{dx} = 4x + 18x^{-4}}}. \end{aligned}$$

5. (3)

$$y = (x - 1)(x^2 - 4).$$

Show that $\frac{dy}{dx} = 3x^2 - 2x - 4$.

Solution

	x^2	-4
x	x^3	$-4x$
-1	$-x^2$	$+4$

$$y = x^3 - x^2 - 4x + 4 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 2x - 4 \times 1 + 0$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 2x - 4}}.$$

6. Differentiate with respect to x:

   (a) $x^4 + 6\sqrt{x}$, (3)

Solution

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^4 + 6\sqrt{x}) = \frac{\mathrm{d}}{\mathrm{d}x}(x^4 + 6x^{\frac{1}{2}})$$
$$= 4x^3 + 6 \times \tfrac{1}{2}x^{-\frac{1}{2}}$$
$$= \underline{\underline{4x^3 + 3x^{-\frac{1}{2}}}}.$$

   (b) $\dfrac{(x+4)^2}{x}$. (4)

Solution

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{(x+4)^2}{x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^2 + 8x + 16}{x}\right)$$
$$= \frac{\mathrm{d}}{\mathrm{d}x}\left(x + 8 + 16x^{-1}\right)$$
$$= 1 + 0 + 16 \times (-x^{-2})$$
$$= \underline{\underline{1 - 16x^{-2}}}.$$

7. Given that (4)

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}},\ x > 0,$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Solution

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4 \times 3x^2 + 0 + 2 \times \tfrac{1}{2}x^{-\frac{1}{2}}$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 12x^2 + x^{-\frac{1}{2}}}}.$$

8. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$. (3)

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Solution

$$y = 4x + 3x^{\frac{3}{2}} - 2x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4 \times 1 + 3 \times \tfrac{3}{2}x^{\frac{1}{2}} - 2 \times 2x$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 4 + \tfrac{9}{2}x^{\frac{1}{2}} - 4x}}.$$

9. Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find

(a) $\frac{\mathrm{d}y}{\mathrm{d}x}$, (2)

Solution

$$y = 3x^2 + 4\sqrt{x} \Rightarrow y = 3x^2 + 4x^{\frac{1}{2}}$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3 \times 2x + 4 \times \tfrac{1}{2}x^{-\frac{1}{2}}$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 6x + 2x^{-\frac{1}{2}}}}.$$

(b) $\dfrac{d^2y}{dx^2}$. (2)

Solution

$$\frac{dy}{dx} = 6x + 2x^{-\frac{1}{2}} \Rightarrow \frac{d^2y}{dx^2} = 6 \times 1 + 2 \times (-\tfrac{1}{2}x^{-\frac{3}{2}})$$
$$\Rightarrow \underline{\underline{\frac{d^2y}{dx^2} = 6 - x^{-\frac{3}{2}}}}.$$

10. (a) Write $\dfrac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$, where p and q are constants. (2)

Solution

$$\frac{2\sqrt{x}+3}{x} = \frac{2x^{\frac{1}{2}}+3}{x} = \underline{\underline{2x^{-\frac{1}{2}} + 3x^{-1}}}.$$

Given that $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term. (4)

Solution

$$y = 5x - 7 + \frac{2\sqrt{x}+3}{x} \Rightarrow y = 5x - 7 + 2x^{-\frac{1}{2}} + 3x^{-1}$$
$$\Rightarrow \frac{dy}{dx} = 5 \times 1 + 0 + 2 \times (-\tfrac{1}{2}x^{-\frac{3}{2}}) + 3 \times (-x^{-2})$$
$$\Rightarrow \underline{\underline{\frac{dy}{dx} = 5 - x^{-\frac{3}{2}} - 3x^{-2}}}.$$

11. (2)

$$f(x) = 3x + x^3.$$

Differentiate to find $f'(x)$.

Solution

$$f(x) = 3x + x^3 \Rightarrow f'(x) = 3 \times 1 + 3x^2 \Rightarrow \underline{\underline{f'(x) = 3 + 3x^2}}.$$

12. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant. (2)

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Solution

$$y = kx^3 - x^2 + x - 5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = k \times 3x^2 - 2x + 1 + 0$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 3kx^2 - 2x + 1}}.$$

13. Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and write down the value of q. (2)

Solution

$$\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}} = \frac{2x^2 - x^{\frac{3}{2}}}{x^{\frac{1}{2}}} = 2x^{\frac{3}{2}} - x,$$

hence, $\underline{\underline{p = \frac{3}{2}}}$ and $\underline{\underline{q = 1}}$

Given that $y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\frac{\mathrm{d}y}{\mathrm{d}x}$, simplifying the coefficient of each term. (4)

Solution

$$y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}} \Rightarrow y = 5x^4 - 3 + 2x^{\frac{3}{2}} - x$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 5 \times 4x^3 + 0 + 2 \times \tfrac{3}{2}x^{\frac{1}{2}} - 1$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 20x^3 + 3x^{\frac{1}{2}} - 1}}.$$

14. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. (3)

Solution

$$y = 2x^3 + \frac{3}{x^2} \Rightarrow y = 2x^3 + 3x^{-2}$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2 \times 3x^2 + 3 \times (-2x^{-3})$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 6x^{-3}}}.$$

15.

$$\mathrm{f}(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}},\ x > 0.$$

(a) Show that $\mathrm{f}(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found. (3)

Solution

$$\mathrm{f}(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}$$
$$= \frac{9 - 24x^{\frac{1}{2}} + 16x}{x^{\frac{1}{2}}}$$
$$= \underline{\underline{9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24}}.$$

(b) Find $\mathrm{f}'(x)$. (3)

Solution

$$\mathrm{f}(x) = 9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24 \Rightarrow \mathrm{f}'(x) = 9 \times (-\tfrac{1}{2}x^{-\frac{3}{2}}) + 16 \times \tfrac{1}{2}x^{-\frac{1}{2}} + 0$$
$$\Rightarrow \underline{\underline{\mathrm{f}'(x) = -\tfrac{9}{2}x^{-\frac{3}{2}} + 8x^{-\frac{1}{2}}}}.$$

16. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. (3)

Solution

$$y = x^4 + x^{\frac{1}{3}} + 3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 + \tfrac{1}{3}x^{-\frac{2}{3}} + 0$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 + \tfrac{1}{3}x^{-\frac{2}{3}}}}.$$

17. The curve C has equation (4)

$$y = \frac{(x+3)(x-8)}{x},\ x > 0.$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form.

Solution

$$y = \frac{(x+3)(x-8)}{x} \Rightarrow y = \frac{x^2 - 5x - 24}{x}$$
$$\Rightarrow y = x - 5 - 24x^{-1}$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 1 + 0 - 24 \times (-x^{-2})$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + 24x^{-2}}}.$$

18. Given that (6)

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x},\ x > 0,$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Solution

$$
\begin{aligned}
& y = 8x^3 - 4\sqrt{x} + \frac{3x^2+2}{x} \\
\Rightarrow\quad & y = 8x^3 - 4x^{\frac{1}{2}} + 3x + 2x^{-1} \\
\Rightarrow\quad & \frac{\mathrm{d}y}{\mathrm{d}x} = 8 \times 3x^2 - 4 \times \tfrac{1}{2}x^{-\frac{1}{2}} + 3 + 2 \times (-x^{-2}) \\
\Rightarrow\quad & \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}}}.
\end{aligned}
$$

19. The curve C has equation (4)

$$y = \tfrac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30,\ x > 0.$$

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Solution

$$
\begin{aligned}
& y = \tfrac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30 \\
\Rightarrow\quad & y = \tfrac{1}{2}x^3 - 9x^{\frac{3}{2}} + 8x^{-1} + 30 \\
\Rightarrow\quad & \frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2} \times 3x^2 - 9 \times \tfrac{3}{2}x^{\frac{1}{2}} + 8 \times (-x^{-2}) + 0 \\
\Rightarrow\quad & \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{3}{2}x^2 - \tfrac{27}{2}x^{\frac{1}{2}} - 8x^{-2}}}.
\end{aligned}
$$

20. Given that $y = 2x^5 + 7 + \dfrac{1}{x^3}$, $x \neq 0$, find, in its simplest form, $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (3)

Solution

$$
\begin{aligned}
y = 2x^5 + 7 + \frac{1}{x^3} &\Rightarrow y = 2x^5 + 7 + x^{-3} \\
&\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2 \times 5x^4 + 0 + (-3x^{-4}) \\
&\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 10x^4 - 3x^{-4}}}.
\end{aligned}
$$

21. The curve C has equation $y = (x+1)(x+3)^2$. (3)

Find $\frac{dy}{dx}$.

Solution

$$y = (x+1)(x+3)^2 = (x+1)(x^2+6x+9)$$

and

	x^2	$+6x$	$+9$
x	x^3	$+6x^2$	$+9x$
$+1$	$+x^2$	$+6x$	$+9$

we have

$$y = x^3 + 7x^2 + 15x + 9 \Rightarrow \frac{dy}{dx} = 3x^2 + 7 \times 2x + 15 \times 1 + 0$$
$$\Rightarrow \frac{dy}{dx} = 3x^2 + 14x + 15.$$

22. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find, in the simplest form, $\frac{dy}{dx}$. (3)

Solution

$$y = x^4 + 6x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = 4x^3 + 6 \times \tfrac{1}{2}x^{-\frac{1}{2}}$$
$$\Rightarrow \frac{dy}{dx} = 4x^3 + 3x^{-\frac{1}{2}}.$$

23. (2)

$$y = x^2(x+2).$$

Find $\frac{dy}{dx}$.

Solution

$$\begin{aligned} y = x^2(x+2) &\Rightarrow y = x^3 + 2x^2 \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2 \times 2x \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 4x.}} \end{aligned}$$

24.

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3.$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving each term in its simplest form. (4)

Solution

$$\begin{aligned} & y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3 \\ \Rightarrow \quad & \frac{\mathrm{d}y}{\mathrm{d}x} = 5 \times 3x^2 - 6 \times \tfrac{4}{3}x^{\frac{1}{3}} + 2 \times 1 + 0 \\ \Rightarrow \quad & \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 15x^2 - 8x^{\frac{1}{3}} + 2.}} \end{aligned}$$

(b) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. (2)

Solution

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} = 15x^2 - 8x^{\frac{1}{3}} + 2 &\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 15 \times 2x - 8 \times \tfrac{1}{3}x^{-\frac{2}{3}} + 0 \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 30x - \tfrac{8}{3}x^{-\frac{2}{3}}.}} \end{aligned}$$

25. The curve C has equation (3)

$$y = 2x - 8\sqrt{x} + 5, \; x \geqslant 0.$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving each term in its simplest form.

Solution

$$\begin{aligned} y = 2x - 8\sqrt{x} + 5 \Rightarrow y &= 2x - 8x^{\frac{1}{2}} + 5 \\ \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 2 \times 1 - 8 \times \tfrac{1}{2}x^{-\frac{1}{2}} + 0 \\ \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 2 - 4x^{-\frac{1}{2}}. \end{aligned}$$

26.

$$\mathrm{f}(x) = \frac{(3 - x^2)^2}{x^2},\ x \neq 0.$$

(a) Show that (3)

$$\mathrm{f}(x) = 9x^{-2} + A + Bx^2,$$

where A and B are constants to be found.

Solution

$$\begin{aligned} \mathrm{f}(x) &= \frac{(3 - x^2)^2}{x^2} \\ &= \frac{9 - 6x^2 + x^4}{x^2} \\ &= 9x^{-2} - 6 + x^2. \end{aligned}$$

(b) Find $\mathrm{f}'(x)$. (2)

Solution

$$\mathrm{f}'(x) = 9 \times (-2x^{-3}) + 0 + 2x = -18x^{-3} + 2x.$$

27. Given that $y = x^3 + 4x + 1$, find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 3$. (4)

Solution

$$y = x^3 + 4x + 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 4 \times 1 + 0 = 3x^2 + 4$$

and

$$x = 3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3 \times 3^2 + 4 = 3 \times 9 + 4 = 27 + 4 = 31.$$

28. Differentiate with respect to x, giving each answer in its simplest form.

(a) $(1-2x)^2$, (3)

Solution

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}[(1-2x)^2] &= \frac{\mathrm{d}}{\mathrm{d}x}(1-4x+4x^2)\\ &= 0-4\times 1+4\times 2x\\ &= \underline{\underline{-4+8x}}.\end{aligned}$$

(b) $\dfrac{x^5+6\sqrt{x}}{2x^2}$. (4)

Solution

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x^5+6\sqrt{x}}{2x^2}\right] &= \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x^5+6x^{\frac{1}{2}}}{2x^2}\right]\\ &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\tfrac{1}{2}x^3+3x^{-\frac{3}{2}}\right)\\ &= \tfrac{1}{2}\times 3x^2+3\times(-\tfrac{3}{2}x^{-\frac{5}{2}})\\ &= \underline{\underline{\tfrac{3}{2}x^2-\tfrac{9}{2}x^{-\frac{5}{2}}}}.\end{aligned}$$

29. Given that $y = 2x^5 + \dfrac{6}{\sqrt{x}}$, find, in the simplest form, $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (3)

Solution

$$\begin{aligned}y = 2x^5+\frac{6}{\sqrt{x}} &\Rightarrow y = 2x^5+6x^{-\frac{1}{2}}\\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2\times 5x^4+6\times(-\tfrac{1}{2}x^{-\frac{3}{2}})\\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 10x^4-3x^{-\frac{3}{2}}}}.\end{aligned}$$

30. Given that $y = 4x^3 - \dfrac{5}{x^2}$, find, in the simplest form, $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (3)

Solution

$$\begin{aligned} y = 4x^3 - \frac{5}{x^2} &\Rightarrow y = 4x^3 - 5x^{-2} \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4 \times 3x^2 - 5 \times (-2x^{-3}) \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 12x^2 + 10x^{-3}}}. \end{aligned}$$

31. The curve C has equation (5)

$$y = \frac{(x^2+4)(x-3)}{2x}.$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form.

Solution

	x^2	$+4$
x	x^3	$+4x$
-3	$-3x^2$	-12

$$\begin{aligned} y = \frac{(x^2+4)(x-3)}{2x} &\Rightarrow y = \frac{x^3 - 3x^2 + 4x - 12}{2x} \\ &\Rightarrow y = \tfrac{1}{2}x^2 - \tfrac{3}{2}x + 2 - 6x^{-1} \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2} \times 2x - \tfrac{3}{2} \times 1 + 0 - 6 \times (-x^{-2}) \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = x - \tfrac{3}{2} + 6x^{-2}}}. \end{aligned}$$

32. Given that $y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Give each term in your answer in its simplest form. (6)

Solution

$$\begin{aligned} y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}} &\Rightarrow y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3x^{\frac{1}{2}}} \\ &\Rightarrow y = 3x^2 + 6x^{\frac{1}{3}} + \tfrac{2}{3}x^{\frac{5}{2}} - \tfrac{7}{3}x^{-\frac{1}{2}} \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3 \times 2x + 6 \times \tfrac{1}{3}x^{-\frac{2}{3}} + \tfrac{2}{3} \times \tfrac{5}{2}x^{\frac{3}{2}} - \tfrac{7}{3} \times (-\tfrac{1}{2}x^{-\frac{3}{2}}) \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 6x + 2x^{-\frac{2}{3}} + \tfrac{5}{3}x^{\frac{3}{2}} + \tfrac{7}{6}x^{-\frac{3}{2}}}}. \end{aligned}$$

33. The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (2)

Solution

$$\begin{aligned} y = 2x^3 + kx^2 + 5x + 6 &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2 \times 3x^2 + k \times 2x + 5 \times 1 + 0 \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 + 2kx + 5}}. \end{aligned}$$

34. Given (5)

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \ x > 0.$$

find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

Solution

$$\begin{aligned} y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4 &\Rightarrow y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} + 4 \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2}x^{-\frac{1}{2}} + 4 \times (-\tfrac{1}{2}x^{-\frac{3}{2}}) + 0 \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}} \end{aligned}$$

and

$$
\begin{aligned}
x = 8 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= \tfrac{1}{2} \times 8^{-\frac{1}{2}} - 2 \times 8^{-\frac{3}{2}} \\
\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{2 \times 8^{\frac{1}{2}}} - \frac{2}{8^{\frac{3}{2}}} \\
\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{2\sqrt{8}} - \frac{2}{8\sqrt{8}} \\
\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{4}{16\sqrt{2}} - \frac{2}{16\sqrt{2}} \\
\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{2}{16\sqrt{2}} \\
\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{8\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= \tfrac{1}{16}\sqrt{2}.
\end{aligned}
$$