

# Dr Oliver Mathematics

## Worked Examples

### Quadratic Equation 2

**From:** CCEA 2019 January M4 Paper 1 (Calculator)

1. A method of trial and improvement will not be accepted.

(8)

$A$  is a fraction whose denominator is 3 more than its numerator.

A new fraction  $B$  is produced.

4 is added to the numerator of  $A$  to give the numerator of  $B$ .

4 is added to the denominator of  $A$  to give the denominator of  $B$ .

Fraction  $B$  is now larger than fraction  $A$  by  $\frac{1}{8}$ .

By setting up and solving a suitable equation find the original fraction  $A$ .

#### Solution

“ $A$  is a fraction whose denominator is 3 more than its numerator” so let the fraction

$$A = \frac{x}{x+3}.$$

For  $B$ , the numerator is

$$x+4,$$

the denominator is

$$(x+3)+4 = x+7,$$

and, hence,

$$B = \frac{x+4}{x+7}.$$

Now, “fraction  $B$  is now larger than fraction  $A$  by  $\frac{1}{8}$ ” and so

$$\frac{x+4}{x+7} - \frac{x}{x+3} = \frac{1}{8}$$

multiply by  $8(x+3)(x+7)$ :

$$\Rightarrow \frac{x+4}{x+7} \times 8(x+3)(x+7) - \frac{x}{x+3} \times 8(x+3)(x+7) = \frac{1}{8} \times 8(x+3)(x+7)$$

$$\Rightarrow 8(x+3)(x+4) - 8x(x+7) = (x+3)(x+7)$$

$$\begin{array}{r|rr}
 \times & x & +3 \\
 \hline
 x & x^2 & +3x \\
 +4 & +4x & +12 \\
 \hline
 \end{array}$$

$$\begin{array}{r|rr}
 \times & x & +3 \\
 \hline
 x & x^2 & +3x \\
 +7 & +7x & +21 \\
 \hline
 \end{array}$$

$$\Rightarrow 8(x^2 + 7x + 12) - 8x(x + 7) = x^2 + 10x + 21$$

$$\Rightarrow 8x^2 + 56x + 96 - 8x^2 - 56x = x^2 + 10x + 21$$

simplify the terms on the LHS:

$$\Rightarrow 96 = x^2 + 10x + 21$$

$$\Rightarrow 0 = x^2 + 10x - 75$$

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array} \right\} \begin{array}{l} +10 \\ -75 \end{array} \Rightarrow +15, -5$$

$$\Rightarrow 0 = (x + 15)(x - 5)$$

$$\Rightarrow x = -15 \text{ or } x = 5.$$

If  $x = -15$ ,

$$A = \frac{-15}{-15+3} = \frac{-15}{-12} = \frac{5}{4}$$

and

$$B = \frac{9}{8};$$

if  $x = 5$ ,

$$A = \frac{5}{8}$$

and

$$B = \frac{9}{12} = \frac{3}{4}.$$

Which is it? Well,

$$\frac{9}{8} - \frac{5}{4} = \frac{9}{8} - \frac{10}{8} = -\frac{1}{8},$$

which fails the test (“Fraction  $B$  is now larger than fraction  $A$  by  $\frac{1}{8}$ ”). Next,

$$\frac{3}{4} - \frac{5}{8} = \frac{6}{8} - \frac{5}{8} = \frac{1}{8},$$

which passes the test.

Hence, the the original fraction  $A$  is

$$\underline{\underline{A = \frac{5}{8}}}.$$