Dr Oliver Mathematics
GCSE Mathematics
2004 June Paper 6H: Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. The manager of a school canteen has made some changes. (2)
   She wants to find out what students think of these changes.
   She uses this question on a questionnaire.

   How much money do you normally spend in the canteen?

   ☐   ☐
   A lot   Not much

   Design a better question for the canteen manager to use.
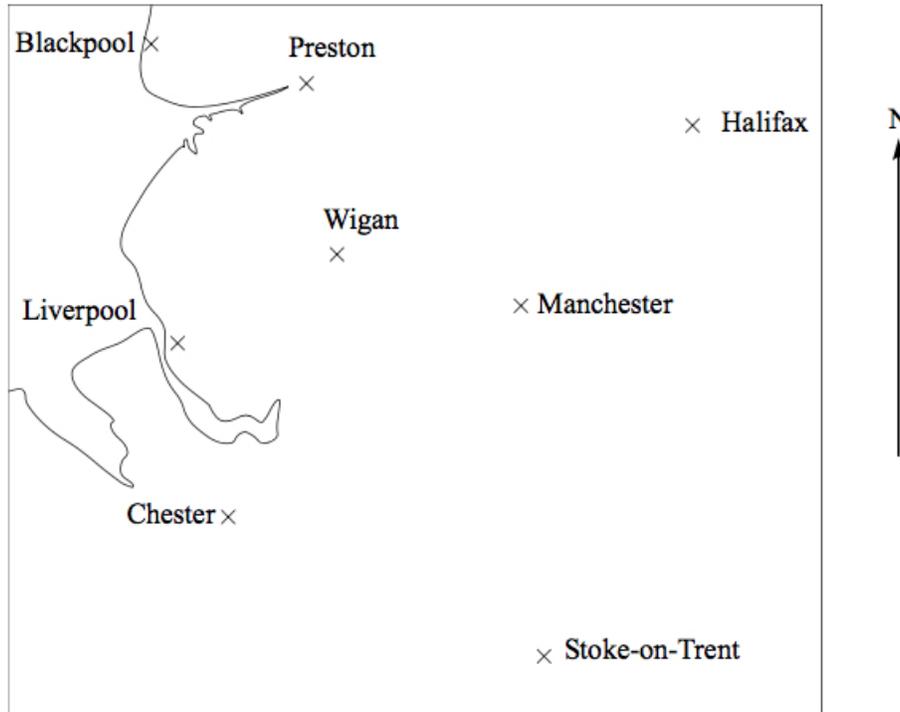   You should include some response boxes.

   Solution

   A suitable question with a time frame, e.g., "Did you use the school canteen today/last week/last month? Tick the appropriate box."
   At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, Never used it, $£0 < \text{money} \leqslant £2$, $£2 < \text{money} \leqslant £5$, $\text{money} \geqslant £5$.

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2. This is a map of part of Northern England. (2)

Scale: 1 cm represents 10 km

A radio station in Manchester transmits programmes.
Its programmes can be received anywhere within a distance of 30 km.
On the diagram, shade the region in which the programmes can be received.

Solution

Shade the interior of the dotted line, centre Manchester, 3 cm.

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3. The table shows the number of computer games sold in a supermarket each month from January to June.

Jan	Feb	Mar	Apr	May	Jun
147	161	238	135	167	250

(a) Work out the three-month moving averages for this information. (2)

Solution

Jan-Mar:
$$\frac{147 + 161 + 238}{3} = \frac{546}{3} = \underline{\underline{182}}.$$

Feb-Apr:
$$\frac{161 + 238 + 135}{3} = \frac{534}{3} = \underline{\underline{178}}.$$

Mar-May:
$$\frac{238 + 135 + 167}{3} = \frac{540}{3} = \underline{\underline{180}}.$$

Apr-Jun:
$$\frac{135 + 167 + 250}{3} = \frac{552}{3} = \underline{\underline{184}}.$$

In a sale, a supermarket took 20% off its normal prices.
On Fun Friday, it took 30% off its sale prices.
Fred says, "That means there was 50% off the normal prices."

(b) Fred is wrong. Explain why. (2)

Solution
$$0.8 \times 0.7 = 0.56;$$
so, 20% and then a 30% reduction equals $\underline{44\% \text{ off}}$.

4. The equation (4)
$$x^3 - 2x = 67$$

has a solution between 4 and 5.
Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **all** your working.

Solution
You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
F(X)= and you type in $X^3 - 2X$; then you press $\boxed{=}$.
Start? and you enter 4; then you press $\boxed{=}$.
End? and you enter 5; then you press $\boxed{=}$.
Step? and enter 0.05 – 1 decimal place divided by 2; then you press $\boxed{=}$.

x	$f(x)$	Comment
4.2	65.688	too low
4.25	68.265	too high

Clearly,
$$4.2 < x < 4.25$$

and the answer is
$$x = 4.2 \text{ (1 dp)}.$$

5. A nanosecond is $0.000\,000\,001$ second.

(a) Write the number $0.000\,000\,001$ in standard form. (1)

Solution
1×10^{-9}.

A computer does a calculation in 5 nanoseconds.

(b) How many of these calculations can the computer do in 1 second? (2)
Give your answer in standard form.

Solution
$$\frac{1}{5 \times 10^{-9}} = 200\,000\,000$$
$$= 2 \times 10^8.$$

6. Use your calculator to work out the value of
$$\frac{6.27 \times 4.52}{4.81 + 9.63}.$$

(a) Write down all the figures on your calculator display. (2)

Solution

$$\frac{6.27 \times 4.52}{4.81 + 9.63} = \frac{28.340\,4}{14.44}$$
$$= \underline{\underline{1.962\,631\,579}} \text{ (FCD)}.$$

(b) Write your answer to part (a) to an appropriate degree of accuracy. (1)

Solution
What to do here? Well, it looks like we round to 2 significant figures (in which case the answer is $\underline{2.0}$) or else we round to 3 significant figures (in which case the answer is $\underline{1.96}$).

7. Here are some patterns made from dots. (2)

Pattern number 1   Pattern number 2   Pattern number 3   Pattern number 4
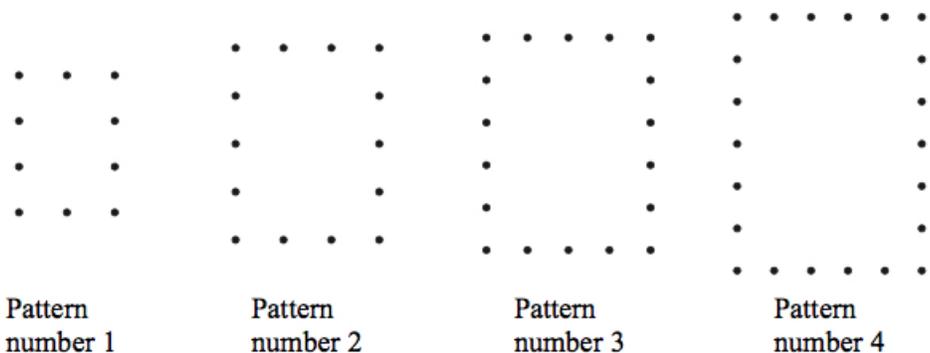
Write down a formula for the number of dots, d, in terms of the Pattern number, n.

Solution
Let the
$$n\text{th term} = an + b.$$

$$\begin{array}{cccc} 10 & 14 & 18 & 22) \\ & 4 & 4 & 4 \end{array}$$

$$\begin{array}{cccc} a + b & 2a + b & 3a + b & 4a + b \\ & a & a & a \end{array}$$

5

We compare terms:

$$a = 4$$

and

$$a + b = 10 \Rightarrow 4 + b = 10$$
$$\Rightarrow b = 6.$$

Hence,

$$n\text{th term} = \underline{\underline{4n + 6}}.$$

8. An ice hockey puck is in the shape of a cylinder with a radius of 3.8 cm and a thickness   (4)
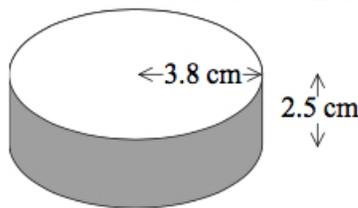   of 2.5 cm.

$\leftarrow$3.8 cm$\rightarrow$ 2.5 cm   Diagram **NOT** accurately drawn

It is made out of rubber with a density of 1.5 grams per cm^3.
Work out the mass of the ice hockey puck.
Give your answer correct to 3 significant figures.

Solution

$$\text{Volume of the hockey puck} = \pi \times 3.8^2 \times 2.5$$
$$= 36.1\pi \text{ cm}^3.$$

Finally,

$$\text{mass} = \text{density} \times \text{volume}$$
$$= 1.5 \times 36.1\pi$$
$$= 170.117\,242\,2 \text{ (FCD)}$$
$$= \underline{\underline{170 \text{ g (3 sf)}}}.$$

9. $DE = 6$ m.
   $EG = 10$ m.
   $FG = 8$ m.
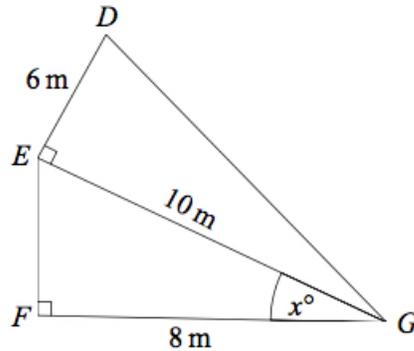

Diagram **NOT**
accurately drawn

Angle $DEG = 90°$.
Angle $EFG = 90°$.

(a) Calculate the length of DG.   (3)
   Give your answer correct to 3 significant figures.

> **Solution**
>
> $$DG = \sqrt{DE^2 + EG^2}$$
> $$= \sqrt{6^2 + 10^2}$$
> $$= 11.661\,903\,79 \text{ (FCD)}$$
> $$= \underline{\underline{11.7 \text{ cm (3 sf)}}}.$$

(b) Calculate the size of the angle marked $x°$.   (3)
   Give your answer correct to one decimal place.

> **Solution**
>
> $$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos x° = \tfrac{8}{10}$$
> $$\Rightarrow x° = 36.869\,897\,65 \text{ (FCD)}$$
> $$= \underline{\underline{36.9° \text{ (1 dp)}}}.$$

10. Solve the simultaneous equations (4)

$$6x - 2y = 33$$
$$4x + 3y = 9.$$

Solution

$$6x - 2y = 33 \quad (1)$$
$$4x + 3y = 9 \quad (2)$$

$$3 \times (1) : 18x - 6y = 99 \quad (3)$$
$$2 \times (2) : 8x + 6y = 18 \quad (4)$$

Now, $(3) + (4)$:

$$26x = 117 \Rightarrow \underline{\underline{x = 4\tfrac{1}{2}}}$$
$$\Rightarrow 27 - 2y = 33$$
$$\Rightarrow 2y = -6$$
$$\Rightarrow \underline{\underline{y = -3}}.$$

11. A 20 Euro note is a rectangle 133 mm long and 72 mm wide. (3)
    A 500 Euro note is a rectangle 160 mm long and 82 mm wide.

Pictures **NOT** accurately drawn

Show that the two rectangles are **not** mathematically similar.

Solution

A 20 Euro note is

$$\frac{133}{72} = 1\frac{61}{72} = 1.84\ldots$$

whereas a 500 Euro note is

$$\frac{160}{82} = 1\frac{39}{41} = 1.95\ldots;$$

hence, they are not mathematically similar.

12. A company bought a van that had a value of £12 000.
Each year the value of the van depreciates by 25%.

(a) Work out the value of the van at the end of three years. (3)

Solution
$$12\,000 \times 0.75^3 = \underline{\underline{£5\,062.50}}.$$

The company bought a new truck.
Each year the value of the truck depreciates by 20%.
The value of the new truck can be multiplied by a single number to find its value at the end of four years.

(b) Find this single number as a decimal. (2)

Solution
$$0.8^4 = \underline{\underline{0.409\,6}}.$$

13. A cone has a volume of 10 m^3. (3)
The vertical height of the cone is 1.5 m.
Calculate the radius of the base of the cone.
Give your answer correct to 3 significant figures.

Solution

$$10 = \tfrac{1}{3}\pi \times r^2 \times 1.5 \Rightarrow r^2 = \frac{30}{1.5\pi}$$
$$\Rightarrow r = \sqrt{\frac{30}{1.5\pi}}$$
$$\Rightarrow r = 2.523\,132\,522 \text{ (FCD)}$$
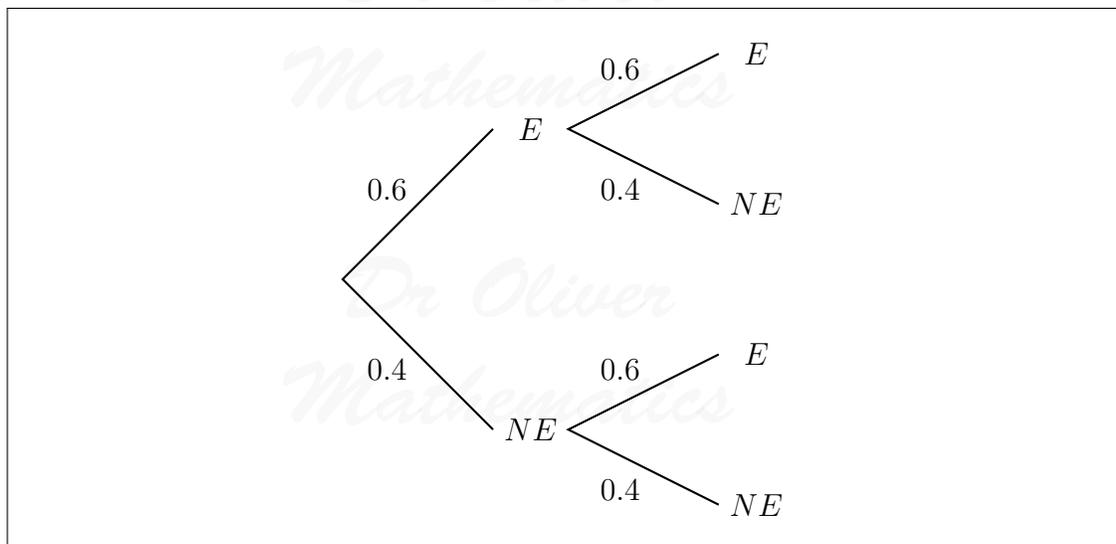$$\Rightarrow \underline{\underline{r = 2.52 \text{ m (3 sf)}}}.$$

14. Amy has 10 CDs in a CD holder.
Amy's favourite group is Edex.
She has 6 Edex CDs in the CD holder.
Amy takes one of these 10 CDs at random.
She writes down whether or not it is an Edex CD.
She puts the CD back in the holder.
Amy again takes one of these 10 CDs at random.

(a) Complete the probability tree diagram. (2)
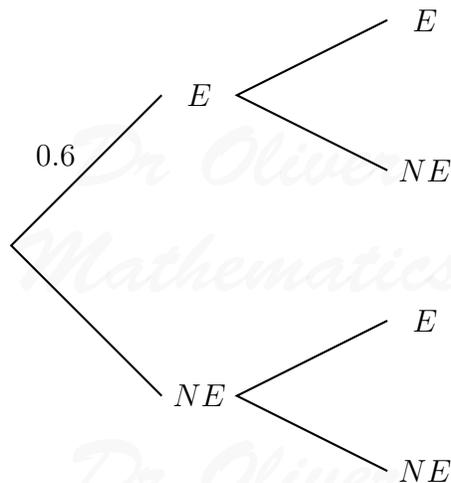
Solution

(b) Find the probability that Amy will pick two Edex CDs. (2)

Solution

$$0.6 \times 0.6 = \underline{0.36}.$$

Amy had 30 CDs.
The mean playing time of these 30 CDs was 42 minutes.
Amy sold 5 of her CDs.
The mean playing time of the 25 CDs left was 42.8 minutes.

(c) Calculate the mean playing time of the 5 CDs that Amy sold. (3)

Solution

$$42 = \frac{(25 \times 42.8) + (5 \times x)}{30} \Rightarrow 1\,260 = 1\,070 + 5x$$
$$\Rightarrow 5x = 190$$
$$\Rightarrow \underline{\underline{x = 38 \text{ minutes}}}.$$

15. The shutter speed, S, of a camera varies inversely as the square of the aperture setting, f.
When $f = 8$, $S = 125$.

(a) Find a formula for S in terms of f. (3)

Solution

$$S \approx \frac{1}{f^2} \Rightarrow S = \frac{k}{f^2}$$

for some constant k. Now,

$$125 = \frac{k}{8^2} \Rightarrow k = 8\,000$$

and so

$$S = \frac{8\,000}{f^2}.$$

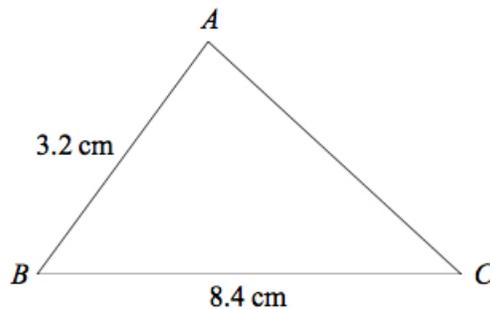(b) Hence, or otherwise, calculate the value of S when $f = 4$. (1)

Solution

$$S = \frac{8\,000}{4^2} = \underline{\underline{500}}.$$

16. $AB = 3.2$ cm. (6)
    $BC = 8.4$ cm.

Diagram **NOT** accurately drawn

The area of triangle ABC is 10 cm².
Calculate the perimeter of triangle ABC.
Give your answer correct to three significant figures.

Solution

$$\tfrac{1}{2} \times 3.2 \times 8.4 \times \sin ABC = 10 \Rightarrow \sin ABC = \tfrac{125}{168}$$
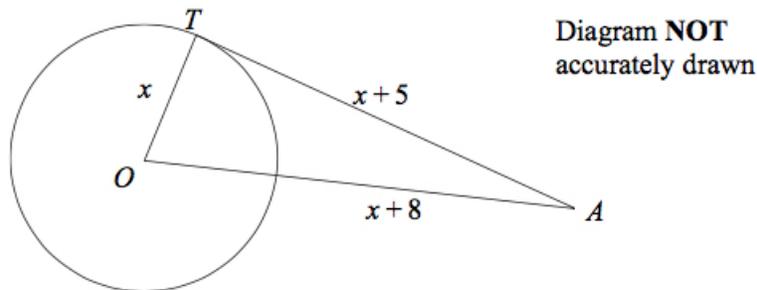$$\Rightarrow ABC = 48.077\,361\,71 \text{ (FCD)}.$$

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Now,

$$b^2 = a^2 + c^2 - 2ac\cos ABC \Rightarrow b^2 = 3.2^2 + 8.4^2 - 2 \times 3.2 \times 8.4 \times \cos ABC$$
$$\Rightarrow b^2 = 44.881\,514\,51 \text{ (FCD)}$$
$$\Rightarrow b = 6.699\,366\,724 \text{ (FCD)},$$

and, so, the perimeter of triangle ABC is

$$3.2 + 8.4 + 6.699\ldots = \underline{\underline{18.3 \text{ cm (3 sf)}}}.$$

17. AT is a tangent at T to a circle, centre O.

Diagram **NOT** accurately drawn

$OT = x$ cm.
$AT = (x+5)$ cm.
$OA = (x+8)$ cm.

(a) Show that $x^2 - 6x - 39 = 0$. (4)

Solution
It is a right-angled triangle (why?) and so

$$(x+8)^2 = x^2 + (x+5)^2 \Rightarrow x^2 + 16x + 64 = x^2 + x^2 + 10x + 25$$
$$\Rightarrow \underline{\underline{x^2 - 6x - 39 = 0}},$$

as required.

(b) Solve the equation $x^2 - 6x - 39 = 0$ to find the radius of the circle. (3)
Give your answer correct to 3 significant figures.

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Solution

$a = 1$, $b = -6$, and $c = -39$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times (-39)}}{2}$$

$$= \frac{6 \pm \sqrt{192}}{2}$$

$$= -3.928\,203\,23 \text{ or } 9.928\,203\,23 \text{ (FCD)};$$

hence, the radius of the circle is $\underline{\underline{9.93 \text{ cm (3 sf)}}}$.

18. The diagram represents a prism. (4)

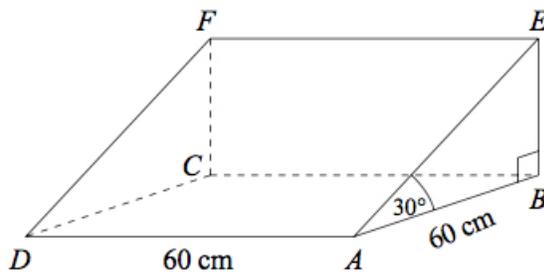Diagram **NOT** accurately drawn

$AEFD$ is a rectangle.
$ABCD$ is a square.
EB and FC are perpendicular to plane $ABCD$.
$AB = 60$ cm.
$AD = 60$ cm.
Angle $ABE = 90°$.
Angle $BAE = 30°$.
Calculate the size of the angle that the line DE makes with the plane $ABCD$.
Give your answer correct to 1 decimal place.

Solution

$$\text{opp} = \text{adj} \times \tan \Rightarrow BE = 60\tan 30°$$

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and

$$DE = \sqrt{DA^2 + AB^2}$$
$$= \sqrt{60^2 + 60^2}$$
$$= 60\sqrt{2}.$$

Finally,

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan x° = \frac{60 \tan 30°}{60\sqrt{2}}$$
$$\Rightarrow x° = 22.207\,654\,3 \text{ (FCD)}$$
$$= \underline{\underline{22.2°}} \text{ (1 dp).}$$

19. Bill said that the line $y = 6$ cuts the curve $x^2 + y^2 = 25$ at two points.

(a) By eliminating y show that Bill is incorrect. (2)

Solution

$$x^2 + 6^2 = 25 \Rightarrow x^2 + 36 = 25$$
$$\Rightarrow x^2 = -11;$$

so, <u>Bill is incorrect</u>.

(b) By eliminating y, find the solutions to the simultaneous equations (6)

$$x^2 + y^2 = 25$$
$$y = 2x - 2.$$

Solution

$$x^2 + (2x - 2)^2 = 25 \Rightarrow x^2 + (4x^2 - 8x + 4) = 25$$
$$\Rightarrow 5x^2 - 8x - 21 = 0$$

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add to: -8
multiply to: $(+5) \times (-21) = -105$ $\Big\}$ $+7, -15$

$$\Rightarrow 5x^2 - 15x + 7x - 21 = 0$$
$$\Rightarrow 5x(x-3) + 7(x-3) = 0$$
$$\Rightarrow (5x+7)(x-3) = 0$$
$$\Rightarrow x = -\tfrac{7}{5} \text{ or } x = 3$$
$$\Rightarrow y = -\tfrac{24}{5} \text{ or } y = 4;$$

hence,

$$x = -1\tfrac{2}{5}, y = -4\tfrac{4}{5} \text{ or } x = 3, y = 4.$$

20. Martin won the 400 metre race in the school sports with a time of 1 minute.
The distance was correct to the nearest centimetre.
The time was correct to the nearest tenth of a second.

(a) Work out the upper bound and the lower bound of Martin's speed in km/h. (5)
Give your answers correct to 5 significant figures.

Solution

We will start by woking out the race distance. The question states "The distance was correct to the nearest centimetre" which means

$$\text{least distance} = 400 - 0.005 = 399.995$$

and

$$\text{upper distance} = 400 + 0.005 = 400.005.$$

So, we have for the race distance,

$$399.995 \leqslant \text{race distance} < 400.005.$$

The question states "The time was correct to the nearest tenth of a second" which means

$$\text{least time} = 60 - 0.05 = 59.95$$

and

$$\text{upper time} = 60 + 0.05 = 60.05.$$

So, we have for the time:

$$59.95 \leqslant \text{time} < 60.05.$$

And we have
$$\frac{399.995}{60.05} < \text{speed (m/s)} < \frac{400.005}{59.95}.$$
But we want km/h and so we multiply by 60×60 and divide by 1000:
$$\frac{399.995 \times 60 \times 60}{60.05 \times 1000} < \text{speed (km/h)} < \frac{400.005 \times 60 \times 60}{59.95 \times 1000}.$$
Finally,
$$\text{lower bound} = 23.979\,716\,9 \text{ (FCD)} = \underline{\underline{23.980 \text{ km/h (5 sf)}}}$$
and
$$\text{upper bound} = 24.020\,316\,93 \text{ (FCD)} = \underline{\underline{24.020 \text{ km/h (5 sf)}}}.$$

(b) Write down an appropriate value for Martin's speed in km/h. (1)
Explain your answer.

Solution
We draw up a table.

	Lower bound	Upper bound	Agree?
1 sf	20	20	Yes
2 sf	24	24	Yes
3 sf	24.0	24.0	Yes
4 sf	23.98	24.02	No

So, an appropriate value for Martin's speed is $\underline{\underline{24.0 \text{ km/h (3 sf)}}}$.

The table shows the number of people in each age group who watched the school sports.

Age group	$0-16$	$17-29$	$30-44$	$45-59$	$60+$
Number of people	177	111	86	82	21

Martin did a survey of these people.
He used a stratified sample of exactly 50 people according to age group.

(c) Work out the number of people from each age group that should have been in his (3)
sample of 50.

Complete the table.

Age group	$0-16$	$17-29$	$30-44$	$45-59$	$60+$
Number of people in sample					

Solution

The number of people is

$$177 + 111 + 86 + 82 + 21 = 477.$$

$\underline{0-16}$:

$$177 \times \frac{50}{477} = 18.55\ldots$$

$\underline{17-29}$:

$$111 \times \frac{50}{477} = 11.63\ldots$$

$\underline{30-44}$:

$$86 \times \frac{50}{477} = 9.01\ldots$$

$\underline{45-59}$:

$$82 \times \frac{50}{477} = 8.59\ldots$$

$\underline{60+}$:

$$21 \times \frac{50}{477} = 2.20\ldots$$

Age group	$0-16$	$17-29$	$30-44$	$45-59$	$60+$
Number of people in sample	19	12	9	9	2

Do they add up to 50? No! So simply round down the number which is greater than or equal to .5; in this case, it is $18.55\ldots$ and we round it down to 18.

Age group	$0-16$	$17-29$	$30-44$	$45-59$	$60+$
Number of people in sample	18	12	9	9	2

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21. (a) Solve (3)

$$\frac{40-x}{3} = 4 + x.$$

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Solution

$$\frac{40 - x}{3} = 4 + x \Rightarrow 40 - x = 3(4 + x)$$
$$\Rightarrow 40 - x = 12 + 3x$$
$$\Rightarrow 28 = 4x$$
$$\Rightarrow \underline{\underline{x = 7}}.$$
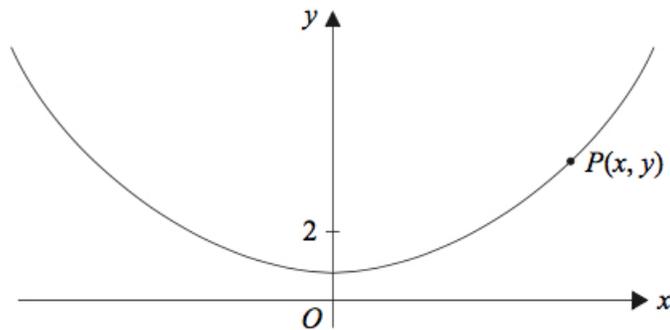
(b) Simplify fully (3)

$$\frac{4x^2 - 6x}{4x^2 - 9}.$$

Solution

$$\frac{4x^2 - 6x}{4x^2 - 9} = \frac{4x^2 - 6x}{(2x)^2 - 3^2}$$
$$= \frac{2x(2x - 3)}{(2x + 3)(2x - 3)}$$
$$= \underline{\underline{\frac{2x}{2x + 3}}}.$$

22. The diagram shows a sketch of a curve. (4)

The point $P(x, y)$ lies on the curve.
The locus of P has the following property: the distance of the point P from the point

$(0, 2)$ is the same as the distance of the point P from the x-axis.
Show that
$$y = \tfrac{1}{4}x^2 + 1.$$

Solution

It helps to work with the *squares* of the distances: the distance of the point P from the point $(0, 2)$ is
$$x^2 + (y - 2)^2$$
and the distance of the point P from the x-axis is y^2. Now,
$$
\begin{aligned}
x^2 + (y - 2)^2 = y^2 &\Rightarrow x^2 + (y^2 - 4y + 4) = y^2 \\
&\Rightarrow x^2 + 4 = 4y \\
&\Rightarrow y = \tfrac{1}{4}x^2 + 1,
\end{aligned}
$$

as required.