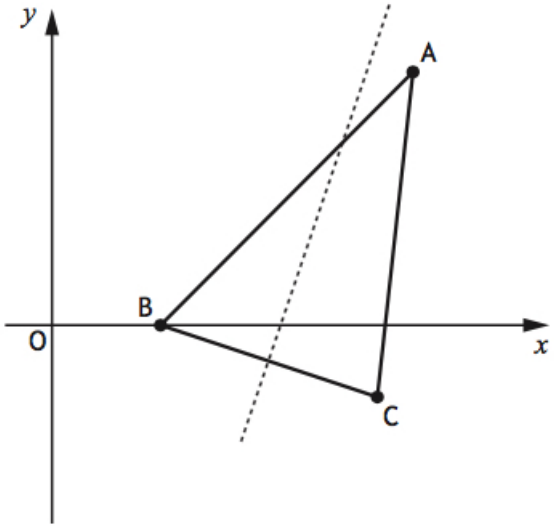
Dr Oliver Mathematics
Mathematics: Higher
2017 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

1. Triangle ABC is shown in the diagram below.



The coordinates of B are $(3, 0)$ and the coordinates of C are $(9, -2)$.
The broken line is the perpendicular bisector of BC.

(a) Find the equation of the perpendicular bisector of BC. (4)

The line AB makes an angle of $45°$ with the positive direction of the x-axis.

(b) Find the equation of AB. (2)

(c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC. (2)

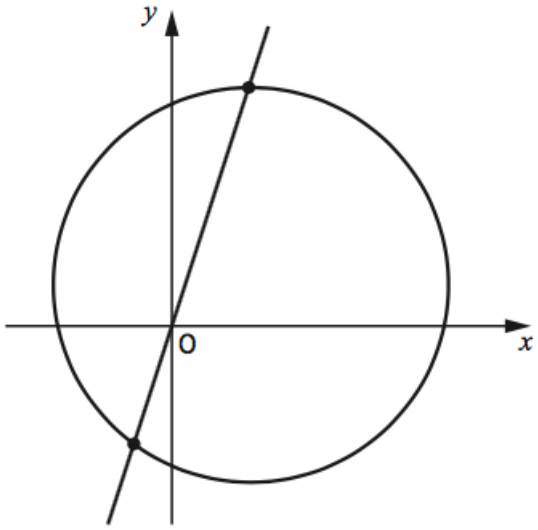
2. (a) Show that $(x - 1)$ is a factor of (2)

$$f(x) = 2x^3 - 5x^2 + x + 2.$$

(b) Hence, or otherwise, solve $f(x) = 0$. (3)

3. The line $y = 3x$ intersects the circle with equation (5)

$$(x-2)^2 + (y-1)^2 = 25.$$



Find the coordinates of the points of intersection.

4. (a) Express (3)

$$3x^2 + 24x + 50$$

in the form

$$a(x+b)^2 + c.$$

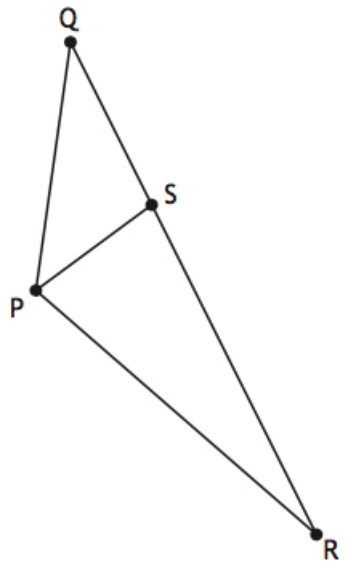
(b) Given that (2)

$$\mathrm{f}(x) = x^3 + 12x^2 + 50x - 11,$$

find $\mathrm{f}'(x)$.

(c) Hence, or otherwise, explain why the curve with equation $y = \mathrm{f}(x)$ is strictly increasing for all values of x. (2)

5. In the diagram, $\overrightarrow{PR} = 9\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i}-9\mathbf{j}+3\mathbf{k}$.



(a) Express $\overrightarrow{PQ}$ in terms of **i**, **j**, and **k**. (2)

The point S divides QR in the ratio $1 : 2$.

(b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$. (2)

(c) Hence, find the size of angle QPS. (5)

6. Solve (5)

$$5\sin x - 4 = 2\cos 2x$$

for $0 \leqslant x < 2\pi$.

7. (a) Find the x-coordinate of the stationary point on the curve with equation (4)

$$y = 6x - 2\sqrt{x^3}.$$

(b) Hence, determine the greatest and least values of y in the interval $1 \leqslant x \leqslant 9$. (3)

8. Sequences may be generated by recurrence relations of the form

$$u_{n+1} = ku_n - 20,\ u_0 = 5 \text{ where } k \in \mathbb{R}.$$

(a) Show that (2)

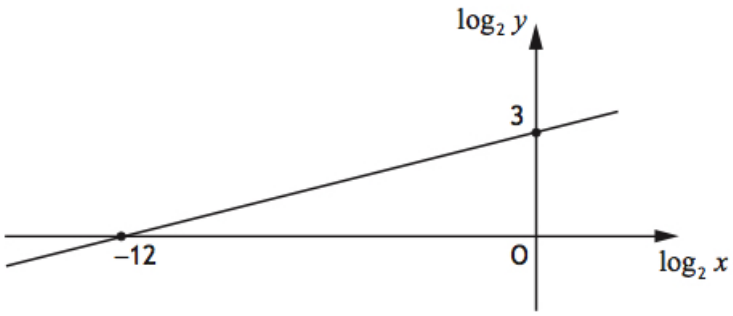
$$u_2 = 5k^2 - 20k - 20.$$

(b) Determine the range of values of k for which $u_2 < u_0$. (4)

9. Two variables, x and y, are connected by the equation (5)

$$y = kx^n.$$

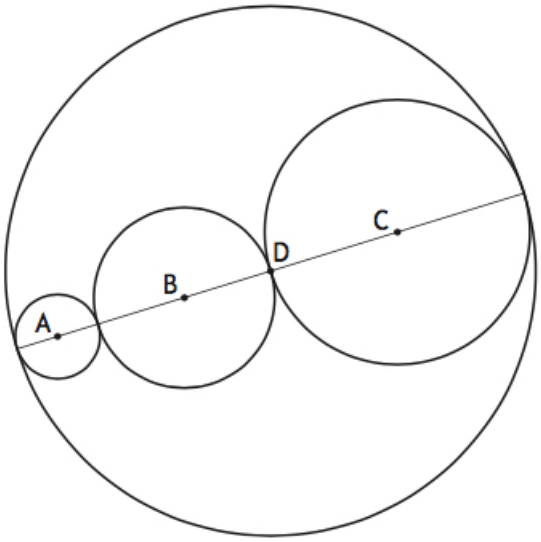
The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.



Find the values of k and n.

10. (a) Show that the points $A(-7, -2)$, $B(2, 1)$, and $C(17, 6)$ are collinear. (3)

Three circles with centres A, B, and C are drawn inside a circle with centre D as shown.



The circles with centres A, B, and C have radii r_A, r_B, and r_C respectively.

- $r_A = \sqrt{10}$,
- $r_B = 2r_A$, and
- $r_C = r_A + r_B$.

(b) Determine the equation of the circle with centre D. (4)

11. (a) Show that (3)

$$\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x \equiv \sin^3 x,$$

where $0 < x < \frac{1}{2}\pi$.

(b) Hence, differentiate (3)

$$\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x,$$

where $0 < x < \frac{1}{2}\pi$.