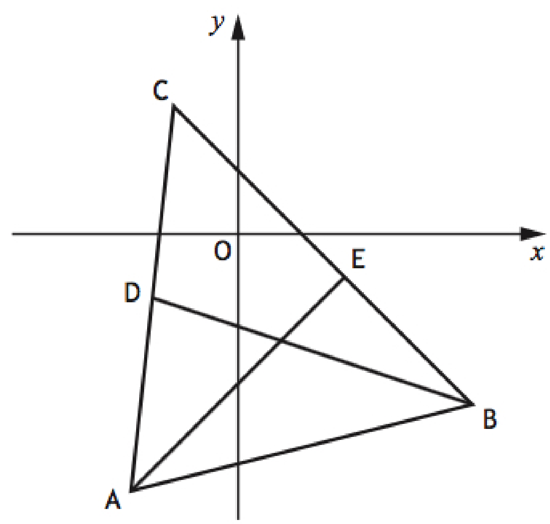
Dr Oliver Mathematics
Mathematics: Higher
2019 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Triangle ABC has vertices $A(-5, -12)$, $B(11, -8)$, and $C(-3, 6)$.



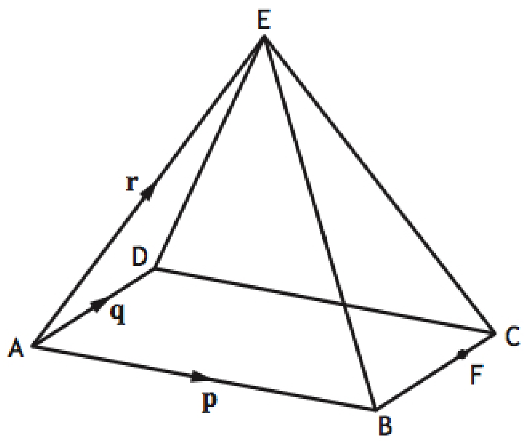
   (a) Find the equation of the median BD. (3)

   (b) Find the equation of the altitude AE. (3)

   (c) Find the coordinates of the point of intersection of BD and AE. (2)

2. Find (4)
$$\int (6\sqrt{x} - 4x^{-3} + 5)\,\mathrm{d}x.$$

3. $ABCDE$ is a rectangular based pyramid.
$\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AD} = \mathbf{q}$, and $\overrightarrow{AE} = \mathbf{r}$.



(a) Express $\overrightarrow{BE}$ in terms of $\mathbf{p}$ and $\mathbf{r}$. (1)

Point F divides BC in the ratio $3:1$.

(b) Express vector $\overrightarrow{EF}$ in terms of $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{r}$. (2)

4. In a forest, the population of a species of mouse is falling by 2.7% each year.

To increase the population scientists plan to release 30 mice into the forest at the end of March each year.

u_n is the estimated population of mice at the start of April, n years after the population was first estimated.

It is known that u_n and u_{n+1} satisfy the recurrence relation

$$u_{n+1} = au_n + b.$$

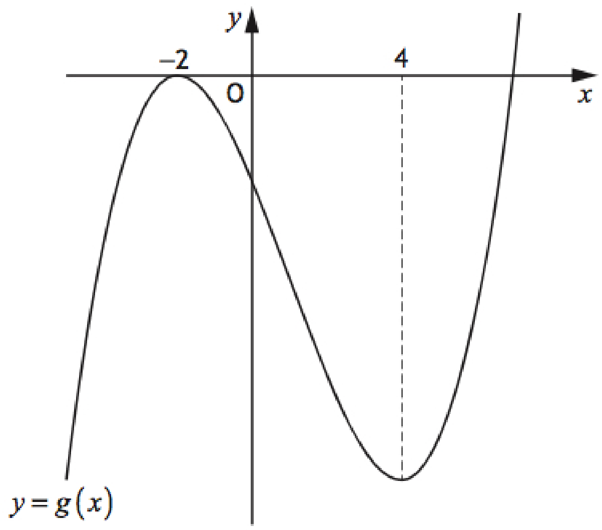
(a) State the values of a and b. (1)

The scientists continue to release this species of mouse each year.

(b) (i) Explain why the estimated population of mice will stabilise in the long term. (1)

(ii) Calculate the long term population to the nearest hundred. (2)

5. The diagram below shows the graph of a cubic function $y = \mathrm{g}(x)$, with stationary points at $x = -2$ and $x = 4$. (2)



Sketch the graph of $y = g'(x)$.

6. (a) Express (4)

$$2\cos x^\circ - 3\sin x^\circ$$

in the form

$$k\cos(x + a)^\circ$$

where $k > 0$ and $0 \leqslant a < 360$.

(b) Hence solve (3)

$$2\cos x^\circ - 3\sin x^\circ = 3$$

for $0 \leqslant x < 360$.

7. (a) Express (3)

$$-6x^2 + 24x - 25$$

in the form

$$p(x + q)^2 + r.$$

(b) Given that (3)

$$f(x) = -2x^3 + 12x^2 - 25x + 9,$$

show that $f(x)$ is strictly decreasing for all $x \in \mathbb{R}$.

8. A function, f, is given by

$$f(x) = \sqrt[3]{x} + 8.$$

The domain of f is $1 \leqslant x \leqslant 1\,000$, $x \in \mathbb{R}$.
The inverse function, f^{-1}, exists.

(a) Find $f^{-1}(x)$. (3)

(b) State the domain of f^{-1}. (1)

9. Electricity on a spacecraft can be produced by a type of nuclear generator. The electrical power produced by this generator can be modelled by

$$P_t = 120e^{-0.0079t},$$

where P_t is the electrical power produced, in watts, after t years.

(a) Determine the electrical power initially produced by the generator. (1)

(b) Calculate how long it takes for the electrical power produced by the generator to reduce by 15%. (4)

10. (a) Show that $(x + 3)$ is a factor of (2)

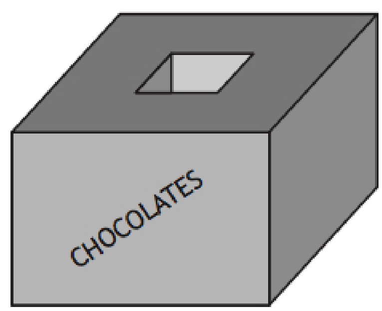
$$3x^4 + 10x^3 + x^2 - 8x - 6.$$

(b) Hence, or otherwise, factorise (5)

$$3x^4 + 10x^3 + x^2 - 8x - 6$$

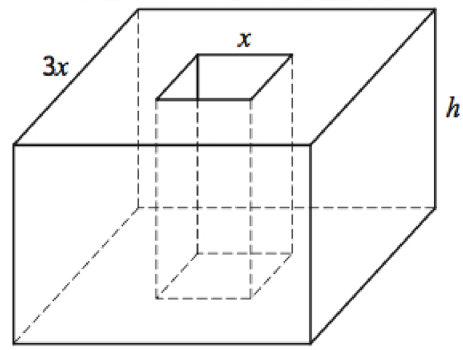
fully.

11. A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.



The box is a cuboid with a cuboid shaped tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres.
- The volume of the box is $2\,000\ \text{cm}^3$.



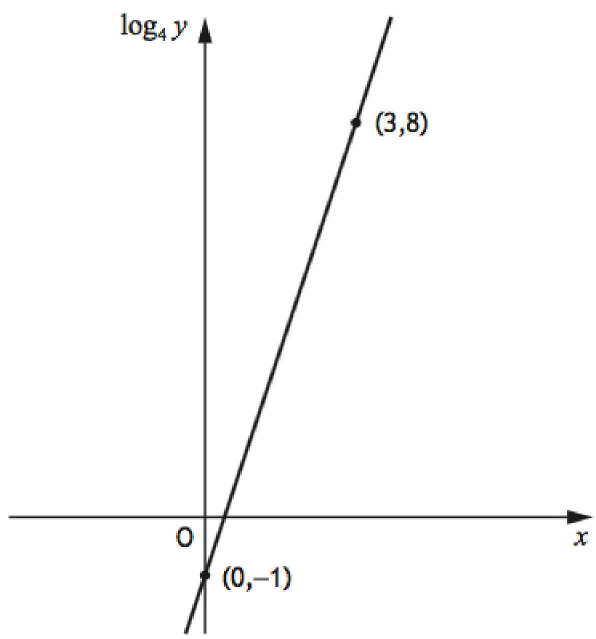
(a) Show that the total surface area, A cm^2, of the box is given by (3)

$$A = 16x^2 + \frac{4\,000}{x}.$$

To minimise the cost of production, the surface area, A, of the box should be as small as possible.

(b) Find the minimum value of A. (6)

12. Two variables, x and y, are connected by the equation $y = ab^x$. (5)
The graph of $\log_4 y$ against x is a straight line as shown.



Find the values of a and b.

13. For a function, f, defined on the set of real numbers, $\mathbb{R}$, it is known that (5)

- the rate of change of f with respect to x is given by

$$3x^2 - 16x + 11.$$

- the graph with equation $y = \mathrm{f}(x)$ crosses the x-axis at $(7, 0)$.

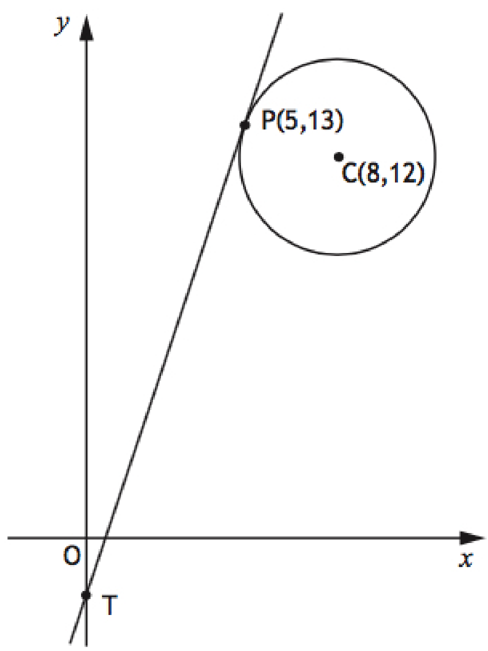
Express $\mathrm{f}(x)$ in terms of x.

14. The vectors $\mathbf{u}$ and $\mathbf{v}$ are such that (4)

- $|\mathbf{u}| = 4$.
- $|\mathbf{v}| = 5$
- $\mathbf{u}.(\mathbf{u} + \mathbf{v}) = 21$.

Determine the size of the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$.

15. A circle has centre $C(8, 12)$.
The point $P(5, 13)$ lies on the circle as shown.



(a) Find the equation of the tangent at P. (3)

The tangent from P meets the y-axis at the point T.

(b) (i) State the coordinates of T. (1)

(ii) Find the equation of the circle that passes through the points C, P, and T. (3)