Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2013 Paper
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.
Final answers should be given correct to three significant figures where appropriate.

Section A

1. (a) Find the gradient of the line, L, whose equation is (2)

$$3x + 2y = 7.$$

   (b) Find the equation of the line which is perpendicular to L and which passes through the point $(3, 1)$. (3)

2. Find the integers that satisfy the inequality (4)

$$-7 < 3x + 1 < 12.$$

3. This year John is 4 times as old as his son Paul. In 5 years' time John will be only 3 times as old as Paul. (4)

   Let the age of Paul now be x years.

   By forming an equation in x and solving it, find Paul's age now.

4. You are given that θ is an acute angle and (3)

$$\sin\theta = \frac{\sqrt{5}}{3}.$$

   Find the exact value of $\tan\theta$.

5. (a) Use calculus to find the stationary points on the curve (5)

$$y = x^3 - \tfrac{3}{2}x^2 - 6x + 3.$$

(b) Sketch the curve on the axes provided showing the stationary points and the point where it cuts the y-axis. (2)

6. Amanda throws 3 fair dice. What is the probability that

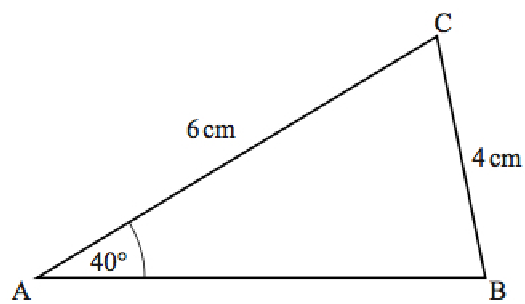
   (a) exactly 2 sixes are thrown, (3)

   (b) at least 1 six is thrown? (3)

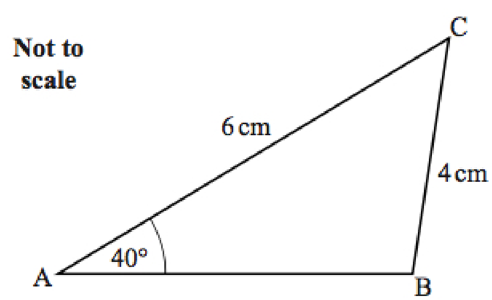
7. John and Jennie are asked to draw a triangle ABC with the following properties: (4)

   - $AC = 6$ cm,
   - $CB = 4$ cm, and
   - the angle $A = 40°$.

   John draws the triangle as shown below



   and Jennie draws the triangle as shown below.

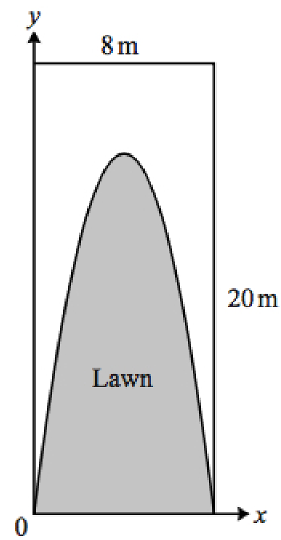


   Calculate the angle B in each case.

8. A mathematical gardener has a garden which is rectangular in shape measuring 20 metres by 8 metres. He wishes to arrange the garden so that approximately half of it is lawn and the rest flower bed. (6)

He sets up a coordinate system as shown in the diagram below and maps out the graph of the curve

$$y = 8x - x^2.$$



Show that the area of the lawn is approximately 53% of the total area.

9. (a) Find the values of the constants a and b such that, for all values of x, (3)

$$x^2 + 8x + 19 \equiv (x + a)^2 + b.$$

(b) Hence state the least value of (2)

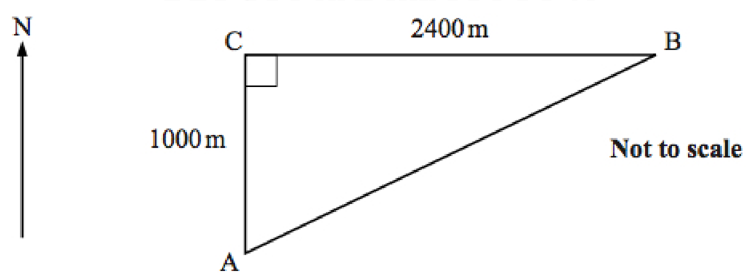
$$x^2 + 8x + 19$$

and the value of x at which this occurs.

(c) Write down the greatest value of (1)

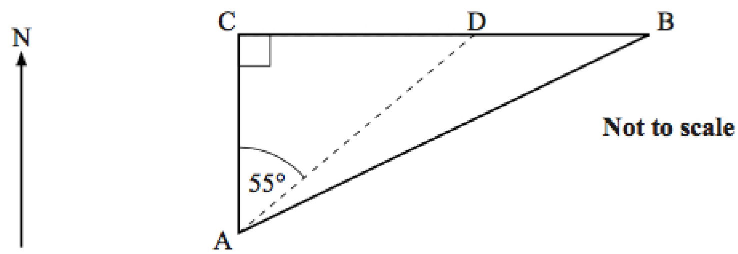
$$\frac{1}{x^2 + 8x + 19}.$$

10. One leg of a cross-country race is from A to B. The checkpoint B is at the end of a wall that runs due east-west, as shown in the diagram. A is a point 1000 m due south of a point C on the wall. $BC = 2400$ m.



(a) What bearing should a runner take to travel from A to B and what is the distance AB? (4)

John sets off from A unable to see the checkpoint, B. He heads out on a bearing of 055° and when he reaches the wall at point D he knows he has to go east along the wall to reach the point B, as shown in the diagram.



(b) How much further than the distance AB does John run? (3)

Section B

11. A circle has equation

$$(x-2)^2 + y^2 = 100.$$

(a) Write down the radius and the coordinates of the centre, C, of this circle. (2)

The line $y = 2x + 6$ cuts the circle at two points, A and B.

(b) Find

(i) the coordinates of A and B, (5)

(ii) the mid-point, M, of AB, (1)

(iii) the length AB. (2)

(c) Hence find the distance of the centre of the circle from the line AB. (2)

12. An object sinks through a thick liquid such that at time t seconds after being released on the surface the depth, s metres, is given by

$$s = 4t^2 - \tfrac{2}{3}t^3 \text{ for } 0 \leqslant t \leqslant 4.$$

(a) Find the formula for the velocity, v metres per second, t seconds after being released. Hence show that the object stops sinking when $t = 4$. (4)

(b) Find

(i) the acceleration of the object when it is released on the surface of the liquid, (4)

(ii) the greatest depth of the object. (2)

(c) Sketch the velocity-time and acceleration-time graphs. (2)

13. A number of students from a group of 20 boys and 30 girls are to be selected to attend a one-day conference.

    The number of girls attending must be at least the same as the number of boys but no more than twice the number of boys.

    Let there be x boys and y girls selected.

    (a) Given that $x > 0$ and $y > 0$, write down four more inequalities to represent the information. (3)

    (b) Plot these inequalities on the grid provided. Indicate the region for which the inequalities hold. Shade the area that is **not** required. (5)

    In order to attend the conference the students need to be given a special uniform. The uniform for the boys costs £40 and the uniform for the girls cost £50. The school has £2 000 to spend on the uniforms.

    (c) By plotting the appropriate line on your graph, find the maximum number of students that could go to the conference. (4)

14. A curve has equation

$$y = 4x^3 - 5x^2 + 1$$

    and passes through the point $A(1, 0)$.

    (a) Find the equation of the normal to the curve at A. (5)

    This normal also cuts the curve in two other points, B and C.

    (b) Show that the x-coordinates of the three points where the normal cuts the curve are given by the equation (2)

$$8x^3 - 10x^2 + x + 1 = 0.$$

    (c) Show that the point $B(\frac{1}{2}, \frac{1}{4})$ satisfies the normal and the curve. (2)

    (d) Find the coordinates of C. (3)