

Dr Oliver Mathematics
GCSE Mathematics
2018 Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

1. (a) Simplify

$$m^3 \times m^4.$$

(1)

Solution

$$m^3 \times m^4 = \underline{\underline{m^7}}.$$

- (b) Simplify

$$(5np^3)^3.$$

(2)

Solution

$$(5np^3)^3 = \underline{\underline{125n^3p^9}}.$$

- (c) Simplify

$$\frac{32q^9r^4}{4q^3r}.$$

(2)

Solution

$$\frac{32q^9r^4}{4q^3r} = \underline{\underline{8q^6r^3}}.$$

2. (a) Find the lowest common multiple (LCM) of 40 and 56.

(2)

Solution

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$$\begin{array}{r|l} & 40 \\ 2 & 20 \\ 2 & 10 \\ 2 & 5 \\ 5 & 1 \end{array}$$

So

$$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5.$$

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$$\begin{array}{r|l} & 56 \\ 2 & 28 \\ 2 & 14 \\ 2 & 7 \\ 7 & 1 \end{array}$$

So

$$56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7.$$

Hence,

$$\text{LCM}(40, 56) = 2^3 \times 5 \times 7 = \underline{\underline{280}}.$$

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$$A = 2^3 \times 3 \times 5 \text{ and } B = 2^2 \times 3 \times 5^2.$$

(b) Write down the highest common factor (HCF) of A and B . (1)

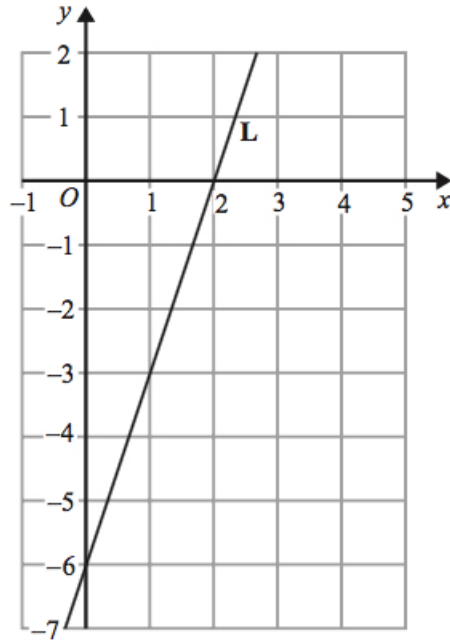
Solution

$$\text{LCM}(A, B) = 2^2 \times 3 \times 5 = \underline{\underline{60}}.$$

3. The line **L** is shown on the grid. (3)

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Find an equation for **L**.

Solution

$$\text{Gradient} = \frac{0 - (-6)}{2 - 0} = 3$$

and the equation for **L** is

$$y - 0 = 3(x - 2) \Rightarrow \underline{\underline{y = 3x - 6.}}$$

4. Raya buys a van for £8 500 plus VAT at 20%. (5)

Raya pays a deposit for the van.

She then pays the rest of the cost in 12 equal payments of £531.25 each month.

Find the ratio of the deposit Raya pays to the total of the 12 equal payments.

Give your answer in its simplest form.

Solution

$$8\,500 \times 1.2 = \pounds 10\,200.$$

Now,

$$12 \times 531.25 = \pounds 6\,375$$

which means she pays a deposit for the van of

$$10\,200 - 6\,375 = \pounds 3\,825.$$

Hence, the ratio of the deposit Raya pays to the total of the 12 equal payments is

$$3\,825 : 6\,375 = \underline{\underline{3 : 5}}.$$

5. (a) Complete the table of values for

(2)

$$y = x^2 - x - 6.$$

x	-3	-2	-1	0	1	2	3
y	6			-6			

Solution

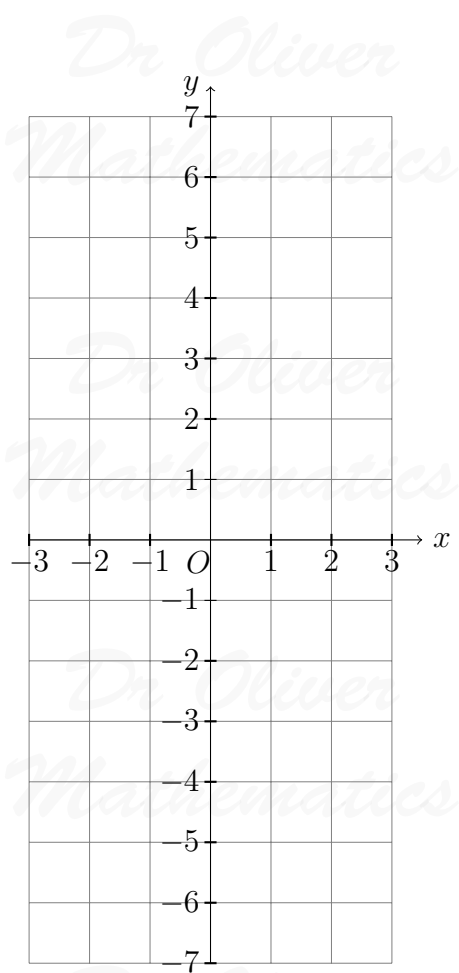
x	-3	-2	-1	0	1	2	3
y	6	<u>0</u>	<u>-4</u>	-6	<u>-6</u>	<u>-4</u>	<u>0</u>

- (b) On the grid, draw the graph of

(2)

$$y = x^2 - x - 6$$

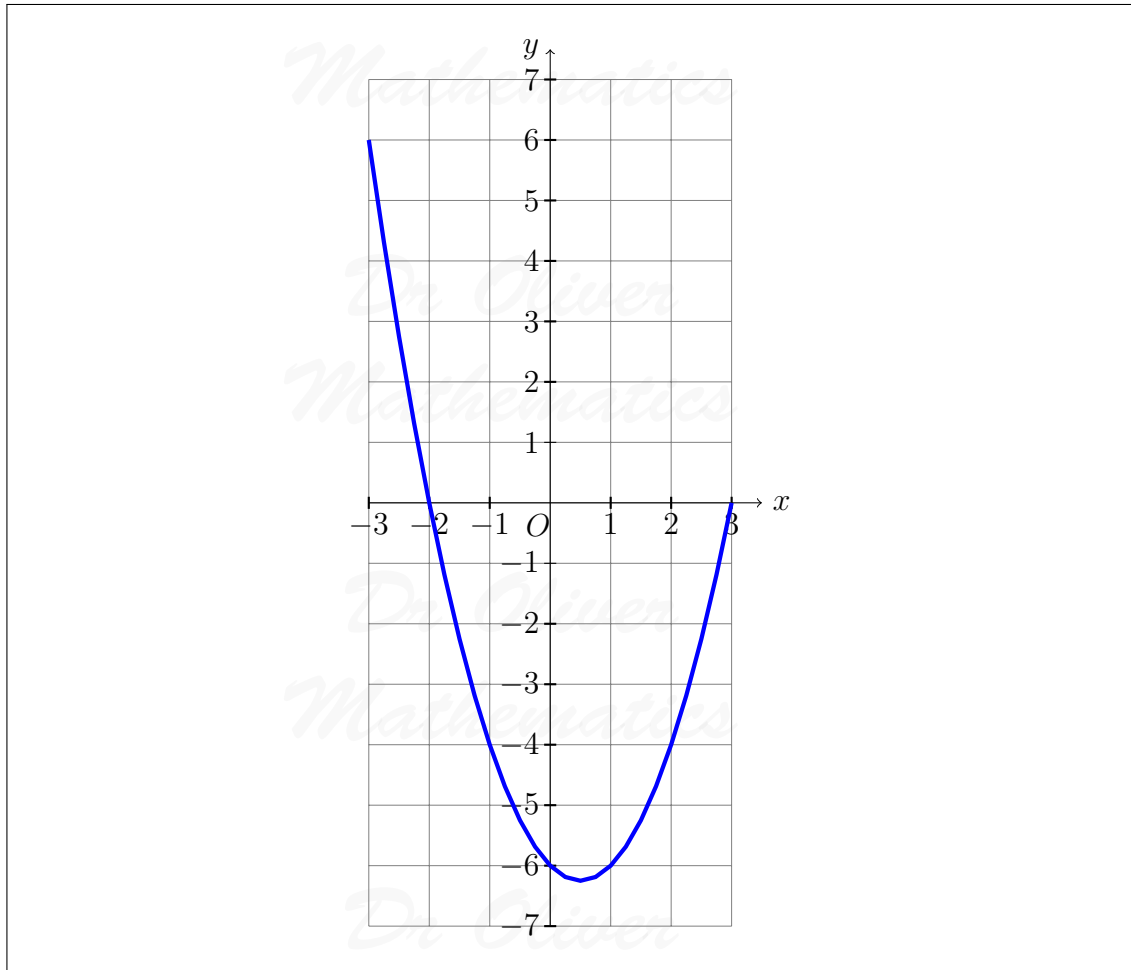
for values of x from -3 to 3.



Solution

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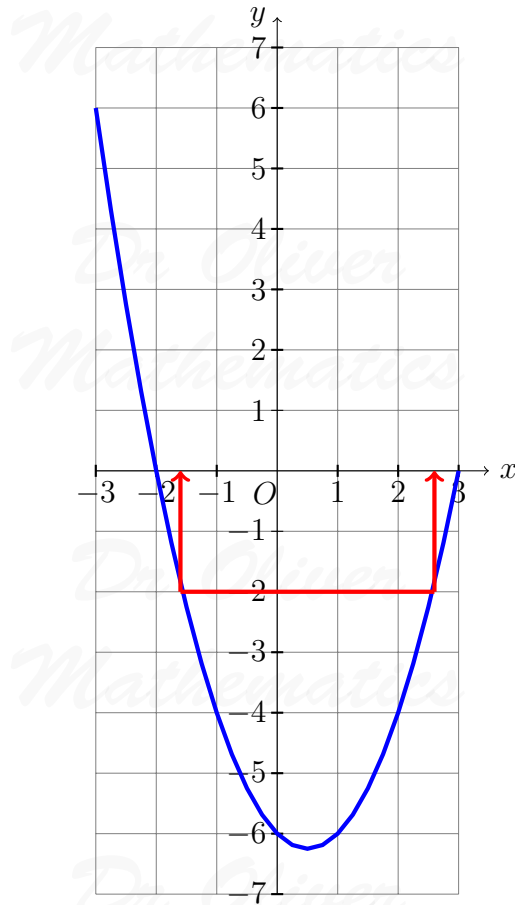


(c) Use your graph to find estimates of the solutions to the equation

(2)

$$x^2 - x - 6 = -2.$$

Solution



Correct read-off: approximately $x = -1.6$ or $x = 2.6$.

6.

(3)

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}.$$

A force of 70 newtons acts on an area of 20 cm^2 .

The force is increased by 10 newtons.

The area is increased by 10 cm^2 .

Helen says, "The pressure decreases by less than 20%."

Is Helen correct?

You must show how you get your answer.

$$\text{Original pressure} = \frac{70}{20} = 3.5 \text{ N/cm}^2.$$

The force is increased by 10 newtons (so 80) and the area is increased by 10 cm² (so 30). Hence,

$$\text{new pressure} = \frac{80}{30} = 2\frac{2}{3} \text{ N/cm}^2$$

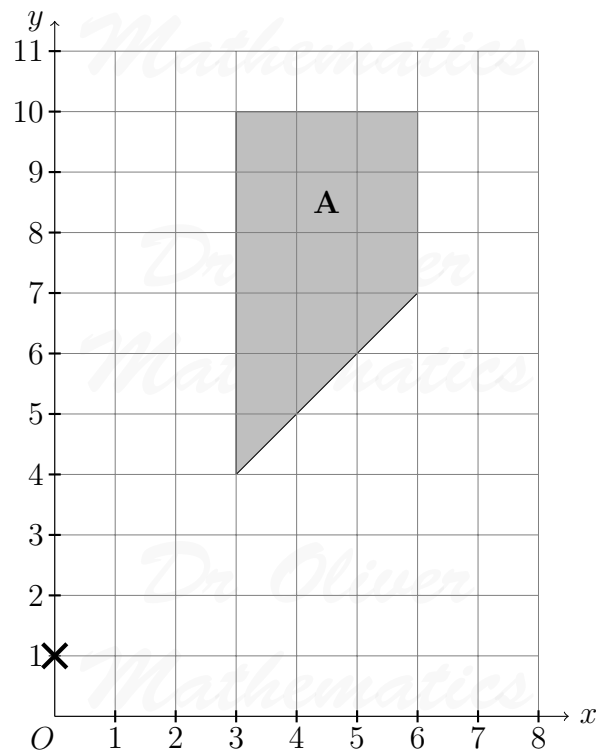
and the pressure decreases by

$$\frac{3.5 - 2\frac{2}{3}}{3.5} \times 100\% = 21\frac{17}{21}\%$$

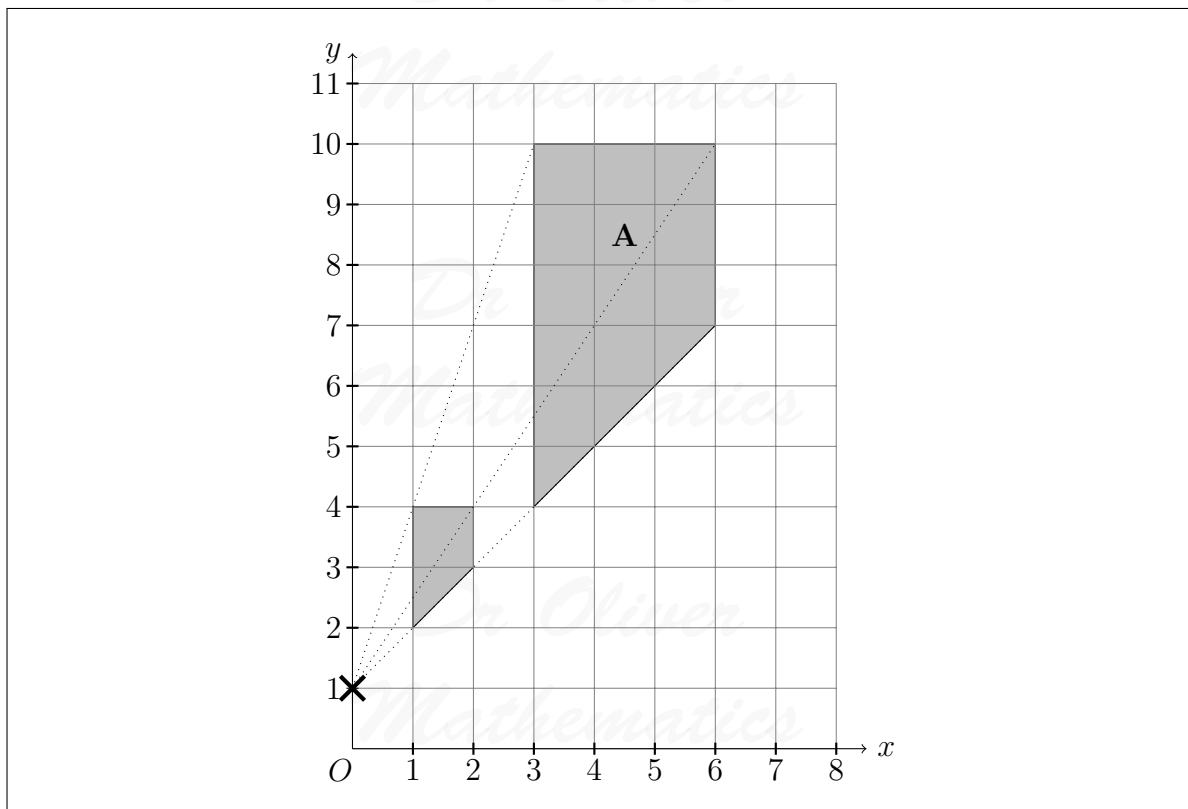
so the claim is wrong.

7. Enlarge shape **A** by scale factor $\frac{1}{3}$, centre $(0, 1)$.

(2)



Solution



8. 60 people were asked if they prefer to go on holiday in Britain or in Spain or in Italy. (4)

38 of the people were male.

11 of the 32 people who said Britain were female.

8 males said Italy.

12 people said Spain.

One of the females is chosen at random.

What is the probability that this female said Spain?

Solution

We make a table ...

	Britain	Spain	Italy	Total
Males	21		8	38
Females	11			
Total	32	12		60

... and we complete the table.

	Britain	Spain	Italy	Total
Males	21	9	8	38
Females	11	3	8	22
Total	32	12	16	60

Hence, the probability that this female said Spain is $\frac{3}{22}$.

9. Jean invests £12 000 in an account paying compound interest for 2 years. (4)

In the first year the rate of interest is $x\%$.

At the end of the first year the value of Jean's investment is £12 336.

In the second year the rate of interest is $\frac{1}{2}x\%$.

What is the value of Jean's investment at the end of 2 years?

Solution

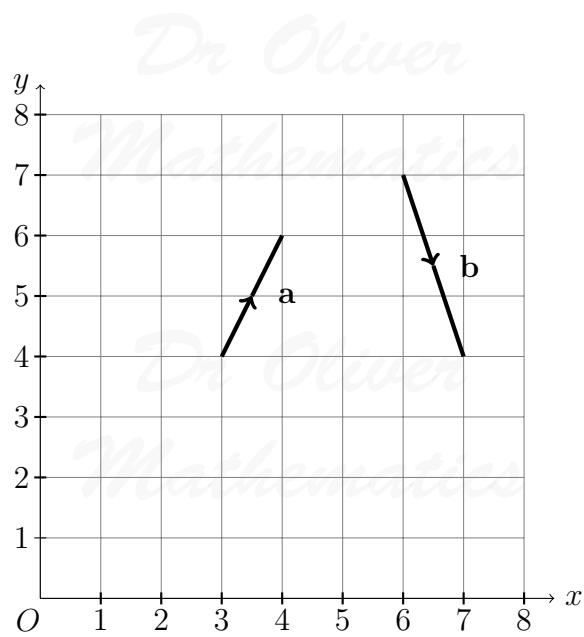
Well,

$$\frac{12\,336}{12\,000} = 1.028$$

which means that the first year the rate of interest is 2.8% and in the second year the rate of interest is 1.4%. Finally, the value of Jean's investment at the end of 2 years is

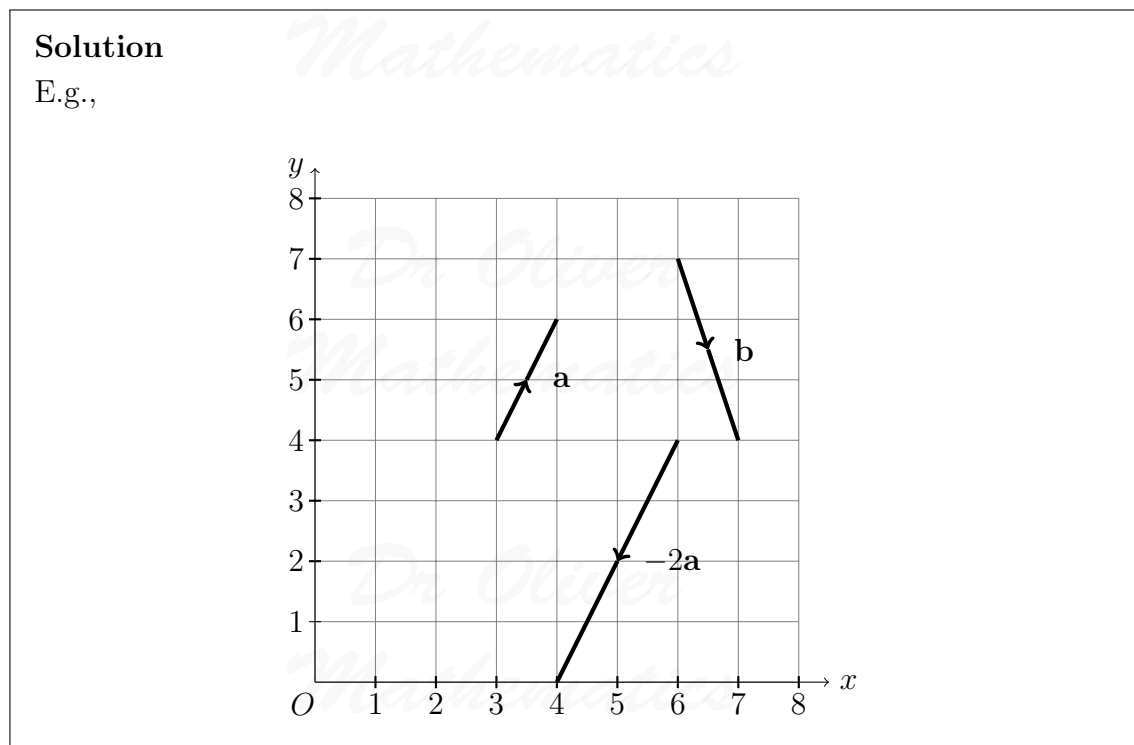
$$\begin{aligned} 12\,336 \times 1.014 &= 12\,508.704 \\ &= \underline{\underline{\pounds 12\,508.70}} \text{ (2 dp)}. \end{aligned}$$

10. The vector **a** and the vector **b** are shown on the grid.



(a) On the grid, draw and label vector $-2\mathbf{a}$.

(1)



(b) Work out $\mathbf{a} + 2\mathbf{b}$ as a column vector.

(2)

Solution

$$\begin{aligned}
 \mathbf{a} + 2\mathbf{b} &= \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ -3 \end{pmatrix} \\
 &= \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -6 \end{pmatrix} \\
 &= \underline{\underline{\begin{pmatrix} 3 \\ -4 \end{pmatrix}}}.
 \end{aligned}$$

11. f and g are functions such that

$$f(x) = \frac{2}{x^2} \text{ and } g(x) = 4x^3.$$

(a) Find $f(-5)$.

(1)

Solution

$$\begin{aligned}
 f(-5) &= \frac{2}{(-5)^2} \\
 &= \underline{\underline{\frac{2}{25}}}.
 \end{aligned}$$

(b) Find $f g(1)$.

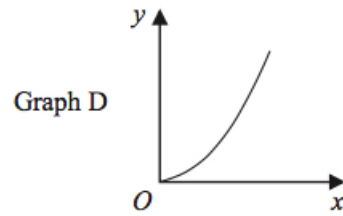
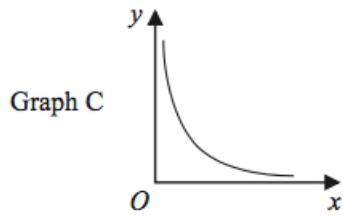
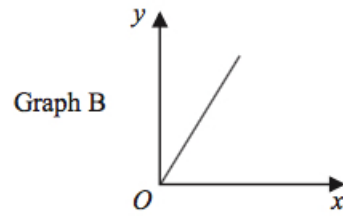
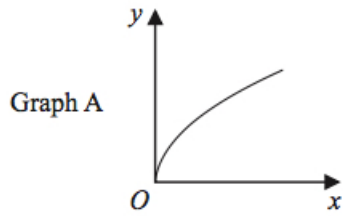
(2)

Solution

$$\begin{aligned}
 f g(x) &= f(g(1)) \\
 &= f(4) \\
 &= \frac{2}{4^2} \\
 &= \frac{2}{16} \\
 &= \underline{\underline{\frac{1}{8}}}.
 \end{aligned}$$

12. The graphs of y against x represent four different types of proportionality.

(2)



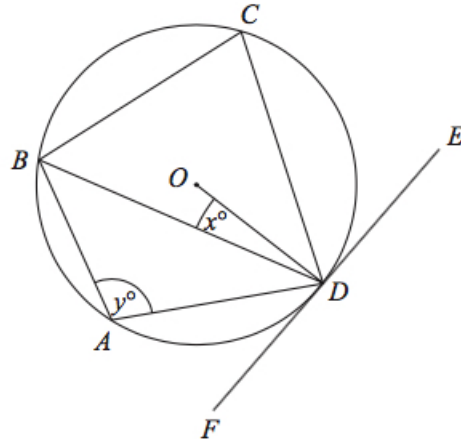
Match each type of proportionality in the table to the correct graph.

Proportionality	Graph Letter
$y \propto x$	
$y \propto x^2$	
$y \propto \sqrt{x}$	
$y \propto \frac{1}{x}$	

Solution

Proportionality	Graph Letter
$y \propto x$	<u>Graph B</u>
$y \propto x^2$	<u>Graph D</u>
$y \propto \sqrt{x}$	<u>Graph A</u>
$y \propto \frac{1}{x}$	<u>Graph C</u>

13. A , B , C , and D are points on the circumference of a circle, centre O .



FDE is a tangent to the circle.

- (a) Show that $y - x = 90$.

You must give a reason for each stage of your working.

(3)

Solution

$\angle BDO = x^\circ$ and $\angle ODE = 90^\circ$ (FDE is a tangent to the circle). Finally,

$$y = x + 90 \Rightarrow \underline{\underline{y - x = 90}}$$

by the alternate segment theorem.

Dylan was asked to give some possible values for x and y .

He said, “ y could be 200 and x could be 110, because $200 - 110 = 90$.”

- (b) Is Dylan correct?

You must give a reason for your answer.

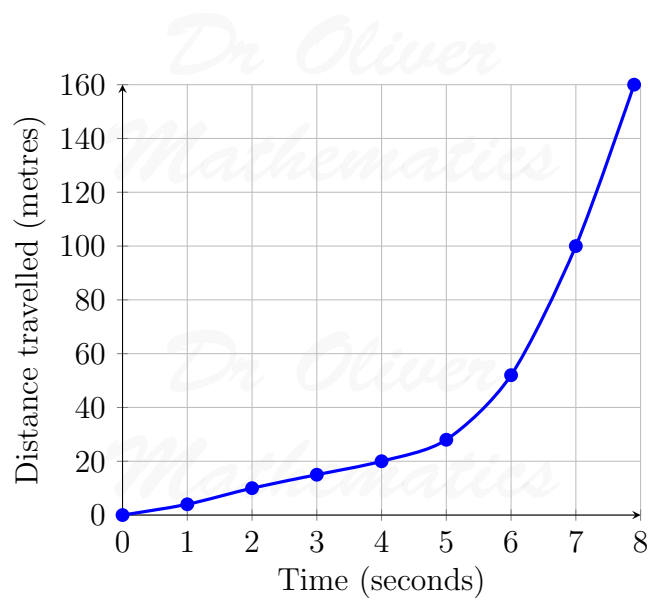
(1)

Solution

No because y is less than 180° .

14. The distance-time graph shows information about part of a car journey.

(3)



Use the graph to estimate the speed of the car at time 5 seconds.

Solution

The speed of the car at time 5 seconds is around

$$\frac{43 - 13}{6 - 4} = \underline{\underline{15 \text{ m/s.}}}$$

15. A darts team is going to play a match on Saturday and on Sunday.

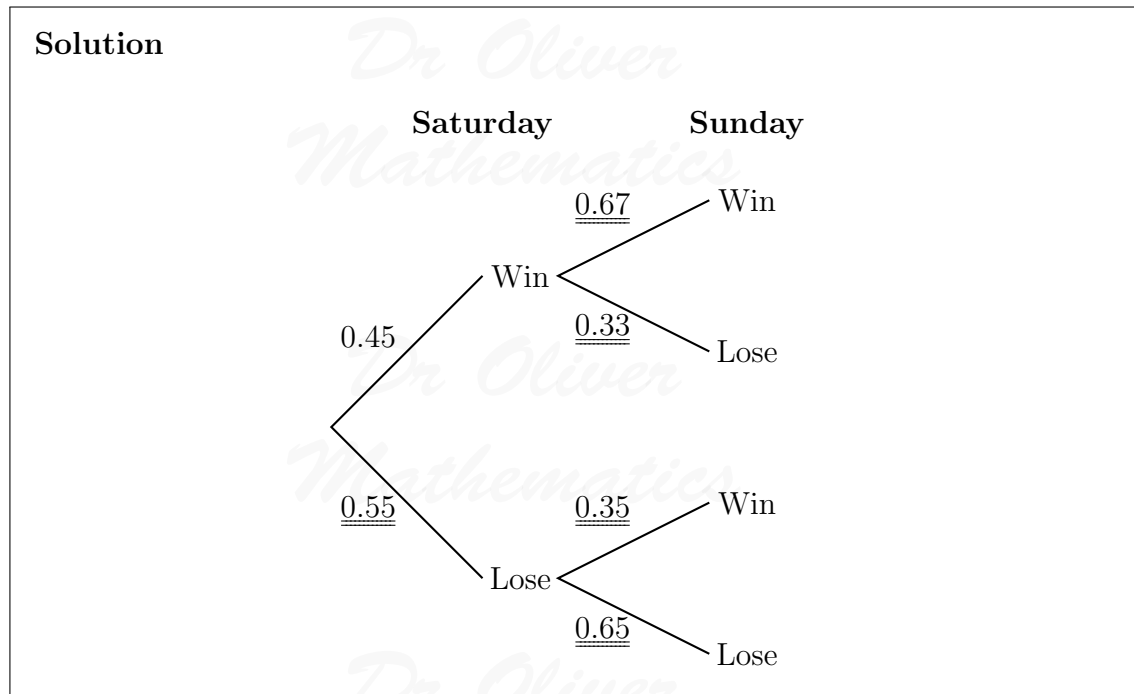
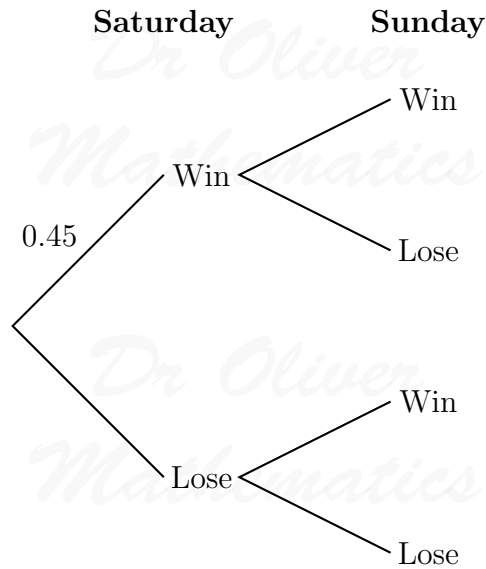
The probability that the team will win on Saturday is 0.45.

If they win on Saturday, the probability that they will win on Sunday is 0.67.

If they do **not** win on Saturday, the probability that they will win on Sunday is 0.35.

(a) Complete the probability tree diagram.

(2)



(b) Find the probability that the team will win exactly one of the two matches.

(3)

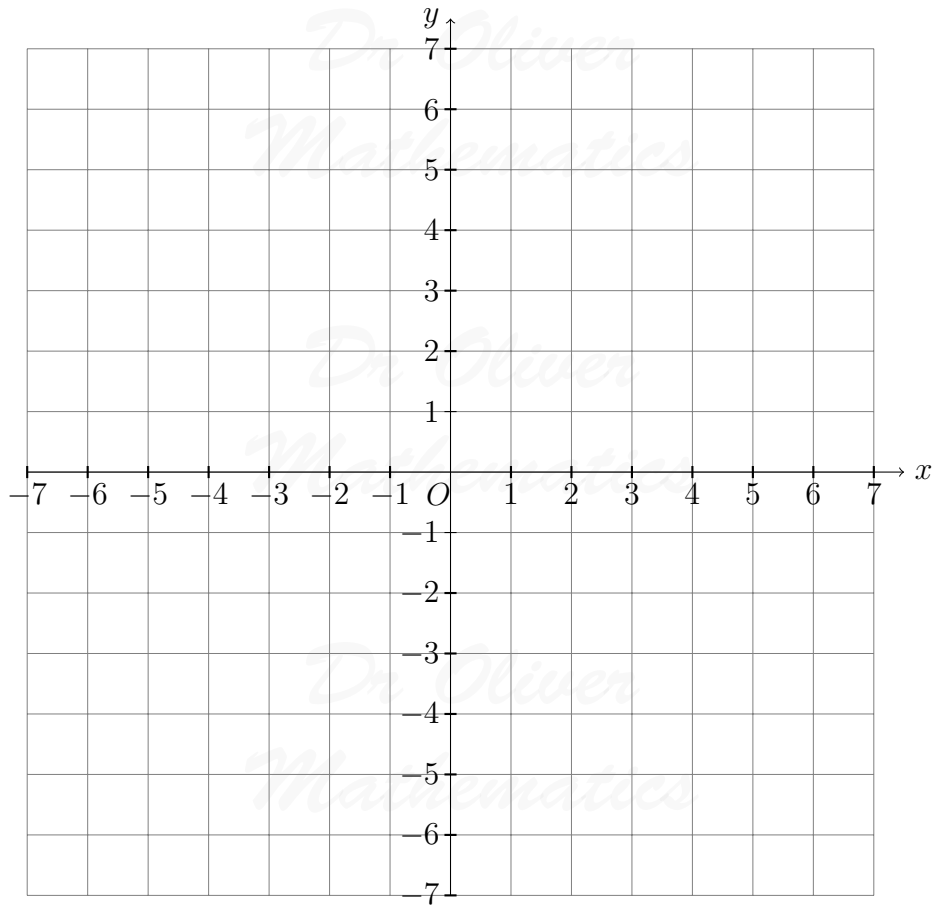
Solution

$$\begin{aligned} P(\text{exactly one of the two matches}) &= P(W, L) + P(L, W) \\ &= (0.45 \times 0.33) + (0.55 \times 0.35) \\ &= 0.1485 + 0.1925 \\ &= \underline{\underline{0.341}}. \end{aligned}$$

16. (a) On the grid, draw the graph of

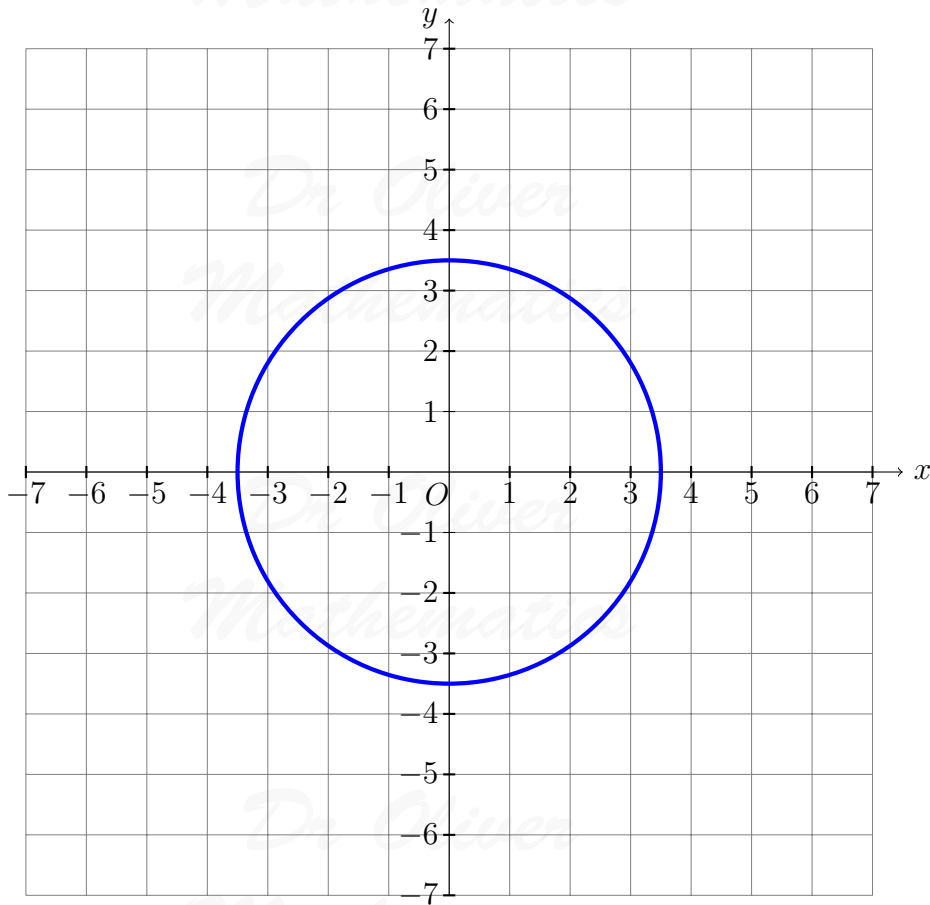
$$x^2 + y^2 = 12.25.$$

(2)



Solution

Draw in $2x + y = 1$ and read off the points of intersection:



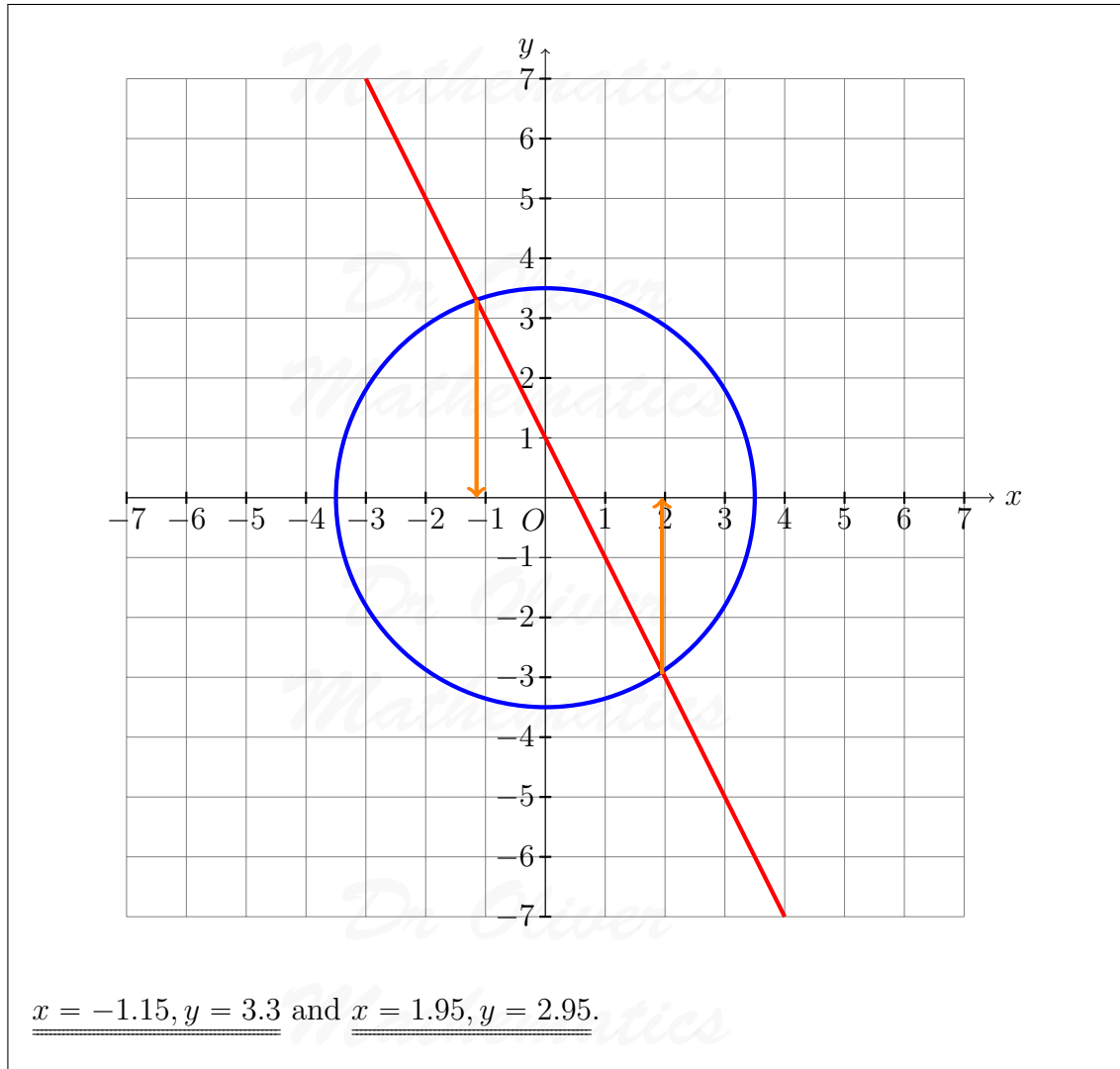
(b) Hence find estimates for the solutions of the simultaneous equations

(3)

$$x^2 + y^2 = 12.25$$

$$2x + y = 1.$$

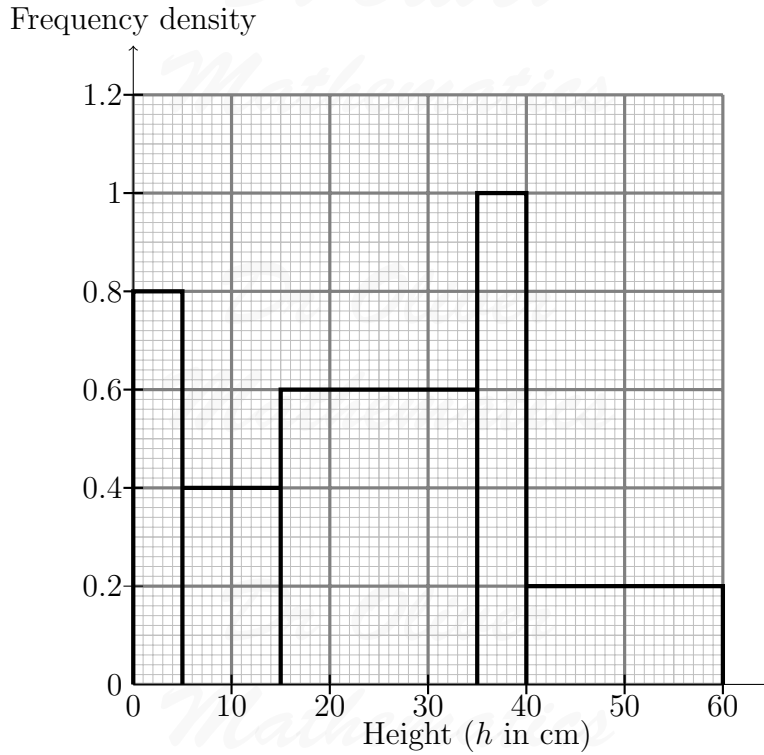
Solution



17. The histogram shows information about the times taken by some students to finish a puzzle.

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(a) Complete the frequency table for this information.

(2)

Time taken (t minutes)	Frequency
$0 < t \leq 5$	4
$5 < t \leq 15$	
$15 < t \leq 25$	
$25 < t \leq 30$	
$30 < t \leq 50$	

Solution

Time taken (t minutes)	Frequency	Width	Frequency Density
$0 < t \leq 5$	4	5	$\frac{4}{5} = 0.8$
$5 < t \leq 15$	<u>4</u>	10	$\frac{4}{10} = 0.4$
$15 < t \leq 25$	<u>6</u>	10	$\frac{6}{10} = 0.6$
$25 < t \leq 30$	<u>5</u>	5	$\frac{5}{5} = 1$
$30 < t \leq 50$	<u>4</u>	20	$\frac{4}{20} = 0.2$

- (b) Find an estimate for the lower quartile of the times taken to finish the puzzle. (2)

Solution

The total frequency is

$$4 + 4 + 6 + 5 + 4 = 23$$

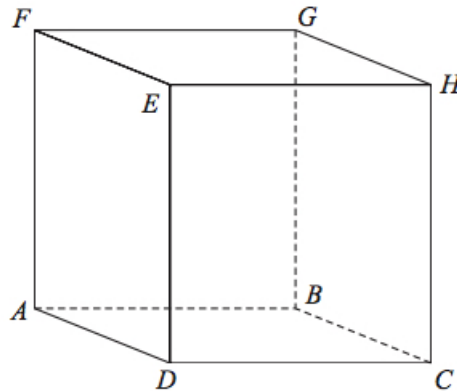
so the lower quartile is the

$$\frac{23 + 1}{4} = 6\text{th}$$

piece of data. Hence, the lower quartile is

$$5 + \frac{2}{4} \times 10 = \underline{\underline{10}}.$$

18. $ABCDEFGH$ is a cuboid. (4)



$$AB = 7.3 \text{ cm.}$$

$$CH = 8.1 \text{ cm.}$$

$$\text{Angle } BCA = 48^\circ.$$

Find the size of the angle between AH and the plane $ABCD$.

Give your answer correct to 1 decimal place.

Solution

Let x° be the angle between AH and the plane $ABCD$. Now,

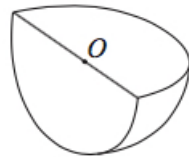
$$\begin{aligned} \sin x &= \frac{\text{adj}}{\text{hyp}} \Rightarrow \sin 48^\circ = \frac{7.3}{AC} \\ &\Rightarrow AC = \frac{7.3}{\sin 48^\circ}. \end{aligned}$$

Finally,

$$\begin{aligned}\tan &= \frac{\text{opp}}{\text{adj}} \Rightarrow \tan x^\circ = \frac{8.1}{\frac{7.3}{\sin 48^\circ}} \\ &\Rightarrow \tan x^\circ = \frac{8.1 \sin 48^\circ}{7.3} \\ &\Rightarrow x = 39.50849231 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 39.5 \text{ (1 dp)}}}.\end{aligned}$$

19. Shape **S** is one quarter of a solid sphere, centre O .

(5)



Shape **S**

The volume of **S** is $576\pi \text{ cm}^3$.

Find the surface area of **S**.

Give your answer correct to 3 significant figures.

You must show your working.

Solution

Let r cm be the radius of the sphere.

$$\begin{aligned}14 \times \frac{4}{3}\pi r^3 &= 576\pi \Rightarrow \frac{1}{3}r^3 = 576 \\ &\Rightarrow r^3 = 1728 \\ &\Rightarrow r = 12.\end{aligned}$$

Now,

$$\begin{aligned}\text{surface area} &= \left(\frac{1}{4} \times 4 \times \pi \times 5.768\dots^2\right) + \left(2 \times \frac{1}{2} \times \pi \times 5.768\dots^2\right) \\ &= 288\pi \\ &= 904.7786842 \text{ (FCD)} \\ &= \underline{\underline{905 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

20. Martin did this question.

Rationalise the denominator of $\frac{14}{2 + \sqrt{3}}$

Here is how he answered the question.

$$\begin{aligned}\frac{14}{2 + \sqrt{3}} &= \frac{14 \times (2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} \\ &= \frac{28 - 14\sqrt{3}}{4 + 2\sqrt{3} - 2\sqrt{3} + 3} \\ &= \frac{28 - 14\sqrt{3}}{7} \\ &= 4 - 2\sqrt{3}\end{aligned}$$

Martin's answer is wrong.

(a) Find Martin's mistake.

(1)

Solution

In the second line, the denominator should be

$$\underline{\underline{4 + 2\sqrt{3} - 2\sqrt{3} - 3}}$$

and it should read

$$\frac{14}{2 + \sqrt{3}} = 28 - 14\sqrt{3}.$$

Sian did this question.

Rationalise the denominator of $\frac{5}{\sqrt{12}}$

Here is how she answered the question.

$$\begin{aligned}\frac{5}{\sqrt{12}} &= \frac{5\sqrt{12}}{\sqrt{12} \times \sqrt{12}} \\ &= \frac{5 \times 3\sqrt{2}}{12} \\ &= \frac{5\sqrt{2}}{4}\end{aligned}$$

Sian's answer is wrong.

(b) Find Sian's mistake.

(1)

Solution

In the second line, the numerator should be

$$\underline{\underline{2\sqrt{3}}}$$

and it should read

$$\frac{5}{\sqrt{12}} = \frac{5\sqrt{3}}{6}.$$

21. Jackson is trying to find the density, in g/cm^3 , of a block of wood. The block of wood is in the shape of a cuboid.

(5)

He measures:

the length as 13.2 cm, correct to the nearest mm,
the width as 16.0 cm, correct to the nearest mm, and
the height as 21.7 cm, correct to the nearest mm.

He measures the mass as 1 970 g, correct to the nearest 5 g.

By considering bounds, work out the density of the wood.
Give your answer to a suitable degree of accuracy.

You must show all your working and give a reason for your final answer.

Solution

$$13.15 \leq \text{length} < 13.25$$

$$15.95 \leq \text{width} < 16.05$$

$$21.65 \leq \text{height} < 21.75$$

$$1\,967.5 \leq \text{mass} < 1\,972.5$$

Hence, the density of the wood is given by

$$\frac{1\,967.5}{13.25 \times 16.05 \times 21.75} < \text{density} < \frac{1\,972.5}{13.15 \times 15.95 \times 21.65}$$
$$\Rightarrow 0.425\,367\,754\,6 < \text{density} < 0.434\,382\,850\,6 \text{ (FCD)}.$$

We draw up a table.

	Lower bound	Upper bound	Agree?
1 sf	0.4	0.4	Yes
2 sf	0.43	0.43	Yes
3 sf	0.425	0.433	No

So, the density of the wood is

$$\underline{\underline{0.43 \text{ g/cm}^3}}$$

because the lower bound and upper bound agree to 2 significant figures but not to 3 significant figures.