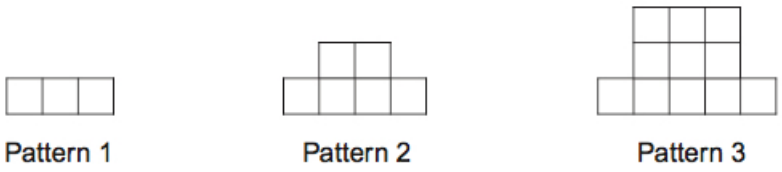
Dr Oliver Mathematics
AQA GCSE Mathematics
2015 November Paper 2: Calculator
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.

1. Here is a sequence of patterns made with squares.



The rule for working out the number of squares in each pattern is

Square the pattern number and then add 2

(a) How many squares are in pattern 7? (1)

Solution

$$\begin{aligned} \text{Pattern 1}: &\quad 3 = 1^2 + 2 \\ \text{Pattern 2}: &\quad 6 = 2^2 + 2 \\ \text{Pattern 3}: &\quad 11 = 3^2 + 2. \end{aligned}$$

So

$$7^2 + 2 = 49 + 2 = \underline{\underline{51}}.$$

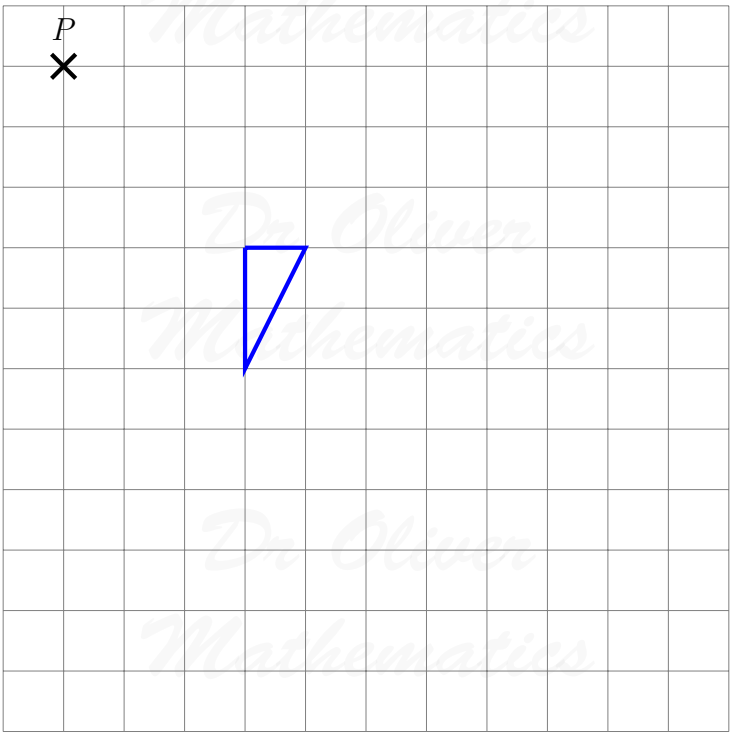
(b) Which pattern has 123 squares? (2)

Solution

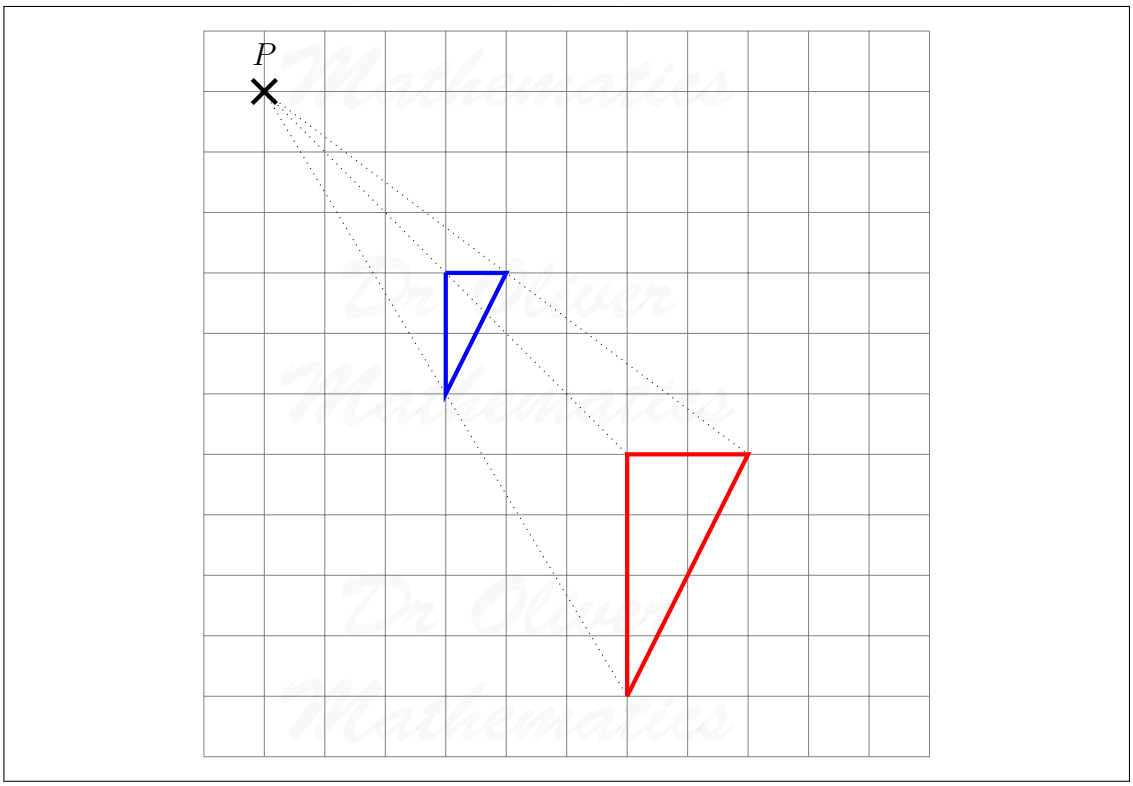
Let the pattern number be n. Then

$$\begin{aligned} n^2 + 2 = 123 &\Rightarrow n^2 = 121 \\ &\Rightarrow \underline{\underline{n = 11}}. \end{aligned}$$

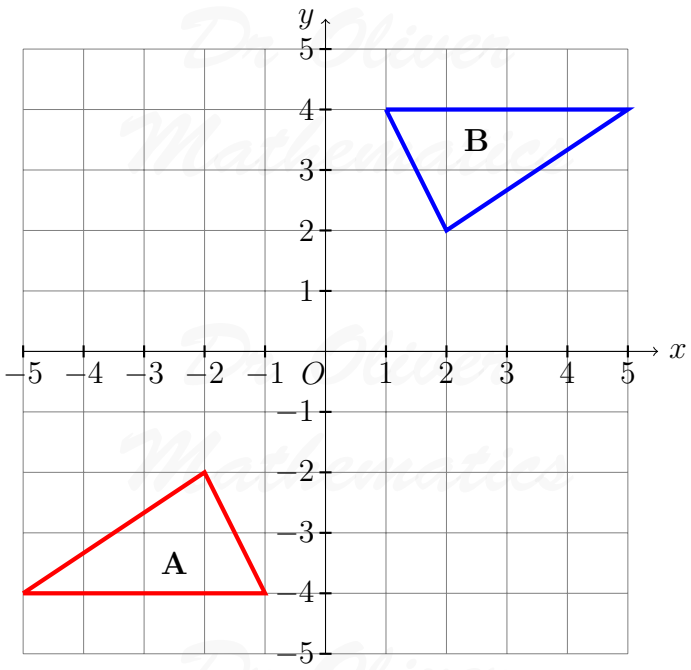
2. (a) Enlarge the triangle by scale factor 2, using point P as the centre of enlargement. (3)



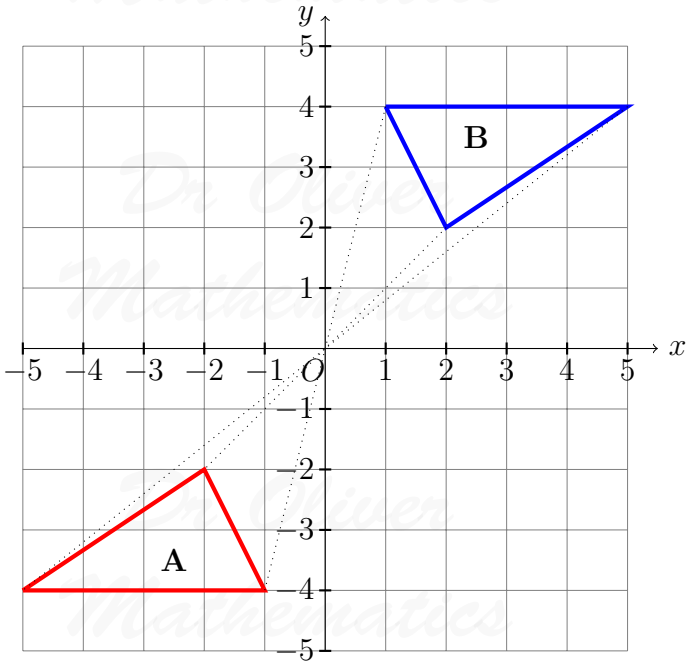
Solution



(b) Describe fully the **single** transformation that maps shape A onto shape B. (3)



Solution



Rotation, through 180°, about (0, 0).

3. A family uses 300 units of gas. (4)

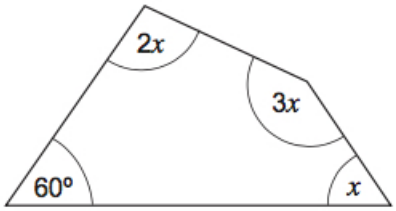
Each unit of gas costs 19p without VAT.
VAT of 5% is added to the bill.

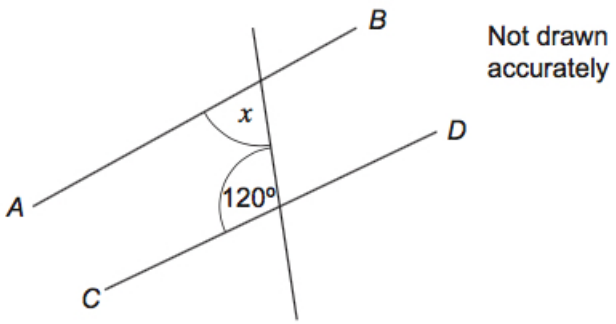
Work out the total gas bill.

Solution

$$\begin{aligned}\text{Total} &= (300 \times 0.19) \times 1.05 \\ &= 57 \times 1.05 \\ &= £59.85.\end{aligned}$$

4. Show that AB is **not** parallel to CD. (4)





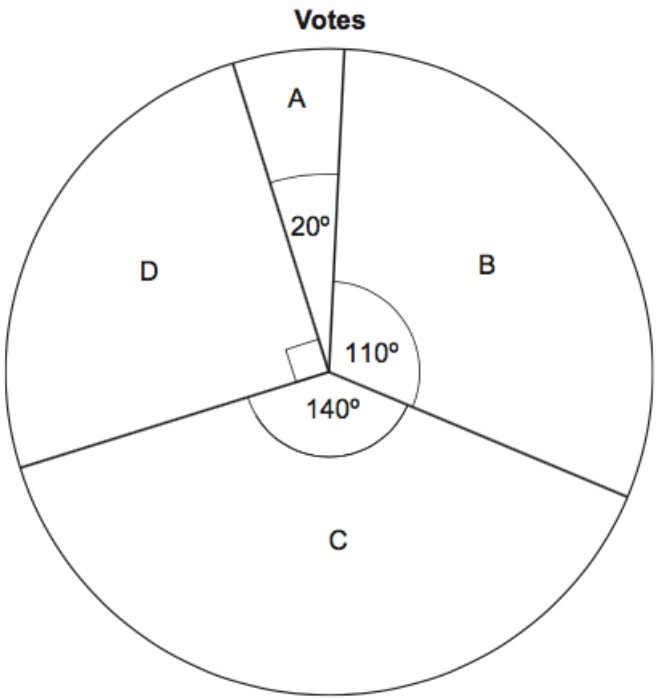
Solution

Well,

$$\begin{aligned} 60 + x + 2x + 3x + x = 360 &\Rightarrow 60 + 6x = 360 \\ &\Rightarrow 6x = 300 \\ &\Rightarrow x = 50; \end{aligned}$$

hence, AB and CD **must cross** to the left of the transversal line as they do not add up to 180°.

5. The pie chart shows information about how people voted in an election. (3)



1 800 people voted for D.

How many **more** people voted for C than B?

Solution

Well,

$$1\,800 \times 4 = 7\,200$$

voted and

$$\begin{aligned}(\tfrac{140}{360} \times 7\,200) - (\tfrac{110}{360} \times 7\,200) &= \tfrac{7\,200}{360}(140 - 110) \\ &= 20 \times 30 \\ &= \underline{\underline{600}}.\end{aligned}$$

6. (a) Solve (3)

$$6x + 4 = 2(2x - 5).$$

Solution

$$\begin{aligned} 6x + 4 = 2(2x - 5) &\Rightarrow 6x + 4 = 4x - 10 \\ &\Rightarrow 2x = -14 \\ &\Rightarrow \underline{\underline{x = -7}}. \end{aligned}$$

(b) Multiply out (2)

$$y(2 - y^3).$$

Solution

$$y(2 - y^3) = \underline{\underline{2y - y^4}}.$$

7. Abby and Judy share some money.
Abby gets 25%.

(a) Write (2)

Abby's share : Judy's share

as a ratio.
Give your answer in its simplest form.

Solution
Well,

$$25 : 75 = \underline{\underline{1 : 3}}.$$

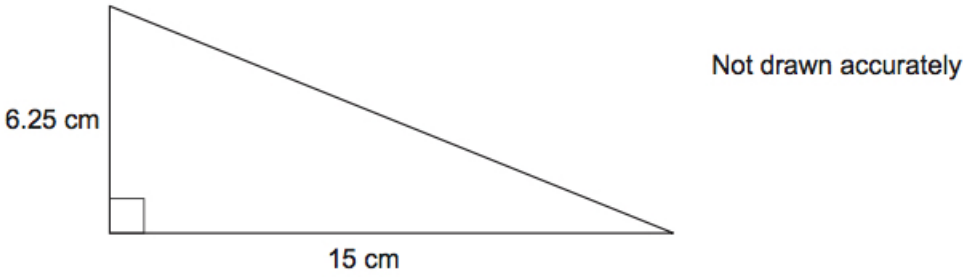
Judy gets £19.50

(b) How much does Abby get? (2)

Solution

$$\frac{19.50}{3} = \underline{\underline{£6.50}}.$$

8. Work out the length of the hypotenuse. (3)



Solution

Let x cm be the length of the hypotenuse. Then

$$x^2 = 6.25^2 + 15^2 \Rightarrow x^2 = 39\tfrac{1}{16} + 225$$
$$\Rightarrow x^2 = 264\tfrac{1}{16}$$
$$\Rightarrow \underline{\underline{x = 16\tfrac{1}{4}}}.$$

9. Here is information about the scores, t, of class A in a test. (4)

Score	Frequency
$0 < t \leqslant 10$	4
$10 < t \leqslant 20$	8
$20 < t \leqslant 30$	9
$30 < t \leqslant 40$	3
$40 < t \leqslant 50$	1

The mean score for class B in the same test is 22.

Dan says, "On average, class A did better than class B."

Is he correct?
You must show your working.

Solution

Score	Frequency	Midpoint	Freq × Midpoint
$0 < t \leqslant 10$	4	5	$4 \times 5 = 20$
$10 < t \leqslant 20$	8	15	$8 \times 15 = 120$
$20 < t \leqslant 30$	9	25	$9 \times 25 = 225$
$30 < t \leqslant 40$	3	35	$3 \times 35 = 105$
$40 < t \leqslant 50$	1	45	$1 \times 45 = 45$
Total	25		515

So,

$$\begin{aligned}\text{mean for class A} &= \frac{\sum fx}{\sum f} \\ &\approx \frac{515}{25} \\ &= 20.6.\end{aligned}$$

Given that the mean score for class B in the same test is 22, class A did less well.

10. a and b are different prime numbers with $a > b$.

(a) Give an example to show that (1)

$$a^2 + b^2$$

could be even.

Solution

E.g., $a = 5$ and $b = 3$:

$$a^2 + b^2 = 5^2 + 3^2 = 25 + 9 = 34.$$

(b) Give an example to show that (1)

$$a^2 + b^2$$

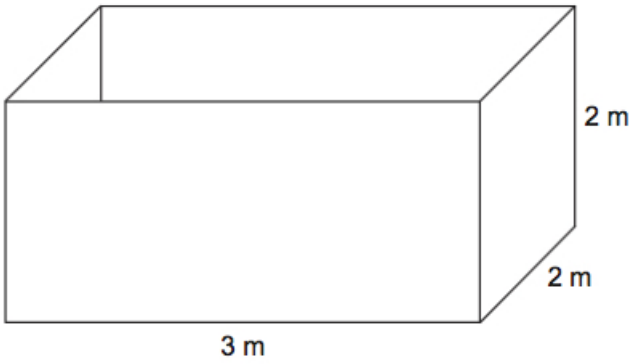
could be odd.

Solution

E.g., $a = 5$ and $b = 2$:

$$a^2 + b^2 = 5^2 + 2^2 = 25 + 4 = 29.$$

11. An empty tank is in the shape of a cuboid as shown. (5)



The tank is to be filled with water at 1.25 litres per second.

$1\ \text{m}^3 = 1\,000$ litres.

Work out the time taken to fill the tank.
Give your answer in hours and minutes.

Solution

Well,

$$\begin{aligned}\text{volume} &= 3 \times 2 \times 2 \\ &= 12\ \text{m}^3 \\ &= 12\,000\ \text{litres}\end{aligned}$$

and

$$\begin{aligned}\text{time} &= \frac{12\,000}{1.25} \\ &= 9\,600\ \text{seconds} \\ &= 160\ \text{minutes} \\ &= \underline{\underline{2\ \text{hours}\ 40\ \text{minutes}}}.\end{aligned}$$

12. (a) Complete the table of values for (2)

$$y = x^3 + 5.$$

x	-3	-2	-1	0	1	2	3
y	-22		4	5	6	13	

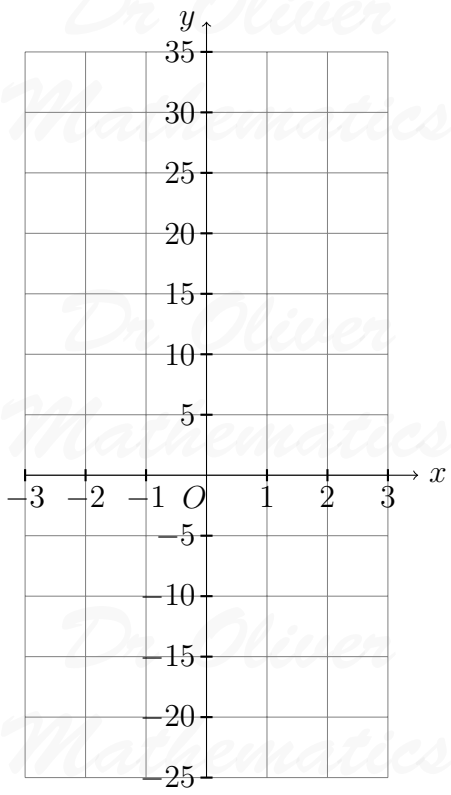
Solution

x	-3	-2	-1	0	1	2	3
y	-22	$\underline{\underline{-3}}$	4	5	6	13	$\underline{\underline{32}}$

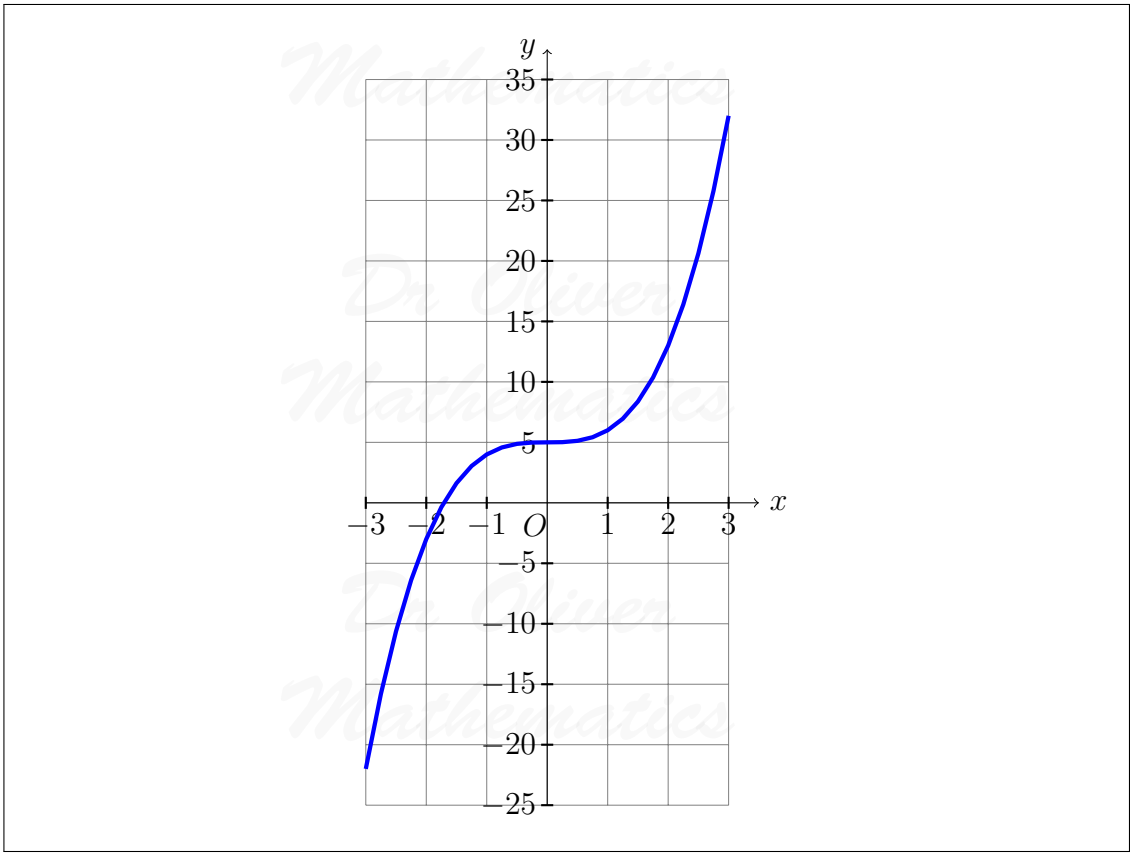
(b) On the grid, draw the graph of (2)

$$y = x^3 + 5$$

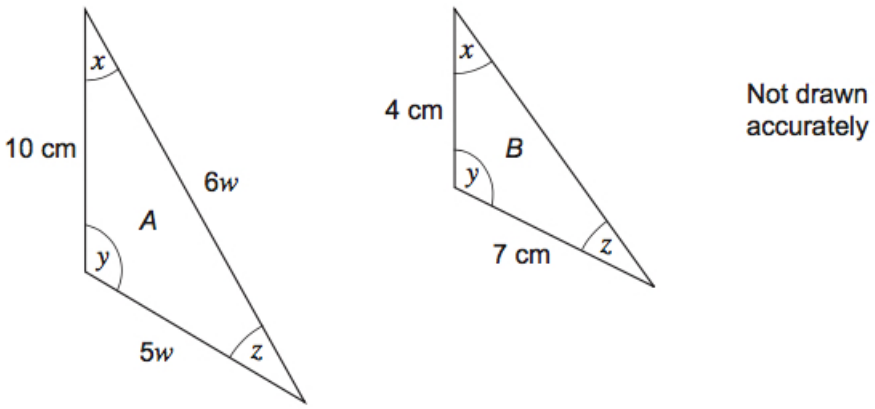
for values of x from -3 to 3.



Solution



13. A and B are similar triangles.



(a) Circle the scale factor from A to B. (1)

-6 $\frac{2}{5}$ $\frac{5}{2}$ 6

Solution

Well,

$$\frac{4}{10} = \tfrac{2}{5}$$

so

$$-6 \qquad \underline{\underline{\tfrac{2}{5}}} \qquad \tfrac{5}{2} \qquad 6$$

(b) Work out the perimeter of triangle B. (4)

Solution

Let the unmarked be x cm. Now,

$$\begin{aligned} \frac{5w}{7} = \tfrac{2}{5} &\Rightarrow 5w = \tfrac{2}{5} \times 7 \\ &\Rightarrow 5w = 2\tfrac{4}{5} \\ &\Rightarrow w = \tfrac{14}{25} \\ &\Rightarrow 6w = 3\tfrac{9}{25}. \end{aligned}$$

and

$$\begin{aligned} \frac{3\frac{9}{25}}{x} = \tfrac{2}{5} &\Rightarrow \frac{x}{3\frac{9}{25}} = \tfrac{5}{2} \\ &\Rightarrow x = \tfrac{5}{2} \times 3\tfrac{9}{25} \\ &\Rightarrow x = 8\tfrac{2}{5}. \end{aligned}$$

Finally,

$$\begin{aligned} \text{perimeter} &= 4 + 7 + 8\tfrac{2}{5} \\ &= \underline{\underline{19\tfrac{2}{5}}} \text{ cm}. \end{aligned}$$

14. (a) Which calculation works out the total amount after decreasing £50 by 8%? (1)
Circle the correct answer.

$$£50 \qquad £50 \times 0.92 \qquad \frac{£50}{0.08} \qquad \frac{£50}{1.08}$$

Solution

£50	£50 × 0.92	$\frac{£50}{0.08}$	$\frac{£50}{1.08}$

(b) Adrian is going on holiday. (3)

He has two bags.

The mass of one bag is 9 kg.
This is 45% of the total mass of the two bags.

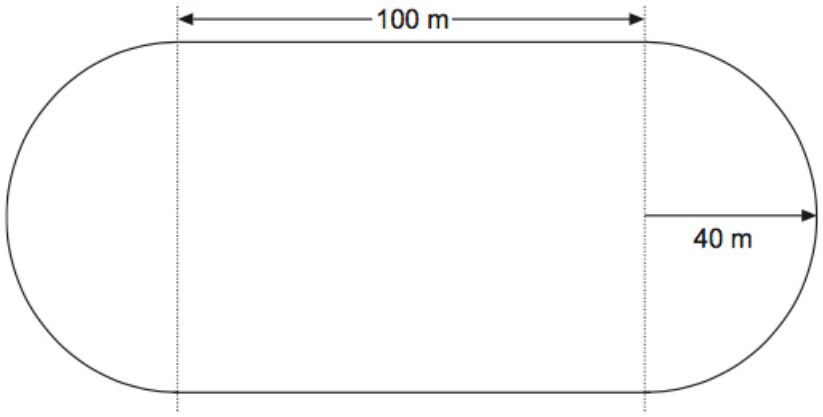
What is the mass of his other bag?

Solution

Let the mass of the other bag be x kg. Then

$$\begin{aligned} 9 : (9 + x) = 45\% : 100\% &\Rightarrow \frac{9 + x}{9} = \frac{100}{45} \\ &\Rightarrow 9 + x = \frac{100}{45} \times 9 \\ &\Rightarrow 9 + x = 20 \\ &\Rightarrow \underline{\underline{x = 11}}. \end{aligned}$$

15. A cycle track has two identical semi-circular ends and two straight sides as shown. (4)



A cyclist completes one lap.

Her average speed is 18 m/s.
Her target time to complete one lap is 30 seconds.

Does she beat her target?
You **must** show your working.

Solution

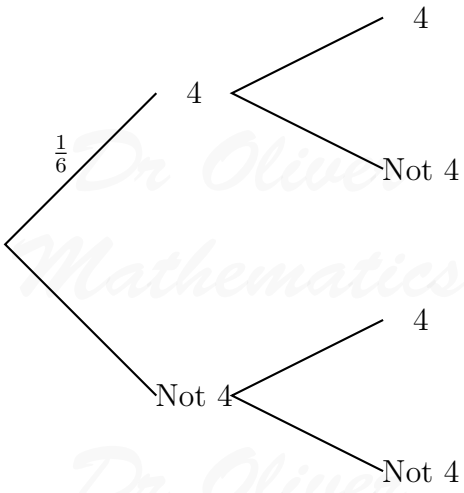
Well, the length is 100, 100, and *one* full circle (why?). Now,

$$\frac{100 + 100 + (2 \times \pi \times 40)}{18} = \frac{200 + 80\pi}{18}$$
$$= 25.073\,745\,13 \text{ (FCD)};$$

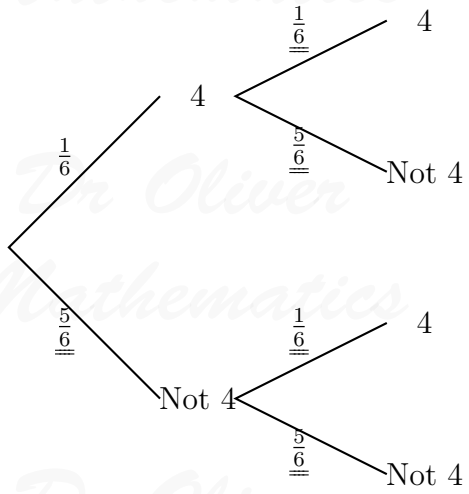
yes, she beats her target.

16. An ordinary fair dice is rolled.

(a) Complete the tree diagram for the dice landing on 4. (1)



Solution

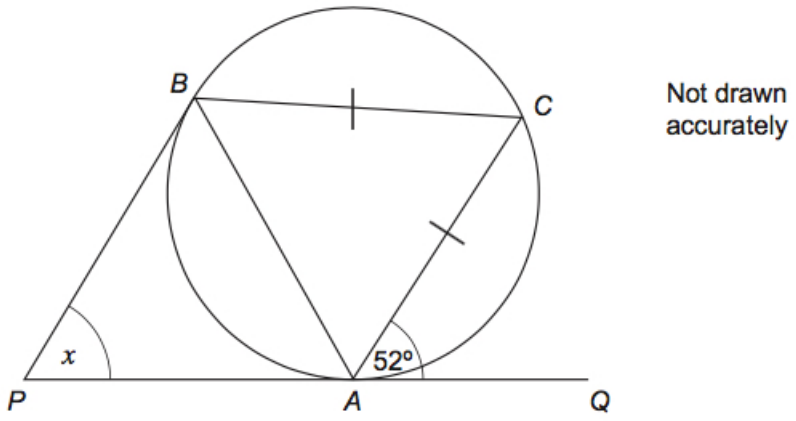


(b) Work out the probability of the dice landing on 4 both times. (2)

Solution

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}.$$

17. PAQ and PB are tangents to the circle. (4)
$AC = BC$.

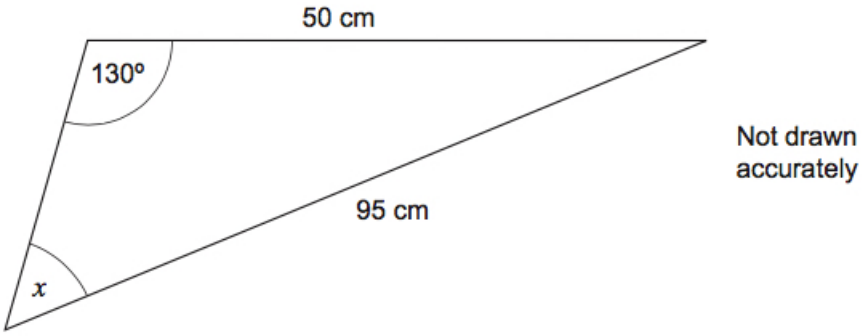


Work out the size of angle x.
You **must** show your working which may be on the diagram.

Solution

$\angle ABC = 52^\circ$ (alternating segment theorem)
$\angle BAC = 52^\circ$ (base angles)
$\angle PAB = 180 - 52 - 52 = 76^\circ$ (straight line)
$\angle PBA = 76^\circ$ (base angles)
$\angle APB = 180 - 76 - 76 = \underline{\underline{28^\circ}}$ (completing the triangle)

18. (a) Work out the size of angle x. (3)

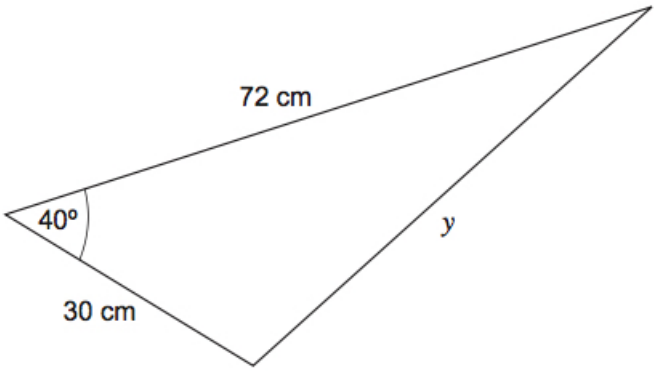


Solution

Sine rule:

$$\frac{\sin x^\circ}{50} = \frac{\sin 130^\circ}{95} \Rightarrow \sin x^\circ = \frac{50 \sin 130^\circ}{95}$$
$$\Rightarrow x^\circ = 23.777\,207\,26^\circ \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x^\circ = 23.8^\circ \text{ (3 sf)}}}.$$

(b) Work out the size of angle y. (3)



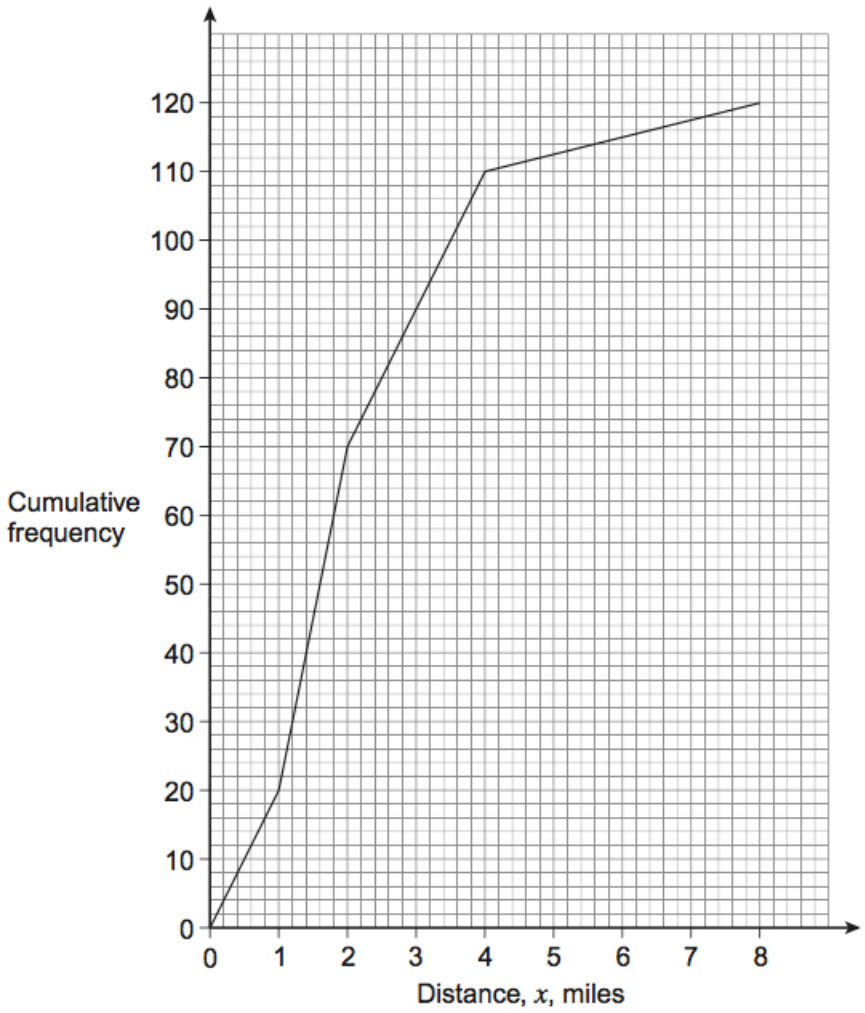
Not drawn accurately

Solution

Cosine rule:

$$\begin{aligned} & y^2 = 30^2 + 72^2 - 2 \times 30 \times 72 \times \cos 40^\circ \\ \Rightarrow \quad & y^2 = 2\,774.688\,006 \text{ (FCD)} \\ \Rightarrow \quad & y = 52.675\,307\,36 \text{ (FCD)} \\ \Rightarrow \quad & \underline{\underline{y = 52.7 \text{ cm (3 sf)}}}. \end{aligned}$$

19. The cumulative frequency diagram shows information about the distances, in miles, that 120 students travel to school.



(a) Work out the interquartile range. (2)

Solution

Read-off the 30th and the 90th value:

$$3 - 1.2 = \underline{\underline{1.8}}.$$

A sample of 25 students is taken from the 120 students.
The sample is stratified by distance travelled using the intervals below.

Distance, x miles	$0 \leqslant x < 1$	$1 \leqslant x < 2$	$2 \leqslant x < 4$	$4 \leqslant x < 8$

(b) Work out the number of students in the sample who are in the $2 \leqslant x < 4$ interval. (4)

Solution

There are $110 - 70 = 40$ who are in the $2 \leqslant x < 4$ interval and the number in the stratified sample is

$$\tfrac{25}{120} \times 40 = 8\tfrac{1}{3};$$

round down: $\underline{\underline{8}}$.

20. (a) Expand and simplify (3)

$$(5x - 2y)(x + 2y).$$

Solution

$\times$	$5x$	$-2y$
x	$5x^2$	$-2xy$
$+2y$	$+10xy$	$-4y^2$

$$(5x - 2y)(x + 2y) = \underline{\underline{5x^2 + 8xy - 4y^2}}.$$

(b) Solve (3)

$$x^2 - 2x - 2 = 0.$$

Give your answers to 1 decimal place.

Solution

We will do this one by completing the square:

$$\begin{aligned} x^2 - 2x - 2 = 0 &\Rightarrow x^2 - 2x = 2 \\ &\Rightarrow x^2 - 2x + 1 = 2 + 1 \\ &\Rightarrow (x - 1)^2 = 3 \\ &\Rightarrow x - 1 = \pm\sqrt{3} \\ &\Rightarrow x = 1 \pm \sqrt{3} \\ &\Rightarrow \underline{\underline{x = -0.7 \text{ or } x = 2.7 \text{ (1 dp)}}}. \end{aligned}$$

(c) Simplify (3)

$$\frac{3x^2 - x - 10}{x^2 - 4}.$$

Solution

$$\left.\begin{array}{rc} \text{add to:} & 1 \\ \text{multiply to:} & (+3) \times (-10) = -30 \end{array}\right\} -5,\ +6$$

E.g.,

$$\begin{aligned} 3x^2 - x - 10 &= 3x^2 - 5x + 6x - 10 \\ &= x(3x - 5) + 2(3x - 5) \\ &= (x + 2)(3x - 5). \end{aligned}$$

$$\left.\begin{array}{rc} \text{add to:} & 0 \\ \text{multiply to:} & -4 \end{array}\right\} -2,\ +2$$

$$x^2 - 4 = (x - 2)(x + 2).$$

Now,

$$\begin{aligned} \frac{3x^2 - x - 10}{x^2 - 4} &= \frac{(x + 2)(3x - 5)}{(x - 2)(x + 2)} \\ &= \underline{\underline{\frac{3x - 5}{x - 2}}}. \end{aligned}$$

21. You are given that (3)

$$x^2 + ax + b \equiv (x - 5)^2 + 7.$$

Work out the values of a and b.

Solution

$\times$	x	-5
x	x^2	$-5x$
-5	$-5x$	$+25$

$$\begin{aligned} (x - 5)^2 + 7 &= (x^2 - 10x + 25) + 7 \\ &= x^2 - 10x + 32; \end{aligned}$$

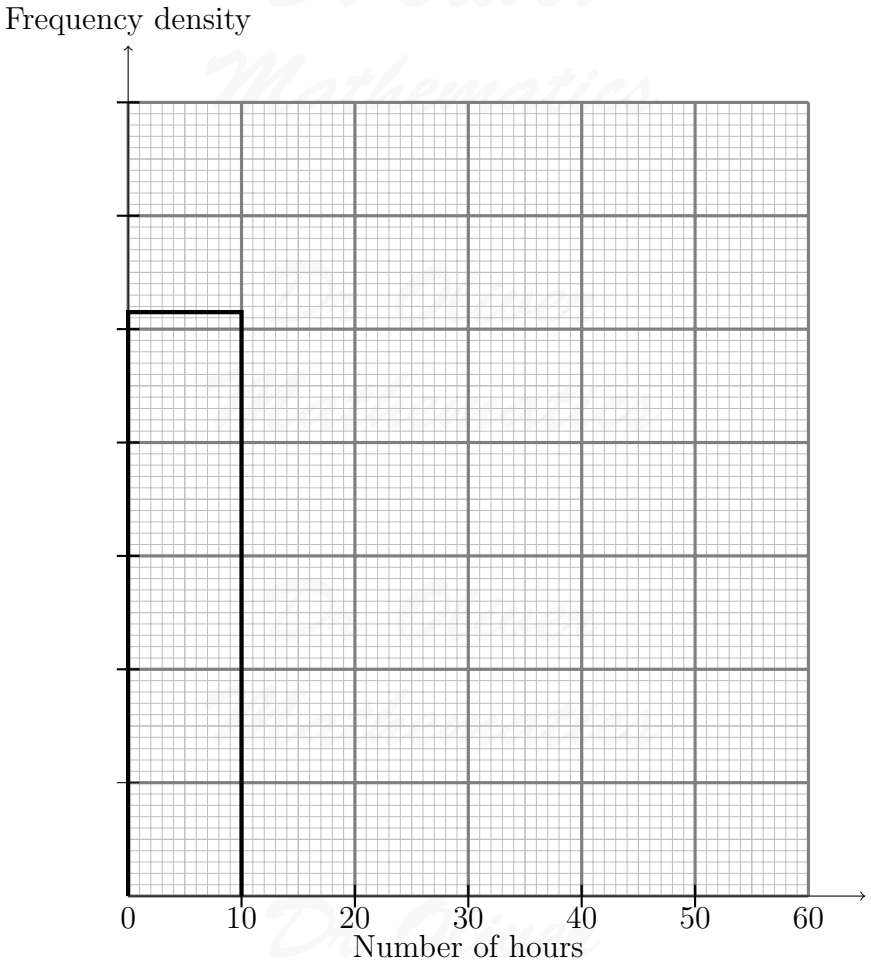
hence, $\underline{\underline{a = -10}}$ and $\underline{\underline{b = 32}}$.

22. 70 people gave information about the number of hours they worked in one week. (6)
The table and histogram show some of that information.

Number of hours, n	Frequency
$0 < n \leqslant 10$	21
$10 < n \leqslant 20$	x
$20 < n \leqslant 40$	y
$40 < n \leqslant 50$	17

$x : y = 3 : 5$.

Complete the histogram.
Remember to label the **scale** on the frequency density axis.



Solution

Well,

$$21 + x + y + 17 = 70 \Rightarrow x + y + 38 = 70$$
$$\Rightarrow x + y = 32$$

Now,

$$3 + 5 = 8$$

and

$$x = \tfrac{3}{8} \times 32 = 12$$
$$y = \tfrac{5}{8} \times 32 = 20.$$

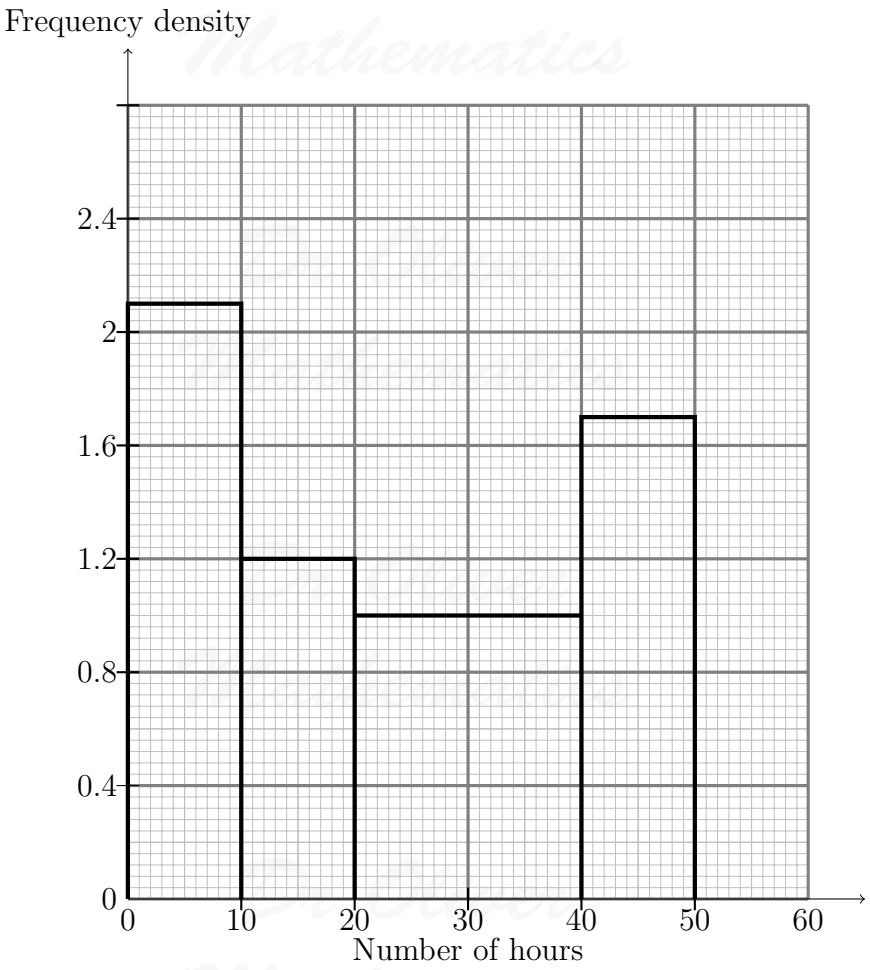
Let us complete the table:

Number of hours, n	Frequency	Width	Frequency Density
$0 < n \leqslant 10$	21	10	$\frac{21}{10} = 2.1$
$10 < n \leqslant 20$	12	10	$\frac{12}{10} = 1.2$
$20 < n \leqslant 40$	20	20	$\frac{20}{20} = 1$
$40 < n \leqslant 50$	17	10	$\frac{17}{10} = 1.7$

52.5 little squares (check this: it is!) is the height of the first bar and its frequency density is 2.1:

$$\begin{aligned} & 52.5 \leftrightarrow 2.1 \\ \Rightarrow \quad & 1 \leftrightarrow \frac{2.1}{52.5} = 0.04 \\ \Rightarrow \quad & 10 \leftrightarrow 0.4. \end{aligned}$$

So we want 10 little squares to represent 0.4 on the frequency density axis. Now the histogram:



23. Solve the simultaneous equations (5)

$$y = 4x + 1$$
$$y = 2x^2 + 7x - 1.$$

Solution

$$2x^2 + 7x - 1 = 4x + 1 \Rightarrow 2x^2 + 3x - 2 = 0$$

add to: $+3$
multiply to: $(+2) \times (-2) = -4$ $\Big\}$ $+4, -1$

$$\begin{aligned}
&\Rightarrow 2x^2 + 4x - x - 2 = 0 \\
&\Rightarrow 2x(x+2) - (x+2) = 0 \\
&\Rightarrow (2x-1)(x+2) = 0 \\
&\Rightarrow 2x - 1 = 0 \text{ or } x + 2 = 0 \\
&\Rightarrow x = \tfrac{1}{2} \text{ or } x = -2 \\
&\Rightarrow y = 3 \text{ or } y = -7;
\end{aligned}$$

hence,

$$\underline{\underline{x = \tfrac{1}{2}, y = 3}} \text{ or } \underline{\underline{x = -2, y = -7}}.$$

24. $x = 400$ to 1 significant figure. (3)
$y = 25$ to 2 significant figures.

Work out the maximum integer value of

$$\frac{x}{y}.$$

Solution

Well,

$$\begin{aligned}
350 &\leqslant x < 450 \\
24.95 &\leqslant y < 25.05.
\end{aligned}$$

Well, the maximum value is

$$\begin{aligned}
\frac{\text{maximum } x}{\text{minimum } y} &= \frac{450}{24.95} \\
&= 18\tfrac{18}{499}
\end{aligned}$$

and the maximum integer value is $\underline{\underline{18}}$.