

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2010 November Paper 1 Variant 1: Calculator
2 hours

The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You must write down all the stages in your working.

1. The equation of a curve is given by

$$y = 2x^2 + ax + 14,$$

where a is a constant.

Given that this equation can also be written as

$$y = 2(x - 3)^2 + b,$$

where b is a constant, find

- (a) the value of a and of b , (2)
(b) the minimum value of y . (1)
2. (a) Sketch, on the same set of axes, the graphs of (2)

$$y = \cos x \text{ and } y = \sin 2x$$

for $0^\circ \leq x \leq 180^\circ$.

- (b) Hence write down the number of solutions of the equation (1)

$$\sin 2x - \cos x = 0$$

for $0^\circ \leq x \leq 180^\circ$.

3. Show that (4)

$$\frac{\cos x}{1 - \sin x} + \frac{\cos x}{1 + \sin x} = 2 \sec x.$$

4. Factorise completely the expression (5)

$$2x^3 - 11x^2 - 20x - 7.$$

5. A curve has the equation

$$y = 2x \sin x + \frac{1}{3}\pi.$$

The curve passes through the point $P(\frac{1}{2}\pi, a)$.

(a) Find, in terms of π , the value of a . (1)

(b) Using your value of a , find the equation of the normal to the curve at P . (5)

6. (a) Find, in ascending powers of x , the first 3 terms in the expansion of (3)

$$(2 - 5x)^6,$$

giving your answer in the form

$$a + bx + cx^2,$$

where a , b , and c are integers.

(b) Find the coefficient of x in the expansion of (3)

$$(2 - 5x)^6(1 + \frac{1}{2}x)^{10}.$$

7. (a) Sets A and B are such that

- $A = \{x : \sin x = 0.5 \text{ for } 0^\circ \leq x \leq 360^\circ\}$,
- $B = \{x : \cos(x - 30)^\circ = -0.5 \text{ for } 0^\circ \leq x \leq 360^\circ\}$.

Find the elements of

(i) A , (2)

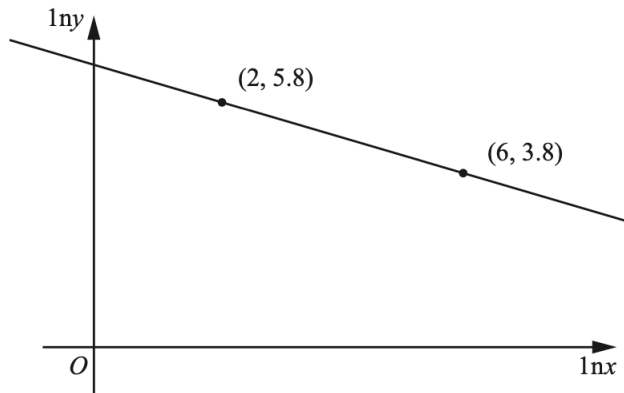
(ii) $A \cup B$. (2)

(b) Set C is such that (3)

$$C = \{x : \sec^2 3x = 1 \text{ for } 0^\circ \leq x \leq 180^\circ\}.$$

Find $n(C)$.

8. Variables x and y are such that, when $\ln y$ is plotted against $\ln x$, a straight line graph passing through the points $(2, 5.8)$ and $(6, 3.8)$ is obtained.



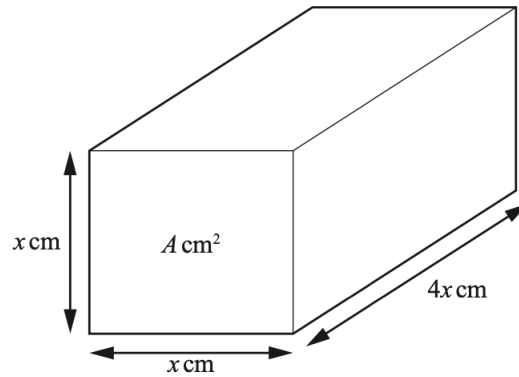
(a) Find the value of $\ln y$ when $\ln x = 0$. (2)

(b) Given that (5)

$$y = Ax^b,$$

find the value of A and of b .

9. The figure shows a rectangular metal block of length $4x$ cm, with a cross-section which is a square of side x cm and area A cm².



The block is heated and the area of the cross-section increases at a constant rate of $0.003 \text{ cm}^2\text{s}^{-1}$.

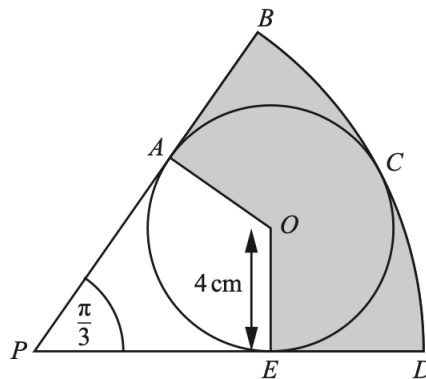
Find

(a) $\frac{dA}{dx}$ in terms of x , (1)

(b) the rate of increase of x when $x = 5$, (3)

(c) the rate of increase of the volume of the block when $x = 5$. (4)

10. The diagram shows a circle, centre O , radius 4 cm, enclosed within a sector $PBCDP$ of a circle, centre P .



- The circle centre O touches the sector at points A , C , and E .
- Angle BPD is $\frac{1}{3}\pi$ radians.

(a) Show that

$$PA = 4\sqrt{3} \text{ cm and } PB = 12 \text{ cm.} \quad (2)$$

Find, to 1 decimal place,

(b) the area of the shaded region, (4)

(c) the perimeter of the shaded region. (4)

11. (a) Find (2)

$$\int \frac{1}{\sqrt{1+x}} dx.$$

(b) Given that (4)

$$y = \frac{2x}{\sqrt{1+x}},$$

show that

$$\frac{dy}{dx} = \frac{A}{\sqrt{1+x}} + \frac{Bx}{(\sqrt{1+x})^3},$$

where A and B are to be found.

(c) Hence find (4)

$$\int \frac{x}{(\sqrt{1+x})^3} dx$$

and evaluate

$$\int_0^3 \frac{x}{(\sqrt{1+x})^3} dx.$$

EITHER

12. A curve is such that

$$\frac{dy}{dx} = 4x^2 - 9.$$

The curve passes through the point $(3, 1)$.

(a) Find the equation of the curve. (4)

The curve has stationary points at A and B .

(b) Find the coordinates of A and of B . (3)

(c) Find the equation of the perpendicular bisector of the line AB . (4)

OR

13. A curve has the equation

$$y = Ae^{2x} + Be^{-x} \text{ where } x \geq 0.$$

At the point where $x = 0$, $y = 50$, and $\frac{dy}{dx} = -20$.

- (a) Show that $A = 10$ and find the value of B . (5)
- (b) Using the values of A and B found in part (a), find the coordinates of the stationary point on the curve. (4)
- (c) Determine the nature of the stationary point, giving a reason for your answer. (2)