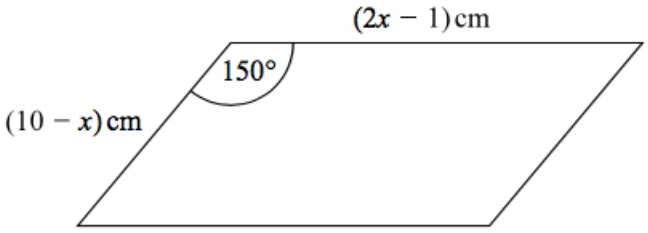
Dr Oliver Mathematics
Worked Examples
Quadratic Inequality 1

From: Edexcel 2019 November Paper 2H (Calculator)

1. The diagram shows a parallelogram.



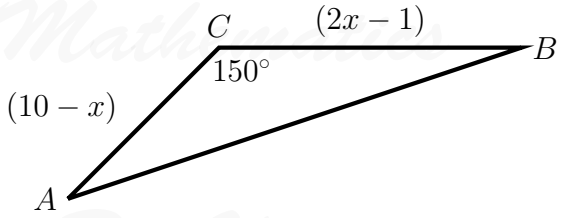
The area of the parallelogram is greater than 15 cm^2.

(a) Show that (3)

$$2x^2 - 21x + 40 < 0.$$

Solution

Cut the parallelogram in half, from bottom left to top right:



Clearly, this is a case where we invoke

$$\text{area of a triangle} = \tfrac{1}{2}ab\sin C.$$

Now,

$$\text{area of this triangle} = \tfrac{1}{2} \times (2x-1) \times (10-x) \times \sin 150^\circ$$

and the

$$\text{area of the parallelogram} = 2 \times \tfrac{1}{2} \times (2x-1) \times (10-x) \times \sin 150^\circ.$$

But

$$\sin 150^\circ = \sin(180 - 150)^\circ = \sin 30^\circ = \tfrac{1}{2}$$

and so

$$\begin{aligned}\text{area of the parallelogram} &= 2 \times \tfrac{1}{2} \times (2x - 1) \times (10 - x) \times \tfrac{1}{2} \\ &= \tfrac{1}{2}(2x - 1)(10 - x).\end{aligned}$$

Next,

$\times$	$2x$	-1
10	$20x$	-10
$-x$	$-2x^2$	$+x$

so

$$\text{area of the parallelogram} = \tfrac{1}{2}(-2x^2 + 21x - 10).$$

So, if the area of the parallelogram is greater than 15 cm^2,

$$\tfrac{1}{2}(-2x^2 + 21x - 10) > 15 \Rightarrow -2x^2 + 21x - 10 > 30$$

(multiplying all four terms by -1)

$$\Rightarrow 2x^2 - 21x + 10 < -30$$

(taking the 30 over to the left-hand side)

$$\Rightarrow \underline{\underline{2x^2 - 21x + 40 < 0}},$$

as required.

(b) Find the range of possible values of x. (3)

Solution

add to: -21
multiply to: $(+2) \times (-40) = -80$
$\Big\}$ $-5, -16$

Now, e.g.,

$$\begin{aligned} 2x^2 - 21x + 40 = 0 &\Rightarrow 2x^2 - 5x - 16x + 40 = 0 \\ &\Rightarrow x(2x-5) - 8(2x-5) = 0 \\ &\Rightarrow (x-8)(2x-5) = 0 \\ &\Rightarrow x - 8 = 0 \text{ or } 2x - 5 = 0 \\ &\Rightarrow x = 8 \text{ or } x = 2\tfrac{1}{2}. \end{aligned}$$

We now do a 'table of signs':

	$x < 2\frac{1}{2}$	$x = 2\frac{1}{2}$	$2\frac{1}{2} < x < 8$	$x = 8$	$x > 8$
$2x - 5$	$-$	0	$+$	$+$	$+$
$x - 8$	$-$	$-$	$-$	0	$+$
$(x-8)(2x-5)$	$+$	0	$-$	0	$+$

Now, we know that we want the terms with $(x-8)(2x-5)$ to be negative. Hence,

$$\underline{\underline{2\tfrac{1}{2} < x < 8}}.$$