# Dr Oliver Mathematics
# GCSE Mathematics
# 2007 June Paper 6H: Calculator
# 2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. A shop sells CDs and DVDs. (2)
   In one week the number of CDs sold and the number of DVDs sold were in the ratio
   $3 : 5$.
   The total number of CDs and DVDs sold in the week was 728.
   Work out the number of CDs sold.

   **Solution**
   $$3 + 5 = 8$$
   and so the the number of CDs sold is
   $$\tfrac{3}{8} \times 728 = \underline{\underline{273}}.$$

2. Here are the first 4 terms of an arithmetic sequence. (2)
   $$2 \quad 5 \quad 8 \quad 11.$$

   Find an expression, in terms of $n$, for the $n$th term of the sequence.

   **Solution**
   Let the
   $$n\text{th term} = an + b.$$

   $$\begin{array}{cccc} 2 & 5 & 8 & 11 \\ & 3 & 3 & 3 \end{array}$$

   $$\begin{array}{cccc} a + b & 2a + b & 3a + b & 4a + b \\ & a & a & a \end{array}$$

We compare terms:
$$a = 3$$
and
$$a + b = 2 \Rightarrow 3 + b = 2$$
$$\Rightarrow b = -1.$$
Hence,
$$n\text{th term} = \underline{\underline{3n - 1}}.$$

3. The equation (4)
$$x^3 - x = 30$$
has a solution between 3 and 4.
Use a trial and improvement method to find this solution.
Give your answer correct to 1 decimal place.
You must show **all** your working.

**Solution**

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
**F(X)=** and you type in $X^3 - X$; then you press $\boxed{=}$.
**Start?** and you enter 3; then you press $\boxed{=}$.
**End?** and you enter 4; then you press $\boxed{=}$.
**Step?** and enter 0.05 – 1 decimal place divided by 2; then you press $\boxed{=}$.

| $x$ | f($x$) | Comment |
|------|--------|---------|
| 3.2 | 29.568 | too low |
| 3.25 | 31.078 | too high |

Clearly,
$$3.2 < x < 3.25$$
and the answer is
$$\underline{\underline{x = 3.2 \text{ (1 dp)}}}.$$

4. Seejal says, "If $a$ and $b$ are prime numbers greater than 2, then $a \times b$ is always an odd (2)
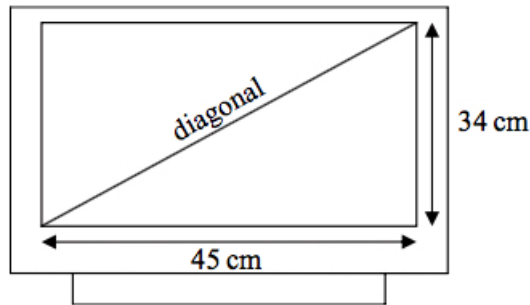number."

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Is Seejal correct?
Give reasons for your answer.

> **Solution**
>
> E.g., 2 is the only even prime number and the product of two odd numbers is odd so <u>she is correct</u>.

5. A rectangular television screen has a width of 45 cm and a height of 34 cm. (4)



Work out the length of the diagonal of the screen.
Give your answer correct to the nearest centimetre.

> **Solution**
>
> $$\begin{aligned} \text{Diagonal} &= \sqrt{45^2 + 34^2} \\ &= 56.400\,354\,61 \text{ (FCD)} \\ &= \underline{\underline{56 \text{ cm (nearest centimetre)}}}. \end{aligned}$$

6. James invested £2 000 for three years in an Internet Savings Account. (3)
   He is paid 5.5% per annum **compound** interest.
   Work out the **total interest** earned after three years.

> **Solution**
>
> $$2\,000 \times 1.055^3 = 2\,348.482\,75$$
>
> and so the total interest earned after three years is <u>£348.48</u>.

7. The map shows part of a lake. In a competition for radio controlled boats, a competitor has to steer a boat so that:

   (i) its path between $AB$ and $CD$ is a straight line, and

   (ii) this path is always the same distance from $A$ as from $B$.

   (a) On the map, draw the path the boat should take. (2)



Scale: 1 cm represents 10 metres
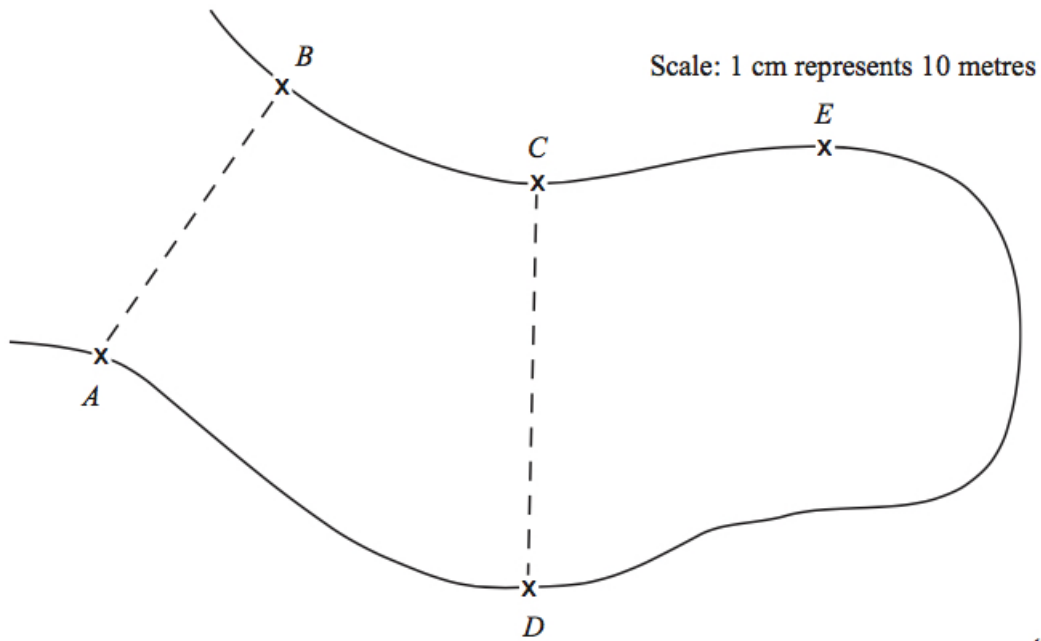
> **Solution**
> Draw a perpendicular bisector from the midpoint of $AB$ until $CD$ is crossed.

There is a practice region for competitors.
The practice region is that part of the lake that is less than 30 metres from point $E$.
The scale of the map is 1 cm represents 10 metres.

   (b) Shade this practice region on the map. (2)

> **Solution**
> Draw a dotted line, 3 cm from $E$ and shade the "lower half".

8. The table gives some information about the time taken by a group of 100 students to complete an IQ test. (4)

| Time ($t$ seconds) | Frequency |
|---|---|
| $60 < t \leqslant 70$ | 12 |
| $70 < t \leqslant 80$ | 22 |
| $80 < t \leqslant 90$ | 23 |
| $90 < t \leqslant 100$ | 24 |
| $100 < t \leqslant 110$ | 19 |

Calculate an estimate for the mean time taken by the students.

**Solution**

| Time ($t$ seconds) | Frequency | Midpoint | Frequency$\times$Midpoint |
|---|---|---|---|
| $60 < t \leqslant 70$ | 12 | 65 | $12 \times 65 = 780$ |
| $70 < t \leqslant 80$ | 22 | 75 | $22 \times 75 = 1\,650$ |
| $80 < t \leqslant 90$ | 23 | 85 | $23 \times 85 = 1\,955$ |
| $90 < t \leqslant 100$ | 24 | 95 | $24 \times 95 = 2\,280$ |
| $100 < t \leqslant 110$ | 19 | 105 | $19 \times 105 = 1\,995$ |
| Total | 100 | | 8\,660 |

An estimate for the mean time taken by the students is

$$\text{mean time} \approx \frac{\sum ft}{\sum t}$$
$$= \frac{8\,660}{100}$$
$$= \underline{86.6 \text{ s}}.$$

9. A plane flies $1\,400$ kilometres in 2 hours 20 minutes. (3)
   Calculate the average speed, in km/h, of the plane.

**Solution**

$$\text{Average speed} = \frac{1\,400}{2\frac{1}{3}}$$
$$= \underline{\underline{600 \text{ km/h}}}.$$

10. The sizes of the angles, in degrees, of the triangle are $2x + 7$, $2x$, $x + 18$.
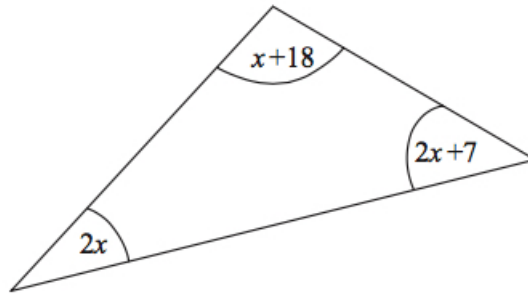


(a) Use this information to write down an equation in terms of $x$.   (2)

> **Solution**
>
> E.g.,
> $$(2x + 7) + 2x + (x + 18) = 180.$$

(b) Use your answer to part (a) to work out the value of $x$.   (2)

> **Solution**
>
> $$(2x + 7) + 2x + (x + 18) = 180 \Rightarrow 5x + 25 = 180$$
> $$\Rightarrow 5x = 155$$
> $$\Rightarrow x = 31.$$

11. (a) Complete the table of values for $y = x^3 + 3x$.   (2)

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $-14$ | | $0$ | | |

> **Solution**
>
> | $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
> |---|---|---|---|---|---|
> | $y$ | $-14$ | $-4$ | $0$ | $4$ | $14$ |
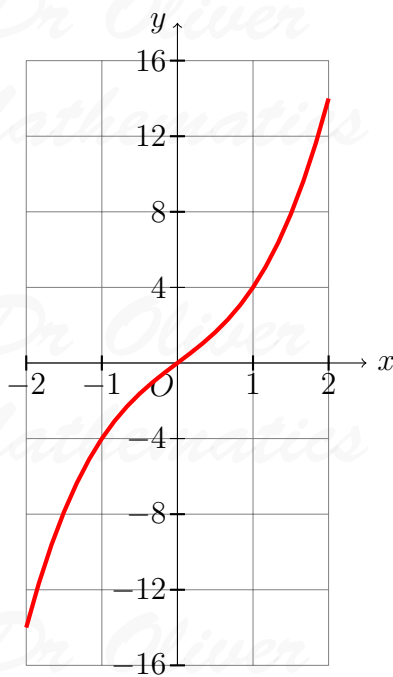
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(b) On the grid, draw the graph of $y = x^3 + 3x$. (2)
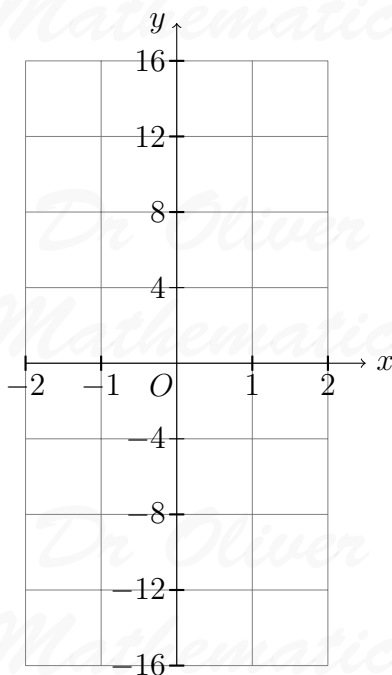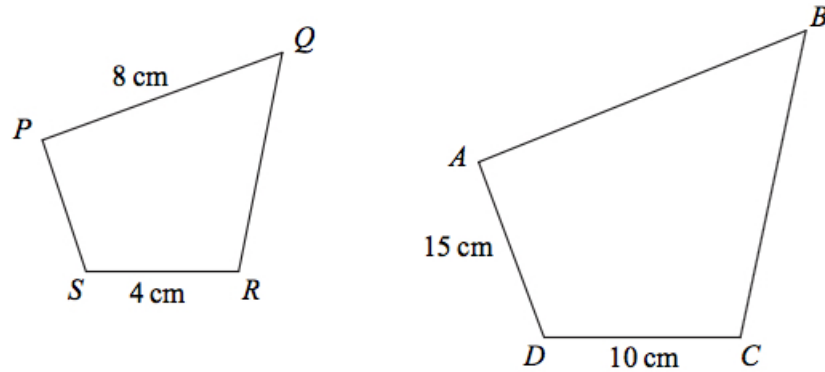


**Solution**

12. The diagram shows two quadrilaterals that are mathematically **similar**.



In quadrilateral $PQRS$, $PQ = 8$ cm, $SR = 4$ cm.
In quadrilateral $ABCD$, $AD = 15$ cm, $DC = 10$ cm.
Angle $PSR =$ angle $ADC$.
Angle $SPQ =$ angle $DAB$.

(a) Calculate the length of $AB$. (2)

**Solution**

$$AB = \frac{10}{4} \times 8$$
$$= \underline{\underline{20 \text{ cm}}}.$$

(b) Calculate the length of $PS$. (2)

**Solution**

$$PS = \frac{4}{10} \times 15$$
$$= \underline{\underline{6 \text{ cm}}}.$$

13. In 2003 the population of Great Britain was $6.0 \times 10^7$.
In 2003 the population of India was $9.9 \times 10^8$.

(a) Work out the difference between the population of India and the population of Great Britain in 2003. (2)
Give your answer in standard form.

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**Solution**

$$\text{Difference} = (9.9 \times 10^8) - (6.0 \times 10^7)$$
$$= 930\,000\,000$$
$$= \underline{\underline{9.3 \times 10^8}}.$$

In 1933 the population of Great Britain was $4.5 \times 10^7$.

(b) Calculate the percentage increase in the population of Great Britain from 1933 to 2003. (3)
Give your answer correct to one decimal place.

**Solution**

$$\text{Population increase} = \frac{6.0 \times 10^7}{4.5 \times 10^7} \times 100$$
$$= 133\tfrac{1}{3}\%,$$

and so the percentage increase is $\underline{\underline{33.3\%}}$ (1 dp).

14. Martin is organising a summer fair. (3)
He needs bread buns and burgers for the barbecue.
Bread buns are sold in packs. Each pack contains 40 bread buns.
Burgers are sold in packs. Each pack contains 24 burgers.
Martin buys exactly the same number of bread buns as burgers.
What is the least number of each pack that Martin buys?

**Solution**

|   | 40 |
|---|----|
| 2 | 20 |
| 2 | 10 |
| 2 | 5  |
| 5 | 1  |

So
$$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5.$$

$$\begin{array}{r|r} & 24 \\ \hline 2 & 12 \\ 2 & 6 \\ 2 & 3 \\ 3 & 1 \end{array}$$

So

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3.$$

Now,

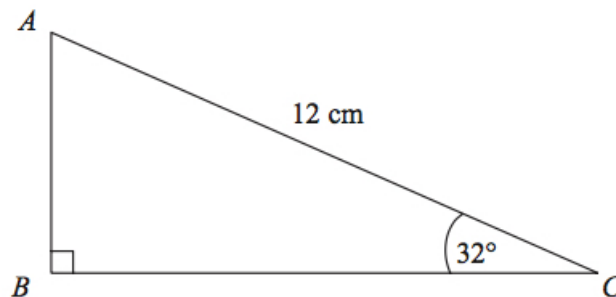$$\mathrm{LCM}(40, 24) = 2^3 \times 3 \times 5$$
$$= 120$$

and he buys <u>3 packs of the bread buns</u> and <u>5 packs of the burgers</u>.

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15. $AC = 12$ cm.
    Angle $ABC = 90°$.
    Angle $ACB = 32°$.

(3)



Calculate the length of $AB$.
Give your answer correct to 3 significant figures.

**Solution**

$$\mathrm{opp} = \mathrm{hyp} \times \sin \Rightarrow AB = 12 \sin 32°$$
$$\Rightarrow AB = 6.359\,031\,171 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{AB = 6.36 \text{ cm} \text{ (3 sf)}}}.$$

16. The mean of eight numbers is 41. (3)
    The mean of two of the numbers is 29.
    What is the mean of the other six numbers?

**Solution**

$$41 = \frac{58 + \text{six numbers}}{8} \Rightarrow 328 = 58 + \text{six numbers}$$
$$\Rightarrow \text{six numbers} = 270$$
$$\Rightarrow \text{mean} = \frac{270}{6}$$
$$\Rightarrow \underline{\text{mean} = 45}.$$

17. (a) Expand and simplify (2)
$$(x + 3)(x - 4).$$

**Solution**

| × | $x$ | $+3$ |
|---|---|---|
| $x$ | $x^2$ | $+3x$ |
| $-4$ | $-4x$ | $-12$ |

Hence,
$$(x + 3)(x - 4) = \underline{\underline{x^2 - x - 12}}.$$

(b) Expand and simplify (2)
$$(2x + 5)(3x - 4).$$

**Solution**

| × | $2x$ | $+5$ |
|---|---|---|
| $3x$ | $6x^2$ | $+15x$ |
| $-4$ | $-8x$ | $-20$ |

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Hence,
$$(2x + 5)(3x - 4) = \underline{\underline{6x^2 + 7x - 20}}.$$

(c) Factorise (2)
$$x^2 + 7x + 10.$$

**Solution**

$$\left.\begin{array}{ll} \text{add to:} & +7 \\ \text{multiply to:} & +10 \end{array}\right\} + 2, +5$$

$$x^2 + 7x + 10 = \underline{\underline{(x + 2)(x + 5)}}.$$

(d) Simplify fully (2)
$$3p^5q \times 4p^3q^2.$$

**Solution**

$$3p^5q \times 4p^3q^2 = \underline{\underline{12p^8q^3}}.$$

(e) $p = 3t + 4(q - t)$. (3)
Find the value of $q$ when $p = 6$ and $t = 5$.

**Solution**

$$6 = 15 + 4(q - 5) \Rightarrow 4(q - 5) = -9$$
$$\Rightarrow q - 5 = -2\tfrac{1}{4}$$
$$\Rightarrow \underline{\underline{q = 2\tfrac{3}{4}}}.$$

18. In a spring, the tension ($T$ newtons) is directly proportional to its extension ($x$ cm). When the tension is 150 newtons, the extension is 6 cm.

(a) Find a formula for $T$ in terms of $x$. (3)

**Solution**

$$T \propto x \Rightarrow T = kx$$

for some $k$. Now,

$$150 = 6k \Rightarrow k = 25$$

and so

$$\underline{\underline{T = 25x}}.$$

(b) Calculate the tension, in newtons, when the extension is 15 cm. (1)

**Solution**

$$T = 25 \times 15 = \underline{\underline{375 \text{ newtons}}}.$$

(c) Calculate the extension, in cm, when the tension is 600 newtons. (1)

**Solution**

$$600 = 25x \Rightarrow \underline{\underline{x = 24 \text{ cm}}}.$$

19. There are three age groups in a competition. (3)
    The table shows the number of competitors in each age group.

| $16 - 18$ years | $19 - 24$ years | $25+$ years |
|:---:|:---:|:---:|
| 120 | 250 | 200 |

John wants to do a survey of the competitors.
He uses a stratified sample of exactly 50 competitors according to each age group. Work out the number of competitors in each age group that should be in his stratified sample of 50.

**Solution**

There are

$$120 + 250 + 200 = 570$$

entrants and we need

| 16 − 18 years | 19 − 24 years | 25+ years |
|:---:|:---:|:---:|
| 120 | 250 | 200 |
| $\frac{120}{570} \times 50 = 10.52\ldots$ | $\frac{250}{570} \times 50 = 21.92\ldots$ | $\frac{200}{570} \times 50 = 17.54\ldots$ |

Finally, we need <u>10 16-18 year olds</u>, <u>22 19-24 year olds</u>, and <u>18 25+ year olds</u>.

20. Solve the equation (5)

$$\frac{x}{2x-3} + \frac{4}{x+1} = 1.$$

**Solution**

Multiply by $(2x-3)(x+1)$:

$$\frac{x}{2x-3} + \frac{4}{x+1} = 1 \Rightarrow x(x+1) + 4(2x-3) = (2x-3)(x+1)$$

| × | $2x$ | $-3$ |
|:---:|:---:|:---:|
| $x$ | $2x^2$ | $-3x$ |
| $+1$ | $+2x$ | $-3$ |

$$\Rightarrow x^2 + x + 8x - 12 = 2x^2 - x - 3$$
$$\Rightarrow x^2 - 10x + 9 = 0$$

$$\left.\begin{array}{l} \text{add to:} \quad -10 \\ \text{multiply to:} \quad +9 \end{array}\right\} -1, -9$$

$$\Rightarrow (x-1)(x-9) = 0$$
$$\Rightarrow \underline{\underline{x = 1 \text{ or } x = 9}}.$$

21. $ABC$ is a triangle. (3)
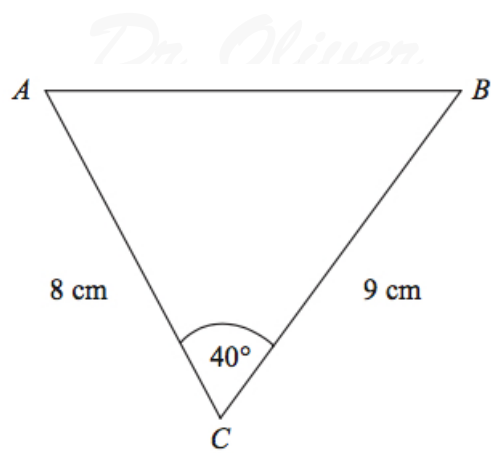
$AC = 8$ cm.
$BC = 9$ cm.
Angle $ACB = 40°$.
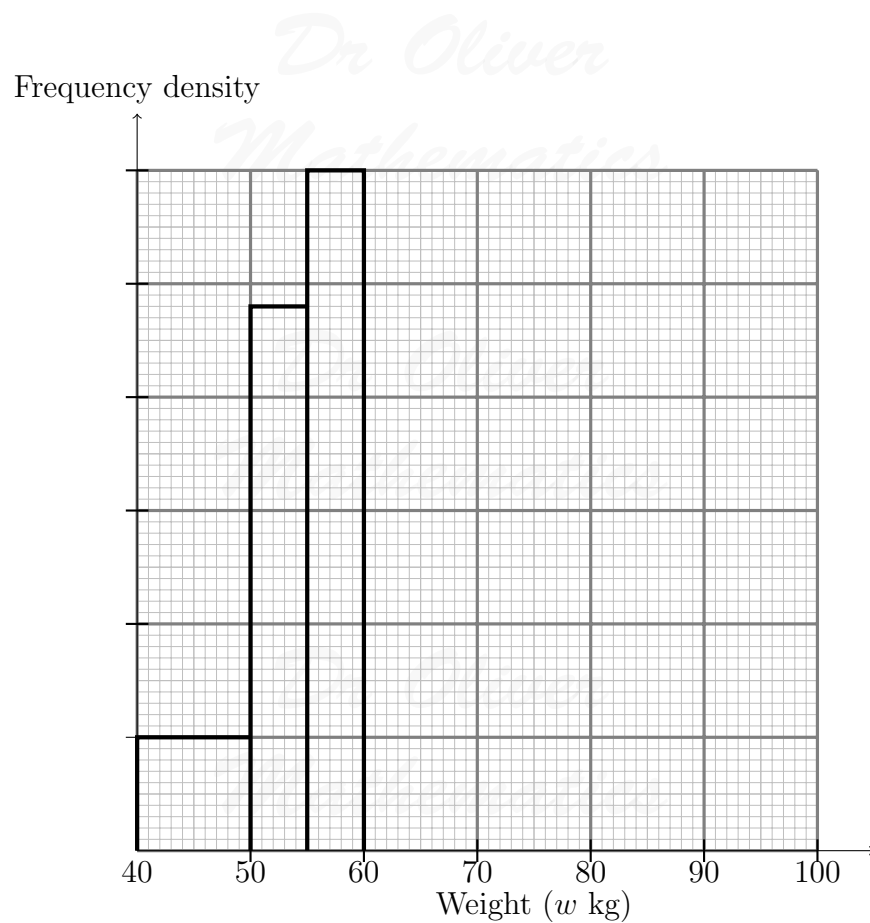Calculate the length of $AB$.
Give your answer correct to 3 significant figures.

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**Solution**

$$c = \sqrt{8^2 + 9^2 - 2 \times 8 \times 9 \times \cos 40°}$$
$$= 5.889\,787\,788 \text{ (FCD)}$$
$$= \underline{\underline{5.89 \text{ cm (3 sf)}}}.$$

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22. The incomplete table and histogram give some information about the weights of people at a keep-fit session.

Frequency density



Weight ($w$ kg)

(a) Use the information in the histogram to complete the frequency table. (2)

| Weight ($w$ kg) | Frequency |
|---|---|
| $40 \leqslant w < 50$ | 10 |
| $50 \leqslant w < 55$ | |
| $55 \leqslant w < 60$ | |
| $60 \leqslant w < 75$ | 15 |
| $75 \leqslant w < 95$ | 8 |

**Solution**

| Weight ($w$ kg) | Frequency | Width | Frequency density |
|---|---|---|---|
| $40 \leqslant w < 50$ | 10 | 10 | $\frac{10}{10} = 1$ |
| $50 \leqslant w < 55$ | <u>24</u> | 5 | $\frac{24}{5} = 4.8$ |
| $55 \leqslant w < 60$ | <u>30</u> | 5 | $\frac{30}{5} = 6$ |
| $60 \leqslant w < 75$ | 15 | 15 | $\frac{15}{15} = 1$ |
| $75 \leqslant w < 95$ | 8 | 20 | $\frac{8}{20} = 0.4$ |

(b) Complete the histogram. (2)

**Solution**
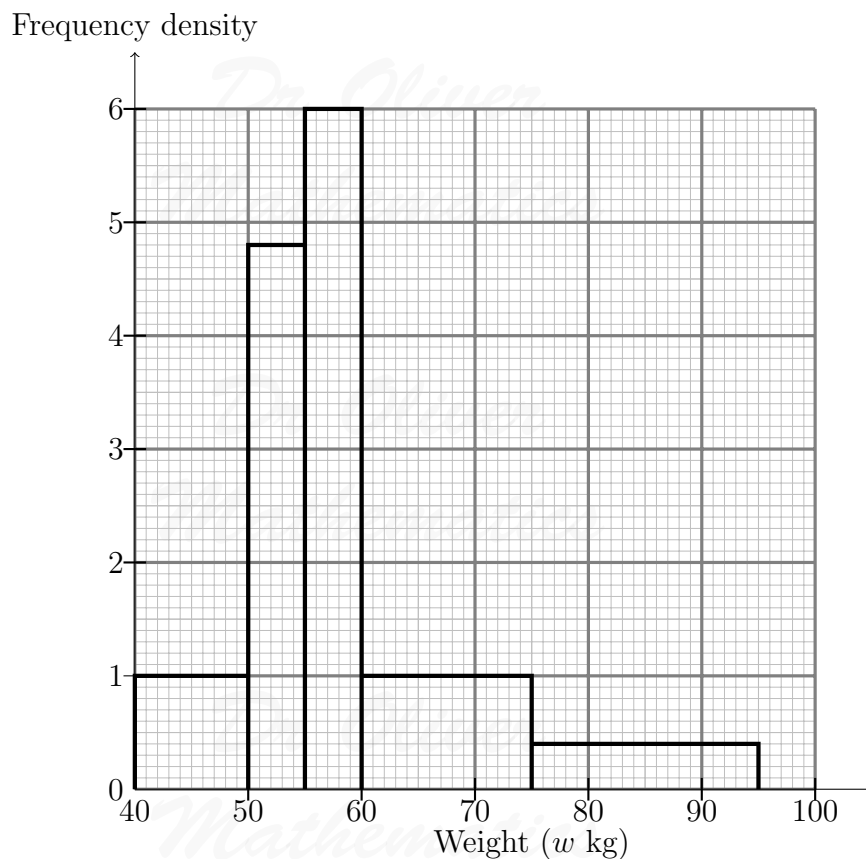


23. Kelly runs a distance of 100 metres in a time of 10.52 seconds.
   The distance of 100 metres was measured to the nearest metre.
   The time of 10.52 seconds was measured to the nearest hundredth of a second.

   (a) Write down the upper bound for the distance of 100 metres. (1)

> **Solution**
>
> $$99.5 \leqslant \text{distance} < 100.5$$
>
> and
>
> $$\text{distance} = \underline{\underline{100.5 \text{ m}}}.$$

(b) Write down the lower bound for the time of 10.52 seconds.                    (1)

> **Solution**
>
> $$10.515 \leqslant \text{time} < 10.525$$
>
> and
>
> $$\text{time} = \underline{\underline{10.515 \text{ s}}}.$$

(c) Calculate the upper bound for Kelly's average speed.                          (2)
   Write down all the figures on your calculator display.

> **Solution**
>
> $$\text{Upper bound} = \frac{100.5}{10.515} = \underline{\underline{9.557\,774\,608 \text{ m/s (FCD)}}}.$$

(d) Calculate the loweer bound for Kelly's average speed.                         (2)
   Write down all the figures on your calculator display.

> **Solution**
>
> $$\text{Lower bound} = \frac{99.5}{10.525} = \underline{\underline{9.453\,681\,71 \text{ m/s (FCD)}}}.$$

24. Martin has a pencil case which contains 4 blue pens and 3 green pens.          (3)
    Martin picks a pen at random from the pencil case.
    He notes its colour, and then replaces it.
    He does this two more times.
    Work out the probability that when Martin takes three pens, exactly two are the same
    colour.

> **Solution**

$$\mathrm{P(exactly\ two)} = 1 - \mathrm{P}(BBB) - \mathrm{P}(GGG)$$
$$= 1 - \left(\tfrac{4}{7} \times \tfrac{4}{7} \times \tfrac{4}{7}\right) - \left(\tfrac{3}{7} \times \tfrac{3}{7} \times \tfrac{3}{7}\right)$$
$$= 1 - \tfrac{64}{343} - \tfrac{27}{343}$$
$$= \underline{\underline{\tfrac{36}{49}}}.$$

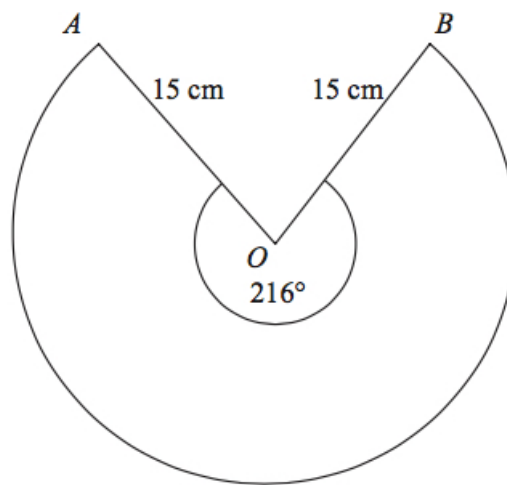25. A cone has a base radius of 5 cm and a vertical height of 8 cm.



8 cm

5 cm
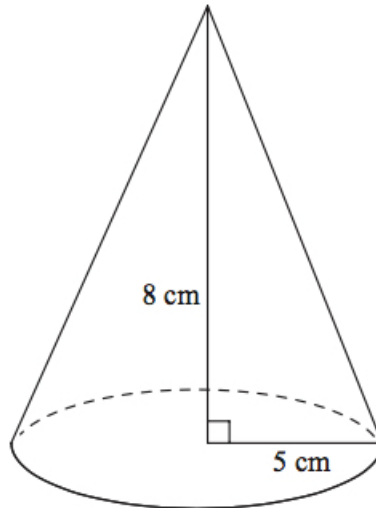
(a) Calculate the volume of the cone. (2)
   Give your answer correct to 3 significant figures.

**Solution**

$$\mathrm{Volume} = \tfrac{1}{3} \times \pi \times 5^2 \times 8$$
$$= 209.439\,510\,2 \ \mathrm{(FCD)}$$
$$= \underline{\underline{209 \ \mathrm{cm}^3 \ \mathrm{(3\ sf)}}}.$$

Here is the net of a different cone.



The net is a sector of a circle, centre $O$, and radius 15 cm.
Reflex angle $AOB = 216°$.
The net makes a cone of slant height 15 cm.

(b) Work out the vertical height of the cone. (4)

**Solution**

The base radius is
$$\frac{216}{360} \times 15 = 9$$
and we call upon our friend Pythagoras:
$$\text{vertical height} = \sqrt{15^2 - 9^2}$$
$$= \underline{\underline{12 \text{ cm}}}.$$